Noise-shield with membrane-type metamaterials for low-frequency sound insulation. Part I: Analytical investigation of the multi-layered assembly

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Introduction

The classical double wall approach attenuates sound very efficiently at higher frequencies, while the low frequency performance of this design can only be enhanced by introducing more mass or increasing the wall spacing, which is not applicable in some cases, e.g. aeronautical engineering. Recently, the so-called membrane-type acoustic metamaterials (MAM) emerged with promising low-frequency transmission loss properties, significantly exceeding the corresponding mass law transmission behaviour inside a certain frequency band due to resonance effects [4]. These compact structures, consisting of a thin membrane with small masses attached, could be used to improve the double wall in the low-frequency range, without adding too much mass to the structure.

In order to design such an improved noise-shield, analytical models can be a helpful tool. However, the analytical models for MAMs available in the literature are either computationally quite fast, but do not take into account the rigidity of the added mass [5], or are very accurate, but require cumbersome root-searching and numerical integration algorithms [1]. Therefore, a new analytical model for the transmission loss of MAMs, combining the advantages of both mentioned models, is derived and verified using a finite element model that is further described in part II of this specific noise-shield topic [2].

Analytical Model

For the analytical calculation of the noise-shield transmission loss $\mathrm{TL} = -20 \log |t|$, the transfer matrix method is employed [3]. In general, the transfer matrix $\mathbf{T}$ gives a relationship of the pressure and velocity amplitudes on each side of an acoustical element (such as partitions, air gaps, or pourous layers). For an air gap with length d and a thin wall with surface mass density m'', the transfer matrices are given by

$$\mathbf{T}^{\mathrm{f}} = \begin{bmatrix} \cos(kd) & \mathrm{i}\mathcal{Z}_0 \sin(kd) \\ \frac{\mathrm{i}}{\mathcal{Z}_0} \sin(kd) & \cos(kd) \end{bmatrix} \text{ and } \mathbf{T}^{\mathrm{w}} = \begin{bmatrix} 1 & \mathrm{i}\omega m'' \\ 0 & 1 \end{bmatrix},$$

respectively, where k is the fluid wave number, $\mathrm{i} = \sqrt{-1}$, and $\mathcal{Z}_0$ is the characteristic impedance of the fluid. In the case of a multi-layered structure with N different layers (here: walls, air gaps, and membranes) connected in series, the transfer matrix $\mathbf{T}$ of the whole structure is given by the product of all layer transfer matrices: $\mathbf{T} = \prod_i^N \mathbf{T}_i$. Then, the transmission factor t of the multi-layered panel can be obtained from the transfer matrix elements T_{11}, T_{12}, T_{21}, and T_{22} through the relationship $t = 2/(T_{11} + T_{12}/\mathcal{Z}_0 + \mathcal{Z}_0 T_{21} + T_{22})$ [3].

To obtain the transfer matrix $\mathbf{T}^{\mathrm{m}}$ of a membrane for low frequencies, the membrane can be regarded as a wall with an effective surface mass density of $\tilde{m}'' = \hat{P}/(-\omega^2 \langle \hat{w} \rangle)$, where $\hat{P}$ is the uniform acoustic pressure amplitude exciting the membrane and $\langle \hat{w} \rangle$ is the resulting surface averaged membrane displacement amplitude [4]. This effective surface mass density can be used just like the surface mass density of a rigid wall in the definition of $\mathbf{T}^{\mathrm{w}}$ to obtain the low-frequency transfer matrix of a membrane.

The displacement amplitude $\hat{w}$ of a membrane is obtained as follows: Consider the square membrane shown in Fig. 1 (left) with the edge length L, surface mass density m''_{m}, uniform tension force per unit length T, and a rigid square mass M placed in the membrane center. The vibration of this membrane, excited by a uniform time-harmonic pressure with amplitude $\hat{P}$, is governed by the non-dimensional Helmholtz equation

$$-k^2 u - \frac{\partial^2 u}{\partial \xi^2} - \frac{\partial^2 u}{\partial \eta^2} = \beta + \gamma, \tag{1}$$

where $\xi = x/L$ and $\eta = y/L$ are the dimensionless coordinates, $u = \hat{w}/L$ is the dimensionless vibration amplitude, $k^2 = (m''_{\mathrm{m}}/T)\omega^2 L^2$ the dimensionless frequency, and $\beta = \hat{P}L/T$ the dimensionless pressure amplitude. γ is an uknown function that represents the interaction force amplitude distribution between the rigid mass and the membrane. This unknown distribution can be approximated by employing a point-matching approach, where a set of I discrete point forces γ_i, distributed at the interface between the mass and the membrane, replaces γ. These point forces lead to a dynamic motion of the rigid mass, which can be described by Newton's second law and the Euler equations according to $u_{\mathrm{S}} = \sum_{i=1}^{I} \gamma_i/(\mu k^2)$, $\alpha_\xi = \sum_{i=1}^{I} \eta'_i \gamma_i/(\mu k^2 \vartheta)$, and $\alpha_\eta = -\sum_{i=1}^{I} \xi'_i \gamma_i/(\mu k^2 \vartheta)$, where u_{S} is the dimensionless displacement amplitude of

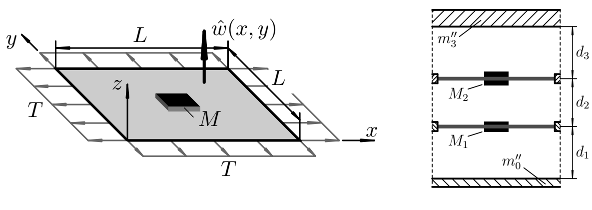


Figure 1: Left: Geometrical definitions of a square membrane centrally loaded with a rigid square mass. Right: Basic structure of a noise-shield design involving membrane layers.

