Frequency and geometry dependent automated optimized meshing algorithm for a boundary element simulation

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Introduction

For the development of a boundary element method (BEM) based room acoustics simulation algorithm called WRWes, which can handle non-locally reacting boundary conditions in a single computational domain [1], three mesh element types are investigated. The WRWes algorithm differs in so far from the classical BEM that it is very flexible with regard to the choice of the mesh and its elements. Furthermore, the use of an adaptive mesh is compared with a standard fixed mesh. This paper gives an overview of the accuracy of the different mesh element types and their computation times.

WRWes algorithm

The used WRWes algorithm can be classified as a boundary element method. Instead of the standard BEM methods that are based on a Kirchoff-Helmholtz integral kernel the WRWes uses a Rayleigh-II integral kernel [2]. The W-matrices describe the transfer paths between the sources and the boundaries ($\mathbf{W}_{bs}$), the sources and receivers ($\mathbf{W}_{ds}$), the boundaries and receivers ($\mathbf{W}_{db}$) and from the boundaries to boundaries ($\mathbf{W}$). The R-matrix ($\mathbf{R}$) contains the boundary properties (complex reflection coefficients and non-locally reacting properties). When a number of sources within an enclosed space is defined in the vector ($\vec{S}$), then the complex pressure at any point within that space can be calculated by equation (1) [1]:

$$\vec{P} = \left[\mathbf{W}_{ds} + \mathbf{W}_{db}\left(\mathbf{I} - \mathbf{RW}\right)^{-1}\mathbf{RW}_{bs}\right]\vec{S} \qquad (1)$$

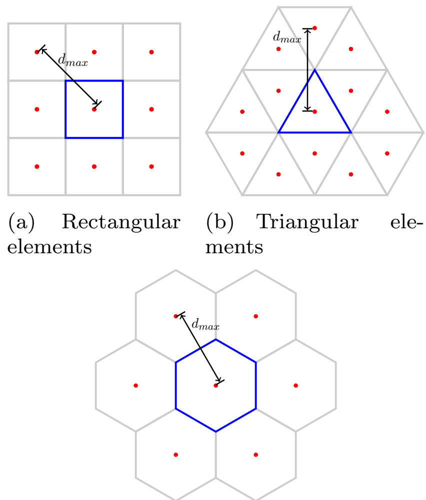
The W-matrices $\mathbf{W}_{db}$ and $\mathbf{W}$ contain the Rayleigh-II kernels and their elements contain a dependency of the mesh. The definition of the W-matrix elements is given in equation (2).

$$W(\Delta r, \omega) = \frac{jk}{2\pi}\left(\frac{1 + jk\Delta r}{\Delta r^2}\right)e^{-jk\Delta r}\cos(\phi)dA \qquad (2)$$

In equation (2) the surface area of the mesh element is indicated by dA and the other part of the equation can be interpreted as a weighting factor. The product of this weighting factor and the mesh element area is interpreted as a point contribution of the mesh element geometric center point. This formulation does not set any requirements on the shape of the mesh element. The only restriction given from the Rayleigh-II theory is that the spatial Nyquist requirement $d_{max} = \frac{c}{2f}$ should be fulfilled. This restricts the distance between the neighboring element center points to a maximum distance d_{max}.

Mesh generation

The simulations are conducted with three types of mesh elements: rectangles, triangles and hexagons as shown in figure 1. The first two are common, but the hexagon



(a) Rectangular elements (b) Triangular elements

(c) Hexagonal elements

Figure 1: Three different mesh types

shape is not used in conventional BEM methods, because of the truncation problems at the edges. The rectangle elements are created by taking four center points at the maximum distance d_{max} under 45 degree angles, the rectangle is then built up from the lines connecting the midpoints of those four points (fig. 1(a)). The triangle elements are constructed by a Delaunay-triangulation algorithm [3], where the skewness is kept as low as possible (fig. 1(b)). The hexagonal elements are created in the same way as the rectangle elements, but instead of four a number of six neighboring center points is used under angles of 60 degrees apart (fig. 1(c)). When elements do not completely fit within the geometry, their intersections with the boundaries are calculated and the elements are truncated accordingly. The geometric center points of the truncated elements are then recalculated. The maximum distance between the neighboring center points is determined by the frequency under consideration. To reduce the number of truncated elements and thereby the number of nodes (center points) in the simulation, the geometry is analyzed by searching for the parallel and perpendicular edges. The mesh is aligned along the parallel and perpendicular edges that combined have the largest overall length.

Simulation set-up

To compare the different meshes a simple box shaped room of size 3.03m x 4.71m x 1.77m with a point source at position (0.7,1.5,0.5), a point receiver at position (1.9,2.1,1.0) and acoustically hard ($R = 1$) boundaries is used for all the simulations. The room is simulated with the WRWes algorithm for the three different meshes as mentioned above in a fixed configuration, where the mesh is defined for the highest simulated frequency (100 Hz) and in an adaptive configuration, where the mesh is adapted for each frequency. For the fixed and adaptive mesh a criterion of 10 nodes per wavelength is chosen. Simulated is a frequency range from 1 to 100 Hz in 1 Hz steps. The calculations are performed on a desktop PC with an 64-bit Intel Core i7 3.4 GHz CPU with 4 cores and 16 GB RAM memory running Windows 7. The BEM simulations are performed with the commercial software package LMS Virtual.Lab v12-SL1 and the WRWes simulation is implemented in Matlab 2013a.

