Comparison of aeroacoustic source term formulations

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Introduction

Since years, noise levels due to the rapid growth of air and ground traffic densities (e.g., airplanes, trains, cars, etc.) have become an issue for urban communities. For low Mach number applications, a separate treatment of fluid and acoustic computations can be performed, and therefore the question of appropriate source term formulations, based on flow computations, arises. An important aspect of the applicability of any formulation is the choice of the numerical method used for the computation of the generated acoustic field, which will be in our case the Finite-Element (FE) method [1, 2]. Since the FE method discretizes the whole computational domain, the used source term formulation results in a right hand side and has to be evaluated on the complete computational domain of the flow. As a consequence, certain practical challenges occur, starting from the interpolation procedure between flow and acoustic grid, over the accuracy of the right hand side computation, up to the actual computation of the acoustic wave propagation. The selected aeroacoustic source term formulations for an acoustic perturbation equation (PE) as well as the inhomogeneous wave equation of Lighthill are investigated by means of theoretical considerations and numerical examples. A special focus will be on the resulting acoustic field, especially in the vicinity or even inside the flow region.

Computational Formulations

Let's start with Lighthill's analogy, which results in the following inhomogeneous wave equation

$$\frac{1}{c_0^2}\frac{\partial^2 p'}{\partial t^2} - \nabla\cdot\nabla p' = \nabla\cdot\nabla\cdot\boldsymbol{T}\,. \tag{1}$$

In (1) p' is an overall, fluctuating pressure (outside the flow region it approaches the acoustic pressure p^{a}), c_0 the speed of sound and $\boldsymbol{T}$ Lighthill's tensor, which computes for low Mach number flows at constant temperature by

$$\boldsymbol{T} = \rho_0 \boldsymbol{u}\otimes\boldsymbol{u} \tag{2}$$

with $\boldsymbol{u}$ the flow velocity and ρ_0 the mean density. Since we assume incompressible flows, the divergence of the momentum equation results in the following equality

$$\nabla\cdot\nabla\cdot\boldsymbol{T} = -\nabla\cdot\nabla p^{\mathrm{ic}} \tag{3}$$

with the incompressible flow pressure p^{ic}. So, instead of the second derivative of Lighthill's tensor, we can also use the Laplacian of the incompressible flow pressure as a source term for Lighthill's inhomogeneous wave equation.

The second approach is based on acoustic perturbation equations [3]. Here, we decompose the overall velocity and pressure in mean, pure flow and pure acoustic parts

$$\boldsymbol{u} = \overline{\boldsymbol{u}} + \boldsymbol{u}^{\mathrm{ic}} + \boldsymbol{u}^{\mathrm{a}}\,; \quad p = \overline{p} + p^{\mathrm{ic}} + p^{\mathrm{a}}\,. \tag{4}$$

This ansatz is substituted into the conservation equations, resulting in perturbation equations (PE) of the following form [4]

$$\begin{aligned} \frac{1}{\rho_0 c_0^2}\left(\frac{\partial p^{\mathrm{a}}}{\partial t} + \nabla\cdot(p^{\mathrm{a}}\overline{\boldsymbol{u}})\right) + \nabla\cdot\boldsymbol{u}^{\mathrm{a}} &= \frac{1}{\rho_0 c_0^2}\frac{Dp^{\mathrm{ic}}}{Dt} \\ \frac{\partial \boldsymbol{u}^{\boldsymbol{a}}}{\partial t} + (\overline{\boldsymbol{u}}\cdot\nabla)\boldsymbol{u}^{\boldsymbol{a}} + (\boldsymbol{u}^{\boldsymbol{a}}\cdot\nabla)\overline{\boldsymbol{u}} + \frac{1}{\rho_0}\nabla p^{\mathrm{a}} &= \boldsymbol{0}\,. \end{aligned} \tag{5}$$

Therefore, for PE the source term is the substantial derivative of the incompressible flow pressure $Dp^{\mathrm{ic}}/Dt = \partial p^{\mathrm{ic}}/\partial t + \overline{\boldsymbol{u}}\cdot\nabla p^{\mathrm{ic}}$.

In a third step, we neglect the mean flow effects in (5) on acoustics and obtain a new aeroacoustic wave equation for the acoustic pressure p^{a}

$$\frac{1}{c_0^2}\frac{\partial^2 p^{\mathrm{a}}}{\partial t^2} - \nabla\cdot\nabla p' = \frac{1}{c_0^2}\frac{\partial^2 p^{\mathrm{ic}}}{\partial t^2}\,. \tag{6}$$

Here, the second time derivative of the fluctuating incompressible flow pressure p^{ic} defines the acoustic source term, and we will call this equation the Aeroacoustic Wave Equation (AWE).

