Numerical simulations of laser cavitation bubbles by the Volume of Fluid method

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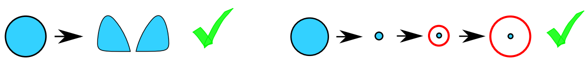
Introduction

For engineering of cavitation problems a main interest lies in the dynamics of bubble clouds. However, this cannot be accurately modelled without knowing the individual dynamics of one cavitation bubble. The motion of a bubble wall is highly nonlinear and partly very rapid (shock waves, erosion of even the hardest materials). If we consider spherical oscillations of the bubble, they are well described by ODE models (e.g. by GILMORE - see below), but deviations from the spherical shape are difficult to capture accurately. Typical non-spherical bubble situations are, for instance, a bubble near an adjacent wall or close to a second bubble, both causing liquid jet flows through the bubble.

Here we present the results of our modelling of laser generated bubbles with the Volume of Fluid Method (VoF) using the software `OpenFOAM` [7] as an alternative to the widely used but restrictive Boundary Integral Method (BIM) (see for instance [1]). Advantages of VoF vs. BIM are:
a) change of topology is rather unproblematic
b) phenomena of compressibility (e.g. shock waves) can be modelled.



The Volume of Fluid method

The VoF method introduces in each computational cell a phase fraction $\alpha \in [0,1]$ to distinguish between gas and liquid phase, as sketched in figure (1). The overall density ρ, for instance, is then expressed in terms of α and the densities of the gas and the liquid: $\rho = (1-\alpha)\,\rho_g + \alpha\,\rho_l$.

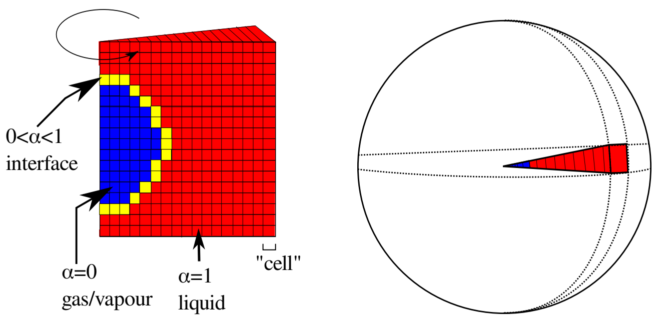


Figure 1: Left: Sketch of space discretization and interface handling. Right: Reducing the problem to 1D spherical symmetry.

Governing equations and numerical methods

The compressible flow of two immiscible fluids, liquid and gas, is described here by the NAVIER-STOKES equation (1), the overall continuity equation (2, left) and the continuity equation (2, right) for each of the fluids, by which an equation for the phase fraction α is obtained:

$$\frac{\partial \rho \vec{U}}{\partial t} + \nabla_{\bullet}(\rho \vec{U} \otimes \vec{U}) = - \nabla p + \nabla_{\bullet}\mathbb{T} + \rho \vec{g} + \int_S \sigma\, \kappa(\vec{x}')\, \vec{n}(\vec{x}')\, \delta(\vec{x}-\vec{x}') dS' \quad (1)$$

$$\frac{\partial \rho}{\partial t} + \nabla_{\bullet}(\rho \vec{U}) = 0, \qquad \frac{\partial \alpha \rho_l}{\partial t} + \nabla \cdot (\alpha \rho_l \vec{U}) = 0, \quad (2)$$

with $p, \rho, \vec{U}$ denoting the pressure, density and velocity field respectively. $\nabla_{\bullet}$ denotes the divergence, ∇ the gradient operator, $\mathbb{T}$ is the viscous stress tensor, $\vec{g}$ the gravitational acceleration, σ the surface tension, taken to be constant, κ the curvature of the interface, $\vec{n}$ the unit normal vector to the interface and $\vec{x}'$ a point on the interface S. Equations (1) and (2) are supplemented with the TAIT-equation for the liquid (3, left) ([6],[3]) and an equation for an adiabatic VAN-DER-WAALS gas neglecting adhesional forces (3, right).

$$\rho_l = \rho_0 \left(\frac{p+B}{p_0+B}\right)^{\frac{1}{\gamma_T}}, \quad \rho_g = \frac{\rho_n}{(1-\beta)\,(p_n/p)^{1/\gamma} + \beta} \quad (3)$$
$$\gamma_T = 7.1, \; B \approx 3046 \cdot 10^5 \text{ Pa}, \quad \beta \approx 0.0015130423, \; \gamma = 1.4$$

Here p_0 and ρ_0 are the pressure and density of water in thermal equilibrium at room temperature, p_n, ρ_n denote the pressure and density of the bubble in thermal and pressure equilibrium and β is related to to the VAN-DER-WAALS co-volume by $V_{Co} = \beta V_n$, V_n = equilibrium volume. For the gas we use the polytropic exponent $\gamma = 1.4$ for air or any two-atomic molecule gas. $\vec{g}$ is set zero.

