A Nitsche Non-Conforming Finite Element Approach towards Computational Acoustics in Rotating Systems

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Introduction

One main source of sound in current products and mashinery originates from rotating parts like vents and propellers causing aeroacoustic and vibroacoustic sound radiation into the surrounding quiescent medium. The applicability of numerical schemes in this context highly depends on their ability to take those rotating bodies into account when computing the wave propagation.

Within this contribution, we extend the Nitsche approach [1] for non-matching finite element grids towards rotating interfaces, discuss occuring instabilities and present two approaches to enable stable and accurate computations of acoustic wave propagation.

Formulation

The Nitsche method is a known approach to connect two disjoint triangulations sharing a common interface for computation by the finite element method. Additionally, in the context of rotating bodies the method should allow one domain to move with respect to the other as depicted in Fig. 1. In this example, the interior domain Ω_1 rotates

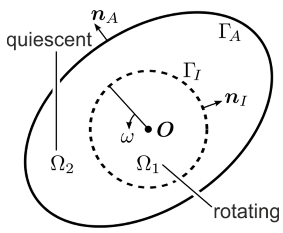

Figure 1: Schematic of multi domain setup.

with a given angular velocity whereas the domain Ω_2 remains steady. Both domains share the common interface Γ_I. In a practical application, the interior region would include the rotating structure like a vent. For a non-moving (quiescent) interior domain, the variational form of the wave equation in two computational domains, including the interface terms given by the Nitsche approach reads as

$$\int_{\Omega_i} \varphi_i \frac{1}{c_0^2} \frac{\partial^2 p_i}{\partial t^2} \, \mathrm{d}\Omega + \int_{\Omega_i} \nabla \varphi_i \cdot \nabla p_i \, \mathrm{d}\Omega + \int_{\Gamma_I} [\varphi] \frac{\partial p_1}{\partial \vec{n}} \, \mathrm{d}\Gamma + \int_{\Gamma_I} \frac{\partial \varphi_1}{\partial \vec{n}} [p] \, \mathrm{d}\Gamma + \underbrace{\beta \sum_{E_I} \frac{1}{h_{E_I}} \int_{\Gamma_{E_I}} [\varphi][p] \, \mathrm{d}\Gamma}_{=P} = 0 \,. \tag{1}$$

Within this symmetry preserving non-matching grid formulation the term

$$[f] = f_1 - f_2 \,,$$

represents a jump of the function between the two adjacent regions.

A straight forward application of this approach to a rotating setup yields instable results as depicted in Fig. 2 showing the results of a monopole sound radiation located on the inner, rotating domain. We notice oscillations of acoustic pressure on the interface thus indicating a overall instable scheme. In the following we investigate two approaches to stabilize the results and to obtain valid results:

1. Motivated by the results in [2] we add an additional penalization term for the time derivative of acoustic pressure as
$$P' = \beta_D \sum_{E_I} \frac{1}{h_{E_I}} \int_{\Gamma_{E_I}} [\varphi] \left[\frac{\partial p}{\partial t} \right] \mathrm{d}\Gamma \,. \tag{2}$$
2. Introduce numerical damping by utilizing a time stepping scheme which attenuates high frequency oscillations like the HHT/α method [3].

Especially the second variant is beneficial as it does not require a change in the variational formulation itself.

Both approaches yield stable results as indicated in Fig. 2 and a more detailed numerical investigation is presented in the following chapter.

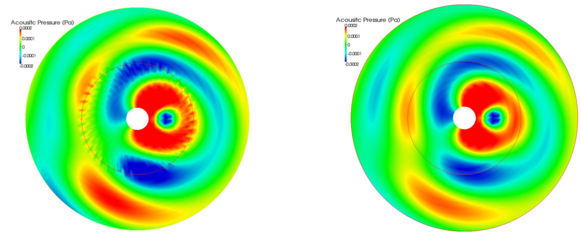

Figure 2: Contours of acoustic pressure and oscillations at the interface (left) and stabilized result (right).

