FEM/ FMBEM coupling for structural-acoustic design sensitivity analysis

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Introduction

Passive noise control by modification of structure geometry moves more and more into the field of vision for designers. This structural-acoustic optimization shows high potential in minimization of radiated noise especially for thin shell geometries. Acoustic design sensitivity analysis can provide information on how the geometry change affects the acoustic performance of the given structure, so it is an important step of the acoustic design and optimization processes [1]. But the sensitivity analysis of the structural-acoustic interaction based on FEM/Conventional BEM algorithm represents the bottleneck in computation efforts. However fast multipole boundary element method (FMBEM) can be applied effectively to accelerate the matrix-vector multiply [2, 3]. In this paper, the coupling algorithm FEM/FMBEM is applied to the structural-acoustic sensitivity analysis based on direct differentiation method, where the design variable can be chosen as fluid and structural density, Poisson's ratio, Young's modulus, structural geometry parameter, etc. A numerical example of a point force excited sphere submerged in water is presented to demonstrate the validity and correctness of the present algorithm.

Evaluation of radiated sound power

For noise radiation into open domains, the emitted sound power P can be expressed as

$$P(\omega) = \frac{1}{2}\int_A \Re\left\{p(\vec{\boldsymbol{y}},\omega)v_f^*(\vec{\boldsymbol{y}},\omega)\right\} dA(\vec{\boldsymbol{y}}) \qquad (1)$$

where p is the sound pressure, v_f is the particle velocity, $()^*$ represents the conjugate complex quantity, and $\Re\{\}$ denotes the real part. Discretization of Eq. (1) leads to a matrix expression for the sound power and is given by

$$P(\omega) = \frac{1}{2}\Re\left\{\boldsymbol{p}^T\boldsymbol{S}\boldsymbol{v}_f^*\right\} \qquad (2)$$

where $\boldsymbol{S}$ denotes the boundary mass matrix and is given by

$$\boldsymbol{S} = \int \boldsymbol{N}_f^T\boldsymbol{N}_f d\Gamma\ . \qquad (3)$$

$\boldsymbol{N}_f$ is a row vector containing the BEM interpolation functions on Γ. The particle velocity of the fluid $\boldsymbol{v_f}$ can be expressed as a function of the structural displacement $\boldsymbol{u}$, as follows

$$\boldsymbol{v_f} = \boldsymbol{S}^{-1}\boldsymbol{C}_{fs}\boldsymbol{u} \qquad (4)$$

where the coupling matrix $\boldsymbol{C}_{fs}$ transforms the structural degrees of freedom to the degrees of freedom of the fluid and is written as

$$\boldsymbol{C}_{fs} = -i\omega\int \boldsymbol{N}_f^T\boldsymbol{n}\boldsymbol{N}_s d\Gamma \qquad (5)$$

$\boldsymbol{N}_s$ are the FEM basis functions and $\boldsymbol{n}$ represents the surface normal vector. By substituting Eq. (4) into Eq. (2), the radiated sound power is written as follows

$$P(\omega) = \frac{1}{2}\Re\left\{\boldsymbol{p}^T\boldsymbol{C}_{fs}\boldsymbol{u}^*\right\} \qquad (6)$$

Differentiating Eq. (6) with respect to the design variable ϑ, the sound power sensitivity is written as

$$\frac{\partial P}{\partial\vartheta} = \frac{1}{2}\Re\left\{\left(\frac{\partial\boldsymbol{p}}{\partial\vartheta}\right)^T\boldsymbol{w}_1 + \boldsymbol{p}^T\frac{\partial\boldsymbol{C}_{fs}}{\partial\vartheta}\boldsymbol{u}^* + \boldsymbol{w}_2^T\left(\frac{\partial\boldsymbol{u}}{\partial\vartheta}\right)^*\right\} \qquad (7)$$

where $\boldsymbol{w}_1 = \boldsymbol{C}_{fs}\boldsymbol{u}^*$ and $\boldsymbol{w}_2^T = \boldsymbol{p}^T\boldsymbol{C}_{fs}$. In what follows, the expression

$$\Re\left\{\boldsymbol{w}_2^T\left(\frac{\partial\boldsymbol{u}}{\partial\vartheta}\right)^*\right\} = \Re\left\{\boldsymbol{w}_2^H\frac{\partial\boldsymbol{u}}{\partial\vartheta}\right\} \qquad (8)$$

will be used. Superscript $()^H$ denotes the conjugate complex transposed. Applying this to Eq. (7), the sound power sensitivity is yielded as

$$\frac{\partial P}{\partial\vartheta} = \frac{1}{2}\Re\left\{\boldsymbol{w}_1^T\frac{\partial\boldsymbol{p}}{\partial\vartheta} + \boldsymbol{p}^T\frac{\partial\boldsymbol{C}_{fs}}{\partial\vartheta}\boldsymbol{u}^* + \boldsymbol{w}_2^H\frac{\partial\boldsymbol{u}}{\partial\vartheta}\right\}\ . \qquad (9)$$

