Contact-free Vibration Measurements with Particle Velocity Probes - Part I: Theory

Thomas Kletschkowski[1], Daniel Sadra[2]

[1] *HAW Hamburg, 20099 Hamburg, E-Mail: thomas.kletschkowski@haw-hamburg.de*

[2] *HAW Hamburg, 20099 Hamburg, E-Mail: daniel.sadra@gmx.de*

Introduction

To identify mode shapes of vibrating structures, it is necessary to perform vibration measurements. In many cases structural sensors (such as accelerometers) are attached to the vibrating surface. As a consequence, the structural response of the system is modified.

To illustrate this effect, it is useful to analyze the first natural frequency of a simply supported beam structure (length: L, constant mass distribution: μ, bending stiffness: EI) with an additional discrete point mass m. The latter may be attached at $L/2$. Assuming a harmonic deflection shape such as

$$w(x,t) = \sin\left(\frac{\pi \cdot x}{L}\right) \cdot q(t), \qquad \text{[m]} \qquad (1)$$

compare [1], with $q(t)$ being a generalized coordinate, the first natural frequency is given by

$$f = \frac{\pi}{2L} \cdot \sqrt{\frac{EI}{L}} \cdot \sqrt{\frac{1}{(\mu L + 2m)}}. \qquad \text{[Hz]} \qquad (2)$$

Considering a length of L = 0.895m, a bending stiffness of EI = 1.621Nm², and a mass distribution of μ = 0.17kg/m, compare [2], the first natural frequency is reduced from 6.06Hz (original system) to 5.52Hz, if an accelerometer (having a mass of m = 0.015kg) is attached to the beam structure.

To avoid such an effect, especially during the test of lightweight structures, it can be advantageous to replace structural measurements with acoustic measurements. For this purpose, it is possible to use particle velocity probes (pu-probes), compare [3], in the very near field of the vibrating structure as discussed in the present paper.

Simplified vibro-acoustical model

In order to analyze the feasibility of contact-free vibration measurements using pu-probes, a simplified vibro-acoustical model, described in [4], has been used to perform numerical investigations. It describes one-dimensional propagation of plane waves though a sound tube. At the right hand side, the tube is terminated by an absorbing boundary with prescribed impedance. The acoustic field results from the harmonic motion of a piston at the left hand side. The latter is driven by an excitation force and connected to a spring-damper combination (parallel connection). All system parameters, taken from [4], are listed in Tab. 1.

Table 1: Physical properties of coupled system, see [4]

Spring-damper-piston system (structure)		
Area of piston cross section	A =	0.000625m
Spring stiffness	k =	7474.75N/m
Mass of piston	m =	0.01kg
Viscosity of damper	b =	0.50kg/s
Sound tube (cavity)		
Length of tube	l =	1.25m
Area of tube cross section	A =	0.000625m
Speed of sound in air	c =	344m/s
Density of air	ρ =	1.205kg/m³

It should be noticed that the first natural frequency of the air-filled tube matches the natural frequency of the undamped spring-piston system (137.7Hz), if a sound hard termination is realized at the right hand side. For this reason, the first two natural frequencies of the sound hard terminated coupled system (128.3Hz and 147.2Hz, see Fig. 1) are typical for a strong vibro-acoustic coupling effect, compare [4].

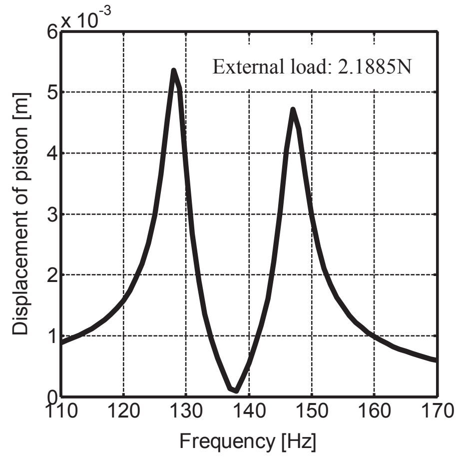

Figure 1: Forced vibration of coupled system, compare [5]

A closed form solution considering arbitrary boundary conditions at the impedance boundary and time-harmonic excitation can also be found in [4] (equation 132–141). Unfortunately it turned out that the above named reference contained typesetting errors. Therefore, we authors would like to recommend two corrections

- Correction 1: Insert ν into equation 135 – see the sine-functions in the denominator,
- Correction 2: Change the numerator in equation 136 (definition of σ) from $-Z_0Z_1$ to $-Z_0Z_2$.

Contact-free vibration measurement

To verify the feasibility of contact-free vibration measurements, the sound tube model has been analyzed. As shown in Fig. 2, the relative deviation between the velocity of the piston and the particle velocity becomes significant, especially at the anti-resonances of the piston (found at 138Hz, 276Hz, and 414Hz), if the distance between piston and pu-probe increases.