Numerical Results

As a demonstrative example, we choose a cylinder in a cross flow, as displayed in Fig. 1. Thereby, the compu-

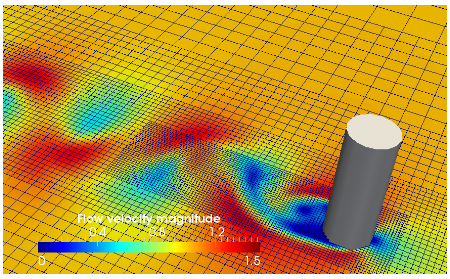

Figure 1: Computational setup for flow computation.

tational grid is just up to the height of the cylinder and together with the boundary conditions (bottom and top as well as span-wise direction symmetry boundary condition), we obtain a pseudo two-dimensional flow field. The diameter of the cylinder D is 1 m resulting with the inflow velocity of 1 m/s and chosen viscosity in a Reynolds number of 250 and Mach number of 0.2. From the flow simulations, we obtain a shedding frequency of 0.2 Hz

(Strouhal number of 0.2). The acoustic mesh is chosen different from the flow mesh, and resolves the wavelength of two times the shedding frequency with 10 finite elements of second order. At the outer boundary of the acoustic domain we add a perfectly matched layer to efficiently absorb the outgoing waves.

Figure 2 displays the acoustic field for the different formulations. One can clearly see that the acoustic field

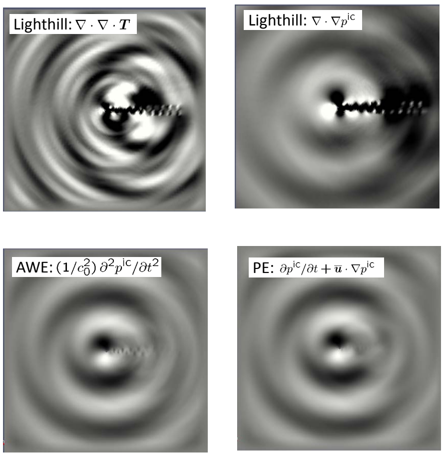

Figure 2: Computed acoustic field with the different formulations

of PE (for comparison with the other formulations we have neglected the convective terms) meets very well the expected dipole structure and is free from dynamic flow disturbances. Furthermore, the acoustic field of AWE is quite similar and exhibits almost no dymanic flow disturbances. Both computations with Lighthill's analogy show flow disturbances, whereby the formulation with the Laplacian of the incompressible flow pressure as source term shows qualitative better result as the classical formulation based on the incompressible flow velocities. To obtain a more detailed and quantitative analysis, we have saved the acoustic pressure along a circle of 20 times the diameter of the cylinder (outside the main flow region) and performed a Fourier transformation. Figure 3 displays the obtained results in a plot over the angle of the circle. We can observe very similar results for all formulations, although PE and AWE exhibits most clearly the expected dipole structure. Performing a further analysis at a circle with just two times the diameter of the cylinder results in the plots displayed in Fig. 4. The obtained results confirms the previous findings according to Fig. 2 and concludes that a perturbation ansatz is necessary to obtain reliable acoustic pressure results in the flow region. Lighthill's acoustic analogy (both incompressible flow velocity or pressure based source terms) models an overall fluctuating pressure in the flow region and contains frequency components, which are not present outside the flow region, where all formulations result in very similar acoustic pressure values.

Acknowledgment

The authors are grateful to Johannes Kreuzinger from Kreuzinger und Manhart Turbulenz GmbH for providing

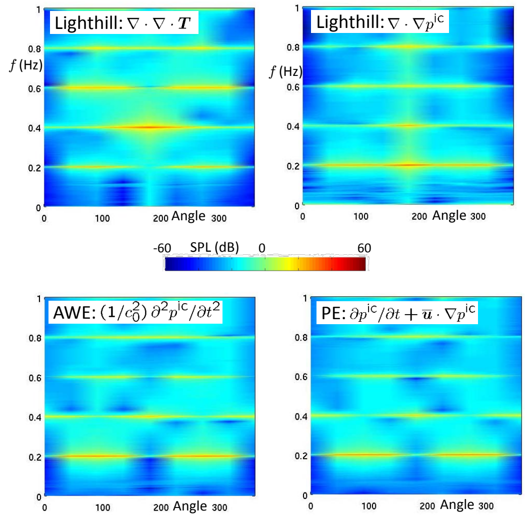

Figure 3: Fourier transformed acoustic pressure along a circle with a diameter of 20 times the diameter of the cylinder.

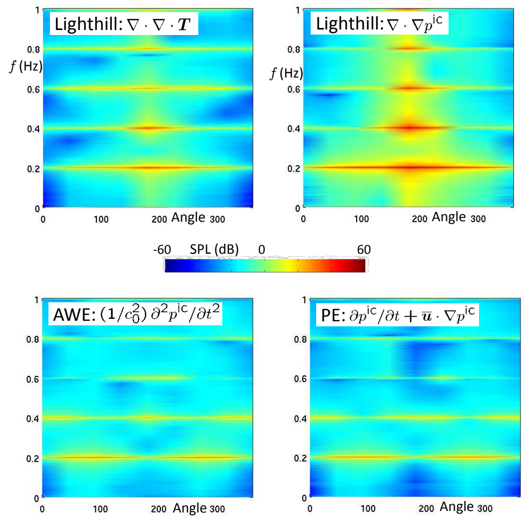

Figure 4: Fourier transformed acoustic pressure along a circle with a diameter of 2 times the diameter of the cylinder.

the flow simulations.

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