Analysis of the acoustic damping of an annular tail pipe.

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Introduction

Several applications require high acoustic damping in combination with low flow resistance. These applications concern for example gas flow machines such as ventilators, turbo-engines, intake and exhaust systems for internal combustion engines. In several cases, absorption is realised by placing resistive materials in the flow, such as fiber materials, foams, perforated plates and metal weavings. They are quite efficient to suppress noise, however they can generate a considerable pressure drop.

In this paper, a damping device has been developed, consisting of a central tube with neglectable flow resistance surrounded by a narrow slit to generate the acoustic damping. The central tube behaves as an acoustic mass which impedance is proportional to frequency. When the frequency has been increased sufficiently, a considerable part of the acoustic flow passes through the slit where it will be damped. In this way, acoustic energy can be dissipated while the flow experiances a low flow resistance. A measurement setup has been developed [1]. The impedance of the damping device will be measured using the two microphone transfer function method according to ISO 10534-2 on an improved measurement wave guide. An electrical equivalent model has been used to validate the measurement results and to extract the resistance of the slit from the measurements. The relation of the slit resistance in terms of frequency and acoustic excitation amplitude has been investigated. A preliminairy non-linear analysis has been performed, which will be further investigated in future research.

Configuration of the damping device

The construction of the damping device is presented in figure 1. A photograph of the device is presented in figure 2. At the left side situates the wave guide through which the waves are incoming. The wave guide consist of a duct with radius $R = 20$ mm. The damping device consists of a central tube with length $L = 45$ mm and radius $r = 15$ mm. Between the central tube and the wave guide wall, the narrow slit is situated. The slit has a length $l = 1.5$ mm and is $t = 0.15$ mm wide. Both the central tube and the slit are connected to the atmosphere at the right side.

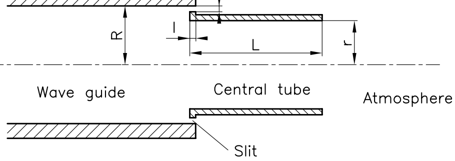

Figure 1: Scheme of the damping device.


Figure 2: Photo of the damping device.

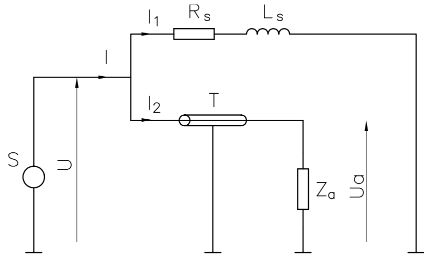

Figure 3: Electrical equivalent circuit of the damping device.

The analysis of the damping device is carried out using an equivalent electrical circuit, which is presented in figure 3. The voltage U represents the pressure p at the left side of the central tube in figure 1. The current I represents the acoustic flow Φ which divides in a current I_1 through the slit and I_2 through the central tube. The slit is represented by its resistance R_s and its acoustic mass L_s. The central tube is represented by the wave guide T with characteristic impedance $Z_c = \frac{\rho\, c}{\pi\, r^2}$ and is closed by the spherical radiator Z_a, representing the atmosphere. In order to determine the acoustic dissipation in the slit, the impedance of each branch will be determined. The impedance of the central tube Z_2 and the impedance Z_1 of the upper branch containing the slit will be:

$$Z_2 = \frac{\cos k\, L + j\, \dfrac{Z_c}{Z_a} \sin k\, L}{\dfrac{j}{Z_c} \sin k\, L + \dfrac{1}{Z_a} \cos k\, L} \quad \text{and} \quad Z_1 = R_s + j\, \omega\, L_s \tag{1}$$

wherein k is the wave number, $j = \sqrt{-1}$, L the length of the central tube, Z_c the characteristic impedance of the central tube and $Z_a = \frac{\rho\, c}{\pi\, r^2} \frac{j\, k\, r}{1 + j\, k\, r}$ the spherical radiator impedance representing the atmosphere. ρ is the air density, c the speed of sound, r the central tube radius, $\omega = 2\, \pi\, f = \frac{k}{c}$ the angle frequency, R_s the slit resistance and $L_s = \frac{\rho\, l}{2\, \pi\, R\, t}$ the acoustical mass of the slit, with $2\, \pi\, R$ the slit circumference and t the slit width. The total impedance of the device will be

$$Z = \frac{Z_1\, Z_2}{Z_1 + Z_2} \tag{2}$$

which will be measured using the two microphone transfer function method according to ISO 10534-2 from which the slit resistance R_s will be determined.

Measurements and analysis

The measurements have been carried out using an improved impedance duct [1]. From the microphone signals, the impedance and the particle velocity are determined. Figure 4 (right) shows the measured particle velocities

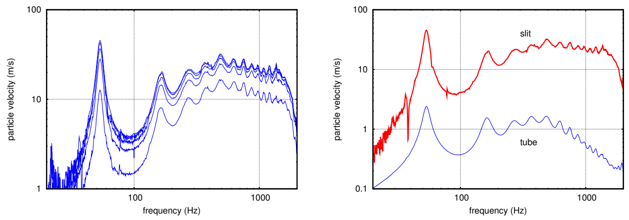

Figure 4: (*left*) Particle velocity through the slit for different loudspeaker excitations. (*right*) Particle velocity through the slit (red line) and before the slit (blue line).