Results

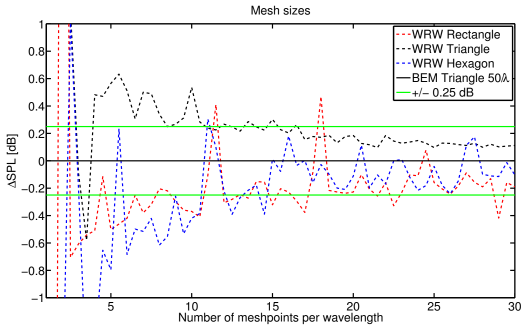


Figure 2: Convergence of the WRWes results for an increasing number of nodes per wavelength for three different mesh types and compared with a BEM reference.

Mesh geometry	Simulation method	Nr. nodes	Nr. elements	Calc. time fixed mesh [min:sec]	Calc. time adaptive mesh [min:sec]
Rectangular	BEM	1930	1928	1:47	-
	WRWes	4060	-	31:15	4:50
Triangular	BEM	2210	4416	5:00	-
	WRWes	11460	-	572:12	87:15
Hexagonal	WRWes	2480	-	8:29	1:32

Figure 3: Computation times

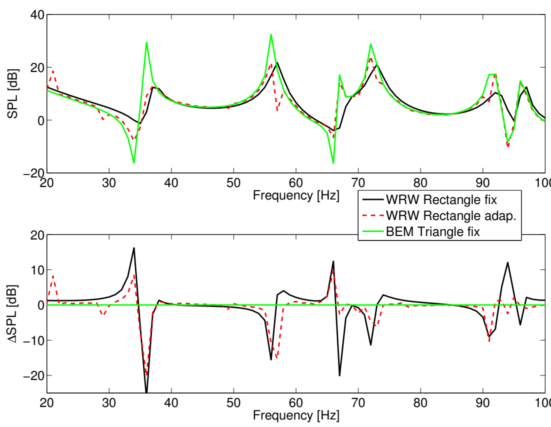


Figure 4: Frequency response with rectangular elements

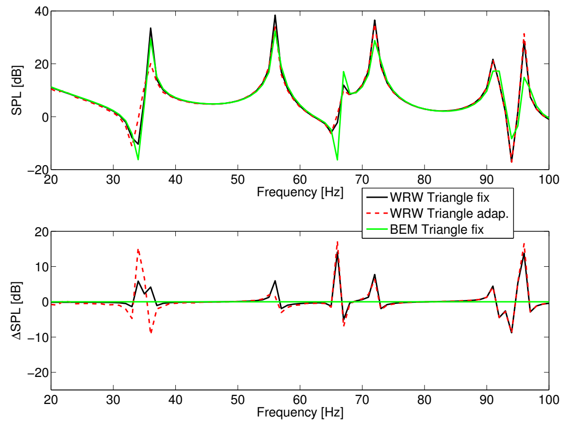


Figure 5: Frequency response with triangular elements

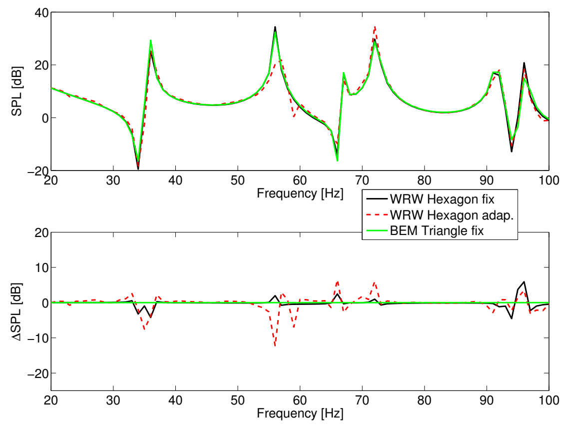


Figure 6: Frequency response with hexagonal elements

Discussion and conclusion

The rectangular mesh is not suitable for the WRWes algorithm, since it results in a very low accuracy and even in the shift of modal frequencies. The triangular mesh shows a good accuracy but is computationally very expensive. The lowest calculation times and highest accuracy can be achieved with the hexagonal mesh. Also the hexagonal adaptive mesh shows the same accuracy as the fixed mesh and is significantly faster.

References

[1] R. Opdam, D. de Vries and M. Vorländer, "Simulation of non-locally reacting boundaries with a single domain boundary element method." Proceedings of Meetings on Acoustics. Vol. 19. No. 1. p. 015100, Acoustical Society of America (2013).

[2] A.J. Berkhout, "Applied Seismic Wave Theory", Elsevier, Amsterdam (1987).

[3] B. Delaunay, "Sur la sphère vide", Bull. Acad. Science USSR VII:Class. Sci. Mat. Nat. 793?800 (1934).