Simulation of the Sound Generation of Lingual Organ Pipes

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Introduction

Lingual organ pipes produce sound by means of the interaction of a vibrating tongue and an acoustic resonator. The pitch of the pipe is determined by the frequency of tongue vibration, whereas the timbre is mostly influenced by the resonator. The two components form a coupled vibroacoustic – fluid dynamic system.

In this contribution a modeling approach for the simulation of the sound generation of lingual organ pipes is presented. The proposed techique incorporates a tongue and a resonator model, which are coupled by means of the flow equations.

Simulation arrangement

The one-dimensional simulation arrangement is depicted in Fig. 1. The pressure in the boot $p_{\mathrm{boot}}(t)$ is the input of the model, whereas the pressure under the tongue $p_{\mathrm{und}}(t)$ and in the shallot $p_{\mathrm{sha}}(t)$ are sought together with the volume velocity $U(t)$ and the displacement of the tongue. The other variables shown in Fig. 1 are derived from the above ones.

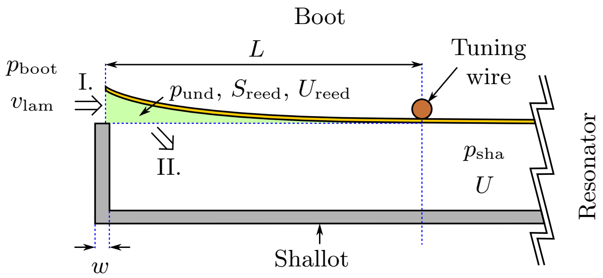

Figure 1: Simulation arrangement

Modal description of tongue vibration

The tongue (also called reed) of a lingual organ pipe can be modeled as a damped Euler – Bernoulli beam with one end clamped [1]. The equation for the displacement y along the tongue $x \in [0, L]$ reads as

$$\rho b h \frac{\partial^2 y(x,t)}{\partial t^2} + \gamma \frac{\partial y(x,t)}{\partial t} + EI \frac{\partial^4 y(x,t)}{\partial x^4} = f(x,t), \quad (1)$$

with b and h and I denoting the width, the thickess, and the second moment of area of the tongue, respectively. The material properties of the tongue are the density ρ, and the Young's modulus E. Damping is represented by the coefficient γ, and f denotes force over unit length.

Equation (1) is solved in a modal manner, using the modal shape functions $\Psi_n(x)$ as

$$y(x,t) = \sum_{n=1}^{\infty} q_n(t)\Psi_n(x), \qquad \int_0^L \Psi_n^2 \,\mathrm{d}x = 1. \quad (2)$$

Thus, a second order partial differential equation is obtained for each weighting factor $q_n(t)$

$$\ddot{q}_n = -2\beta_n\omega_n\dot{q}_n - \omega_n^2 q_n - \frac{1}{\rho b h} f_n, \qquad f_n = \int_0^L \Psi_n f \,\mathrm{d}x. \quad (3)$$

The mode shapes $\Psi_n(x)$, the corresponding eigenfrequencies ω_n, and damping coefficients β_n are known *a priori* from the geometry and material properties. The forces acting on the tongue are as follows:

- The intrinsic force f_0 is assumed to keep the tongue in its equilibrium shape and position.
- The pressure forces are denoted by f_{und} and f_{boot}.
- The Bernoulli force f_{bern} pulls the tongue towards the shallot, depending on the laminar velocity v_{lam}. To obtain the latter, laminar flow is assumed in the gap between the tongue and the shallot [1].
- When the tongue hits the shallot, i.e. $y < 0$, a restoring force $f_{\mathrm{rest}} = -yEw/h$ pushes it away.

Representation of the acoustic system

The acoustic system consisting of the shallot and the resonator is represented by a one-dimensonsal axisymmetric model [2]. The model gives the input impedance $Z_{\mathrm{in}}(\omega)$, from which the time domain reflection function $r(t)$ is obtained using the inverse Fourier transform

$$r(t) = \mathcal{F}^{-1}\left\{\frac{Z_{\mathrm{in}}(\omega) - Z_0}{Z_{\mathrm{in}}(\omega) + Z_0}\right\}. \quad (4)$$

Then the pressure inside the shallot p_{sha} is expressed (with $*$ denoting convolution) as [3]

$$p_{\mathrm{sha}}(t) = Z_0 U(t) + r(t) * \left[p_{\mathrm{sha}}(t) + Z_0 U(t)\right], \quad (5)$$

with Z_0 representing the acoustic plane wave impedance of the shallot and $U(t)$ denoting the sum of the acoustic (U_{ac}) and reed volume velocity (U_{reed}).