through the slit at different loudspeaker excitations (10, 20, 30, 60 and 100V). From these measurements, the dependence of the slit resistance to the particle velocity will be investigated. The slit resistance R_s can be considered as the sum of a linear part (constant resistance) and a non-linear part, which appear to be proportional to the particle velocity u through the slit. The linear part R_{sa} (see expression (3)) is determined by the viscosity of the air η. The non-linear resistance is caused by the loss of

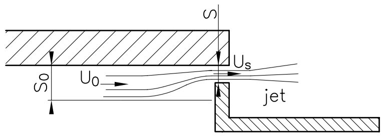

Figure 5: Contraction of the flow through the slit.

kinetic energy of the fluid. Figure 5 shows the situation. When the wave arrives at the slit, the flow has to contract from the surface S_0 to S to pass through the slit. This is represented by the contraction coefficient $\sigma = \frac{S}{S_0} = \frac{U_0}{U_s}$. Figure 4 (right) shows the increase of the particle velocity where U_0 is the velocity before the slit in thin blue line and U_s is the velocity in the slit in thick red line. This increase in particle velocity results in an increase of the flow kinetic energy W. Once the flow is passed through the slit, it forms a jet behind it wherein the kinetic energy is dissipated. The same process happens when the flow direction reverses.

$$R_{sa} = \frac{12\,\eta\,l}{2\,\pi\,R\,t^3} \qquad W = \int_0^{T/2} \rho\,S\,u_s\,\frac{u_s^2 - u_0^2}{2}\,dt \qquad (3)$$

The kinetic energy W presented in expression (3) is valid when the flow through the slit is uniform. However, analysis of the Reynolds number points out that the flow in the slit will be laminar and consequently the slit velocity u will be parabolically distributed as $u(y) = \frac{3}{2}(1 - \frac{y^2}{(h/2)^2})u_s$. Introducing this parabolic velocity distribution in expression (3) results in:

$$W = \int_0^{T/2} \rho\,L \int_{-\frac{h}{2}}^{\frac{h}{2}} \frac{u(y)^3 - u_0^2\,u(y)}{2}\,dy\,dt \qquad (4)$$

wherein L is the slit circumference. The evaluation of the inner integral of the velocity distribution results in:

$$W = \int_0^{T/2} \rho\,L\,\frac{h}{2}\,u_s\,(\alpha\,u_s^2 - u_0^2)\,dt \quad \text{with} \quad \alpha = \frac{54}{16} \qquad (5)$$

The loudspeaker excitation causes a sinusoidal particle velocity u_0 at the reference section and consequently a sinusoidal particle velocity u_s in the slit:

$$u_0 = u\,\sin\omega t \qquad u_s = \frac{u\,\sin\omega t}{\sigma} \qquad (6)$$

Introducing these velocities u_0 ans u_s into expression (5) results ultimately in the slit resistance R_s:

$$R_s = \frac{12\,\eta\,l}{2\,\pi\,R\,t^3} + \frac{4}{3\,\pi}\,\frac{\rho}{S}\,\frac{\alpha - \sigma^2}{\sigma}\,u \qquad (7)$$

Figure 6 presents the slit resistance measurements (thick red lines) in terms of particle velocity for different frequencies. The slit resistances in thin blue line are calculated

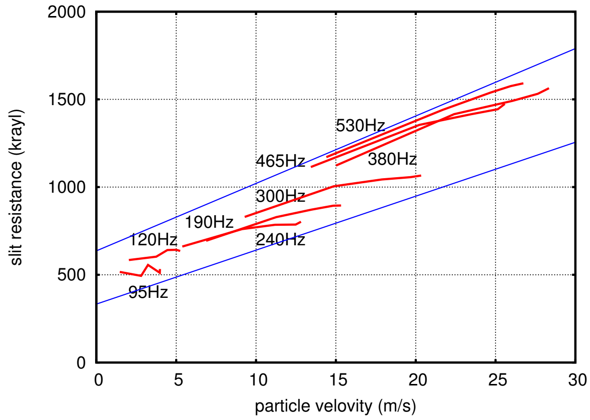

Figure 6: Measurements of the slit acoustical resistance over the plot of expression (7).

from expression (7). The two lines are calculated with a slit width variation of 20 μm. The measured slit resistances are situated between the two calculated lines. From expression (7), the non-linear part of the slit resistance appears to be proportional to the particle velocity and independent of frequency. The analysis provides a good estimation of the measured values.

Conclusion

A damping device with neglectable flow resistance, consisting of a central tube surrounded by a narrow slit, has been investigated. The impedance of the acoustic mass of the central tube increases with frequency. At higher frequencies, a considerable part of the acoustic flow passes through the slit, where it will be damped. The damping mechanism is two fold: a linear part of the slit resistance in which the viscosity of the air is involved and a non-linear part wherein the slit resistance is proportional to the particle velocity of the air through the slit. In this part, the loss of kinetic energy of the pulsating flow due the abrupt cross-section jumps before and after the slit causes additional flow induced damping. Further research will be carried out to investigate the formation of the jet behind the slit.

References

[1] Boonen R., Sas P., Vandenbulck E., " Determination of the acoustic damping of an annular tail-pipe.", proc. of the ISMA2010 Conference including USD2010, (2010), Leuven, Belgium, pp. 47-58, (available at *www.isma-isaac.be/publications*)