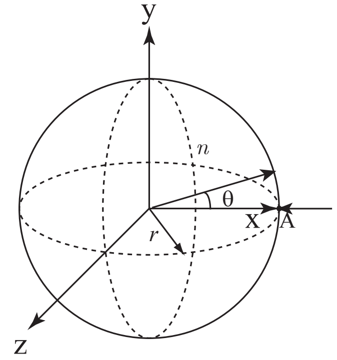

Abbildung 1: Spherical shell excited by a single force at point A

Elastic spherical shell excited by unit force

The spherical shell is excited by a single concentrated force F at a point A, see configuration in Fig. 1 [4]. The angle θ represents the central angle between the calculated point and the point of excitation, i.e. point A.

The finite element model of the structure is created in ANSYS. The structural domain is meshed using four-node shell elements (SHELL63). An approximate element length of $0.35m$ leads to a mesh of 5960 triangular shell elements. The boundary element surface mesh coincides with the finite element mesh. Constant boundary elements are used for the analysis here. The radiated sound power is evaluated based on the displacement and the sound pressure on the wet surface of the structure.

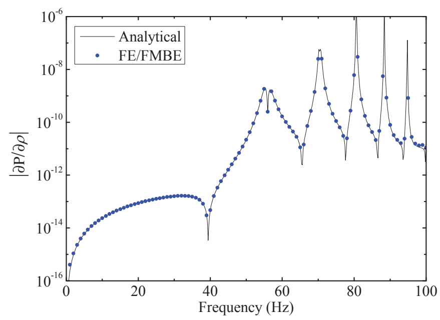

Abbildung 2: Sound power sensitivity (magnitude) with respect to the fluid density

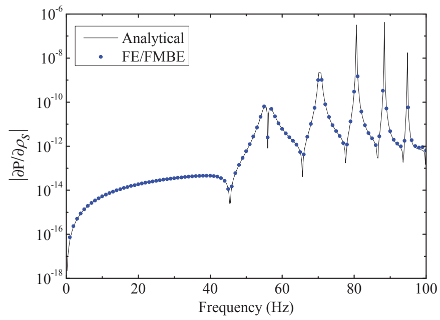

Abbildung 3: Sound power sensitivity (magnitude) with respect to the structural density ρ_s

Figure 2 shows the sound power sensitivity with respect to the fluid density. While the sound power remains rather insensitive in the low frequency range, the sensitivity goes up in the vicinity of resonance peaks. The numerical solution agrees well with the analytical solution.

Figures 3 present the sensitivity of the radiated sound power with respect to the structural density. Very similar to the fluid density, the sound power sensitivity with respect to the structural density are quite small in the low frequency range and exhibit peaks in the vicinity of resonances. Again, numerical and analytical solutions agree very well with each other.

Figure 4 shows the sound power sensitivity with respect

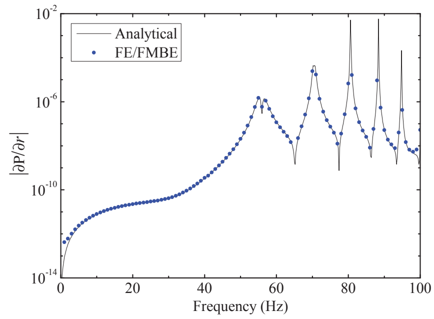

Abbildung 4: Sound power sensitivity (magnitude) with respect to the radius r of the sphere

to the radius of the sphere. Qualitatively, the solution looks very similar to the other sensitivities. Again, analytical and numerical solution match very well.

Conclusions

This paper has presented a formulation for the sound power sensitivity computation of fully coupled structural-acoustic systems, where the design variable can be chosen arbitrarily, e.g. material data of fluid and structure, shape modifying parameters and other features such as shell thickness. While direct differentiation allows an analytic sensitivity evaluation for one parameter, i.e. the fluid density, a semi-analytic approach is used for the structural parameters. A numerical example of a point force excited sphere submerged in water is presented to demonstrate the validity and correctness of the present algorithm.

Literatur

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