the mass center of gravity, α_ξ and α_η are the mass rotations about the corresponding axes, $\mu = M/(m''_\mathrm{m} L^2)$ is the mass ratio, and $\vartheta = j_\mathrm{M}^2/L^2$ is the dimensionless form of the mass radius of gyration j_M. The coupling between the membrane and the mass is established by enforcing the continuity of displacements at each point force location:

$$u(\xi_i, \eta_i) \stackrel{!}{=} u_\mathrm{M}(\xi'_i, \eta'_i) = u_\mathrm{S} - \alpha_\eta \xi'_i + \alpha_\xi \eta'_i. \qquad (2)$$

The Helmholtz equation (1) is solved by approximating u with a finite series of membrane eigenfunctions $\Phi(\xi, \eta)$, which are given by a product of sine functions:

$$u \approx \sum_{n=1}^{N} q_n \Phi_n = \sum_{n_x=1}^{N_x} \sum_{n_y=1}^{N_y} q_n \sin(n_x \pi \xi) \sin(n_y \pi \eta), \qquad (3)$$

where $N = N_x N_y$ and $n = N_y(n_x - 1) + n_y$. Inserting expression (3) into eq. (1), multiplying by an arbitrary eigenfunction Φ_m and integrating over the membrane surface as well as replacing u in eq. (2) by (3) yields the following coupled system of equations:

$$\begin{bmatrix} \mathbf{C} - k^2\mathbf{M} & -\mathbf{L} \\ -\mathbf{L}^T & \frac{1}{\mu k^2}\mathbf{G} \end{bmatrix} \begin{pmatrix} \mathbf{q} \\ \gamma \end{pmatrix} = \beta \begin{pmatrix} \mathbf{b} \\ \mathbf{0} \end{pmatrix}. \qquad (4)$$

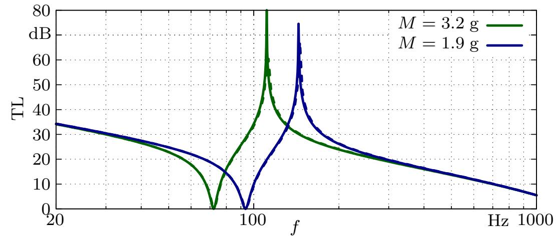
In (4), $\mathbf{C}$ and $\mathbf{M}$ are both diagonal with $c_{nn} = \pi^2/4(n_x^2 + n_y^2)$ and $m_{nn} = 1/4$, respectively, $\mathbf{L}$ is rectangular with $l_{ni} = \Phi_n(\xi_i, \eta_i)$, $\mathbf{G}$ is square with $g_{ij} = 1 + (\xi'_i \xi'_j + \eta'_i \eta'_j)/\vartheta$, and $\mathbf{b}$ is a vector with $b_n = \pi^2 n_x n_y/4$, for odd n_x and n_y, and $b_n = 0$, else. Thus, eq. (4) can be solved for the membrane mode participation factors $\mathbf{q}$ to obtain the membrane displacement u from eq. (3). Performing the surface averaging of u and introducing the normalized vector $\mathbf{q}^* = \mathbf{q}/\beta$, the membrane effective surface mass density is finally obtained from $\tilde{m}'' = -m''_\mathrm{m}/(k^2 \mathbf{b}^T \mathbf{q}^*)$.

Verification

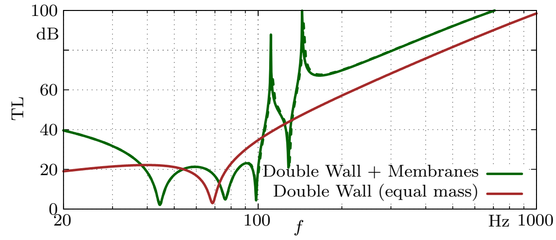
The analytical model presented in this contribution is verified by comparing the predicted transmission loss for two individual membrane configurations and the multi-layered noise-shield design to numerical results from the finite element model described in part II [2].

For the membranes, the following properties were chosen: $m''_\mathrm{m} = 31.75$ g/m^2, $T = 160$ N/m, and $L = 50$ mm. Two different added masses are analyzed with $M_1 = 1.9$ g and $M_2 = 3.2$ g, both with an edge length of $l = 10$ mm. To simulate conditions of an aircraft flying at a cruise height of approximately 35'000 ft, the fluid characteristic impedance was set to $\mathcal{Z}_0 = 136.5$ kg/m^{2}s. Comparing the analytically predicted results with the numerical results in Fig. 2a shows that the analytical model exactly reproduces the numerically obtained transmission loss.

For the verification of the multi-layered structure, the noise-shield design given in Fig. 1 (right) is considered with the same membrane and fluid properties, as well as $m''_0 = 5.4$ kg/m^2, $m''_3 = 13.5$ kg/m^2, $d_1 = d_3 = 20$ mm, and $d_2 = 10$ mm. Again, the analytical and numerical results in Fig. 2b show excellent agreement. Furthermore, the results are compared to a classical double wall



(a) Individual membranes.



(b) Multi-layered panel with membranes.

Figure 2: Transmission loss results for the individual membranes and the multi-layered panel from the analytical model (solid curves) and the numerical model (dashed curves).

with equal mass, which illustrates the influence of the two additional membrane layers. These additional layers lead to a significant improvement over the double wall for the frequency range above 100 Hz. However, one has to accept that the transmission loss is negatively affected below this frequency and between the two peaks due to additional resonances that develop between the layers.

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