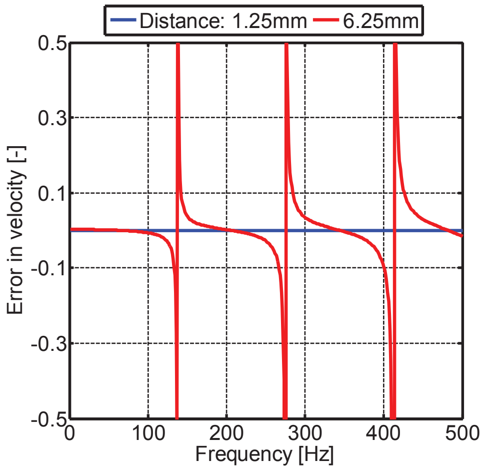

Figure 2: Relative error in velocity measurements, see [5]

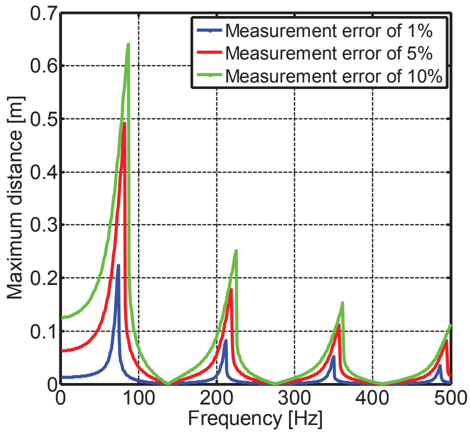

Figure 3: Absolute values of relative error, compare [5]

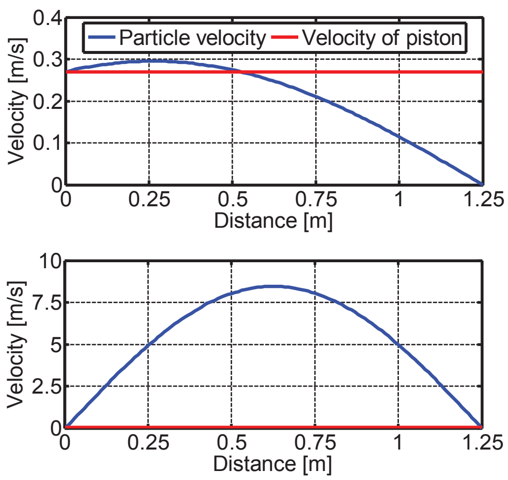

Figure 4: Mode shapes for 87Hz (top) and 138Hz (below)

Upper bounds for the distance between pu-probe and piston that guarantee a defined deviation between the structural velocity and the particle velocity are shown in Fig. 3. It can be seen that a distance up to 0.5m is acceptable, if (i) a relative error of 5% can be tolerated, (ii) and the spatial gradient is small, compare Fig. 4 (top). Otherwise, compare Fig. 4 (below), a small distance is required.

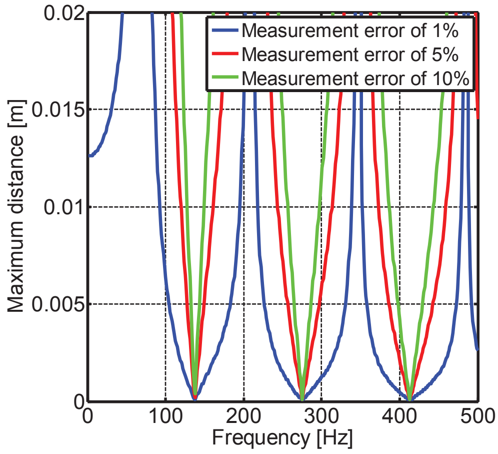

Figure 5: Absolute values of relative error, compare [5]

If the maximum distance between the pu-probe and the piston is set to 0.01m, see Fig. 5, and an absolute value for the relative error of 10% can be accepted, it is possible to perform contact-free vibration measurements in a sub-band of in total 322Hz below 400Hz, as shown in the last row of Tab. 2.

Table 2: Frequency bands of defined errors (1cm distance)

Absolute value of relative error: 1%		
0-92Hz	195-218Hz (Δ =23Hz)	338-352Hz (Δ =14Hz)
Absolute values of relative error: 5%		
0-120Hz	161-244Hz (Δ =83Hz)	313-373Hz (Δ =60Hz)
Absolute values of relative error: 10%		
0-127Hz	150-256Hz (Δ =106Hz)	297-386Hz (Δ =89Hz)

Literature

[1] Brommundt, E., Sachau, D.: Schwingungslehre mit Maschinendynamik, Teubner, 2008

[2] Kletschkowski, T.: Identification of structural parameters based on acoustic measurements. Proc. of AIA-DAGA 2013, 18-21 March 2013, Merano, Italy

[3] de Bree, H.-E.: The microflown – an particle velocity sensor. Acoustics Australia (2003), pp 91-94, Vol. 31 No. 3

[4] Chargin, M., Gartmeier, O.: A Finite element procedure for calculating fluid-structure interaction using MSC/NASTRAN. California: NASA, December 1990. NASA Technical Memorandum 102857

[5] Sadra, D.: Contact-free vibration measurements with particle velocity probes. Master-thesis, Hamburg University of Applied Sciences, 2013