The standard solver *compressibleInterFoam* (vers. 2.1.1) of `OpenFOAM`, which employs the VoF method to solve equations (1) and (2) , has been adapted to the problem by implementing the equations of state (3) and the following additional changes:

- a regulation for the total mass inside the bubble to hold it constant,
- a minimum pressure $p_{\min}$ for eq. (3, right) that must be nearly zero.

Results and validation with the Gilmore model

In order to fine-tune the numerical setting we concentrate on the collapse of a single, spherical bubble. For validation of our VoF results we use the widely known Gilmore ODE-model for a spherical bubble in a compressible liquid ([4], [5, p.3]) with bubble radius R, the enthalpy in the liquid H and the speed of sound C at the bubble wall:

$$\left(1-\frac{\dot{R}}{C}\right)R\ddot{R}+\frac{3}{2}\left(1-\frac{1}{3}\frac{\dot{R}}{C}\right)\dot{R}^2=\left(1+\frac{\dot{R}}{C}\right)H+\left(1-\frac{\dot{R}}{C}\right)\frac{R}{C}\frac{\mathrm{d}H}{\mathrm{d}t}$$

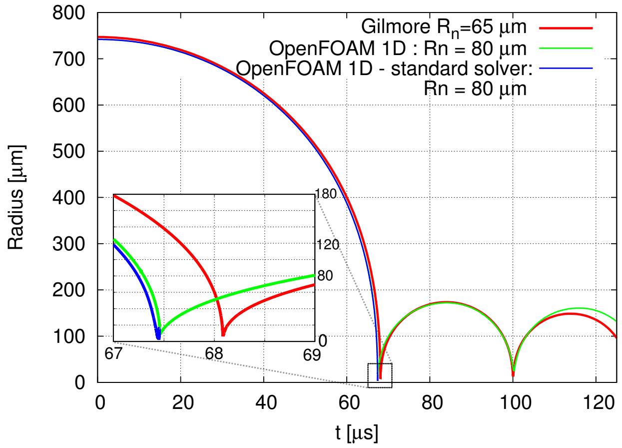


Figure 2: Comparison of 1D test cases with the modified `OpenFOAM` (green) and the standard `OpenFOAM` solver (blue line, both $R_n = 80\,\mu\text{m}$) and the Gilmore model (red, $R_n = 65\,\mu\text{m}$)

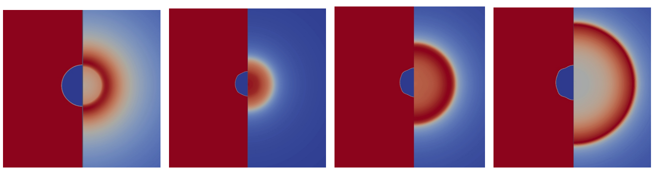


Figure 3: Four images of a time series of a collapsing bubble, a few nanoseconds around the collapse moment. Start radius $R_{\text{max}} = 747\,\mu\text{m}$ and equilibrium radius $R_n = 160\,\mu\text{m}$. Left side is the phase parameter α (blue = gas) and right side is the pressure (red = high, rescaled for each time step, max. $\approx 10^8$ Pa). Time develops from left to right.

The simulations were started at the maximum bubble radius. In `OpenFOAM` the velocity field $\vec{U}$ was initially set to zero and initial data for the pressure were obtained from the analytical formula given in [8, p. 27]. The bubble spans a wide range of radii during evolution, which have to be resolved properly. Also shock waves are emitted that have to be captured adequately, and pressure reflections at the outer boundary have to be minimized. As a mesh that could cope with these demands, we used an improved version of that proposed in [2]. It has a radially symmetric cell ordering for the axisymmetric mesh in order to provide very high resolution combined with an adequate overall size.

Figure (2) shows the radius of the bubble vs. time for a strong collapse, comparing the 1D simulations with our adapted solver to a solution of the Gilmore equation as well as an evolution with the original *compressibleInterFoam* solver. The original solver, assuming isothermal conditions in the bubble and linear equations of state, crashes around R_{min}. The deviation of the adapted solver and the Gilmore solution in the equilibrium radii R_n is subject to current investigations.

For another example from our `OpenFOAM` model, figure (3) presents the calculated phase boundaries and pressure data resolving the shock wave on the axisymmetric mesh.

Conclusion and outlook

By adapting the compressible two-phase solver *compressibleInterFoam* of `OpenFOAM` vers. 2.1.1 and fine-tuning the numerical settings, simulations of a strongly collapsing, single cavitation bubble with high accuracy were performed and were compared to the Gilmore model, following the bubble evolution over the first collapse and rebound.

Our findings will serve as a solid basis for the computationally even more demanding cases of bubble collapse near a rigid boundary or multi-bubble collapses as well as for further developments regarding the inclusion of condensation and evaporation. Concernig the remaining deviation of the equilibrium radii R_n between the models, we suppose that this is due to the neglected physics inside the bubble in the Gilmore model. Further work is necessary to clarify this.

Acknowledgement: The financial support by the Austrian Federal Ministry of Economy, Family and Youth and the National Foundation for Research, Technology and Development is gratefully acknowledged.

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