Three dimensional benchmark

For numerical benchmarking we choose a computational setup as shown in Fig. 3. Depicted are three domains. One cylindrical in the center of the domain which will be subjected to rotation, a quiescent exterior domain for wave propagation and on the outside a perfectly matched layer (PML) to avoid reflections of acoustic waves on the domain boundary. Initially, at $t = 0$ an acoustic pres-

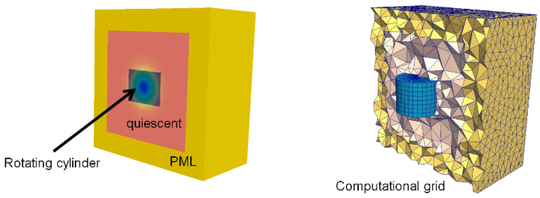

Figure 3: Computational setup for the three dimensional benchmark.

sure distribution is prescribed in the interior, rotating domain as well as inside the exterior, quiescent domain. For the transient pulse propagation, an analytical solution can be given [4] which also enables the investigation of the numerical error. Within the simulations, the interior domain rotates with Mach 0.2 on its lateral area. Although the analytical solution is only given for a quiescent medium, the symmetry of the setup allows to compare the solutions for the case in which the inner cylinder is rotating.

Evaluation of the potential acoustic energy inside the domain given by

$$W_p = \int_\Omega \frac{1}{2} \frac{p^2}{\rho_0 c_0^2} \, \mathrm{d}\Omega \; ,$$

yields the time evolution depicted in Fig. 4. Clearly visi-

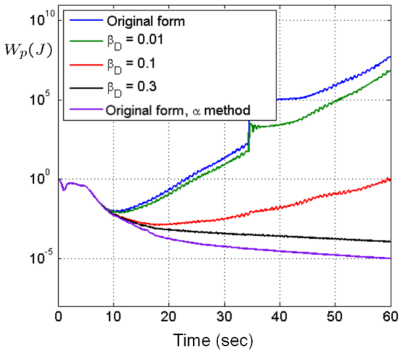

Figure 4: Time evolution of acoustic energy in the propagation domain.

ble is the unphysical increase of energy in the original formulation and for an inappropriate choice of β_D (see (2)). For $\beta_D \geq 0.3$ and for the α time stepping method, we obtain the expected decay in the energy as the pulse is leaving the computational domain and is absorbed by the perfectly matched layer (PML). At the end of the simulation run, the interior domain was rotated 5 times around its axis.

Finally we evaluate the deviation of numerical results to the analytical solution p_a by utilizing the formula

$$E_h = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (p_i - p_\mathrm{a})^2} \; ,$$

for the case of moving and quiescent interior domain with N being the number of finite element nodes in the inner regions. We utilize here the α method as it gives stable results with second order accuracy in the time domain. The results for different values of β are given in Fig. 5 indicating only a insignificant difference for all values of β investigated.

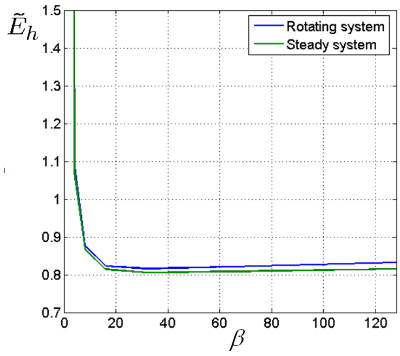

Figure 5: Deviation from analytical solution for a variation of β parameters.

Conclusion

We have presented the application of Nitsches method for non-matching grids to problems of rotating geometries in acoustic simulations. Both investigated stabilization approaches give valid results, retain the symmetry in the resulting algebraic system of equations and do not significantly increase the computational effort. Applications to more complex geometries and to problems in the field of computational aeroacoustics are currently investigated.

References

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