The flow equations

The simulation domain has two transition regions, numbered as I. and II. in Figure 1. Both are described in a simplified, one-dimensional manner, following [4].

In the contraction region (I.), the flow is described by conservation of energy

$$p_{\mathrm{boot}}(t) - p_{\mathrm{und}}(t) = \frac{\rho_0}{2}\frac{U^2(t)}{S^2_{\mathrm{reed}}(t)} + \frac{\rho_0 w}{S_{\mathrm{reed}}(t)}\frac{\partial U(t)}{\partial t}, \quad (6)$$

with ρ_0 denoting the equilibrium density of air and w representing the width of the shallot walls. In the expansion region (II.) conservation of momentum holds:

$$p_{\mathrm{und}}(t) - p_{\mathrm{sha}}(t) = \frac{\rho_0 U^2(t)}{S^2_{\mathrm{sha}}} - \frac{\rho_0 U^2(t)}{S_{\mathrm{sha}} S_{\mathrm{reed}}(t)}. \quad (7)$$

The above equations are valid under the assumption that $S_{\mathrm{sha}} \gg S_{\mathrm{reed}}$, with S_{sha} denoting the inner cross section area of the shallot and $S_{\mathrm{reed}}(t)$ representing the open surface between the shallot and the tongue.

Implementation

The equations are solved for each time step as follows.

1. Solve (3) to obtain q_n and $\dot{q}_n$.
2. Evaluate the open surface under the reed S_{reed} and the volume velocity generated by the tongue movement U_{reed}.
3. Solve the acoustic feedback equation and the flow equations simultaneously, i.e. substitute (5) into the sum of eqs. (6) and (7) and solve for U. Then calculate p_{und} and p_{sha} using (5) and (7).

The restoring force f_{rest} is very large when the reed hits the shallot, which can destabilize the simulation. To ensure stability, the time step size is reduced when the hit occurs and the reduced step size is maintained until the tongue leaves the shallot.

Results and discussion

In the transient phase, depicted in Fig. 2, it is observable that due to the increasing pressure in the boot the tounge moves to a new equilibrium position, nearer to the shallot. Then, small oscillations develop and the amplitude of the movement grows. When the tongue first hits the shallot ($t \approx 180$ ms) the amplitude stabilizes and the motion becomes steady.

In the steady state, shown in Fig. 3, the effect of the acoustic resonance in the shallot is clearly observable. Each time the tongue hits the shallot, a strong negative pressure pulse occurs, which is reflected from the open end of the shallot. The period in the steady state corresponds to the reed vibration period T_{reed}, whereas the period of the smaller oscillations corresponds to that of the first longitudinal acoustic mode, denoted by T_{acou}.

The transient and steady state simulation results are in very good qualitative agreement with measurements found in [1]. Since the number of model parameters are very high and some of them are hard to obtain from measurements, a quantitative comparison is yet to be performed.

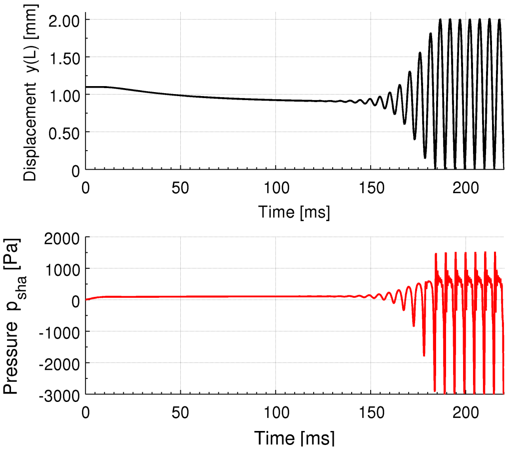

Figure 2: Simulation results – transient

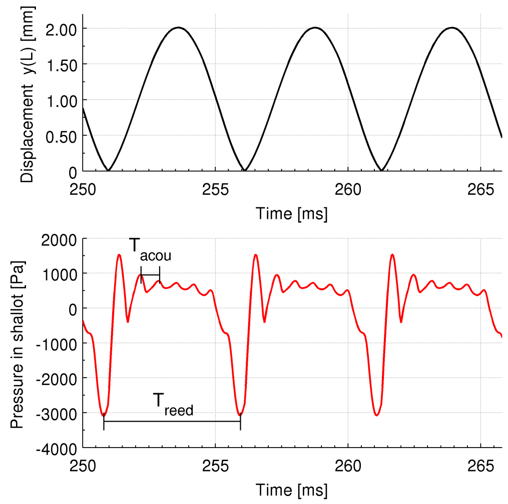

Figure 3: Simulation results – steady state

Acknowledgments

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