Parameter Identification for a Nonlinear Point-to-Point Model in the Context of Parametric Transductions

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Introduction

During the propagation of intense ultrasonic acoustic waves in fluid media, new frequencies are generated due to the nonlinearity of the fluid media. Among others, low frequency components are generated. These low frequency components are of particular interest for technical applications for the following reasons: The nonlinearly generated low frequency components are virtually radiated by an aperture which is much larger than the aperture of the physical transducer. This way, a high directivity can be obtained in the low frequency regime using physically small transducers. Furthermore, nearly the same bandwidth of the radiated ultrasonic wave is available in the low frequency regime. Thus, a high relative bandwidth can be created using transducers with a high Q factor. Additionally, since the channel attenuation is reduced at low centre frequencies, the nonlinearly generated low frequency wave can propagate over long distances.

The virtual aperture is commonly denoted as 'parametric acoustic array' [1]. The generation of low frequency acoustic waves by the exploitation of nonlinear effects is accordingly denoted as 'parametric transduction'. For further reading, see [2] and the references therein.

In [3], the authors describe the inherent signal distortions in the parametric transduction using a grey box modelling approach. The present paper addresses the parameter identification and the experimental validation of the derived model. Therefore, the model is extended in order to obtain an input/output relationship of a parametric transduction system. Besides the distortions of the nonlinear signal generation, the input/output relationship takes the transmitter and the receiver distortions as well as the environmental propagation conditions into consideration. In more detail, a model consisting of an moving-average, auto-regressive (ARMA)-process [4] followed by a generalized Hammerstein-Model [5] will be discussed. The model is linear in its parameters so that the identification can be done using direct estimation techniques. Hence, a least-squares approach is used in this paper.

Using a parametric transduction system in air, first experiments are conducted in order to test the identification approach. The identification results indicate the validity of the proposed model. Exemplary, the parametric transduction of a Gaussian pulse is considered. A comparison of the model output with measurements results shows a good agreement.

Notation: Column vectors are denoted by bold lowercase letters and matrices are denoted by capital bold letters. The superscripts $^{\mathrm{T}}$ and $^{-1}$ denote the transposition and the matrix inverse, respectively.

Channel Model

Nonlinear Signal Generation in Fluid Media

In [3], the authors discuss a physical modelling approach of the parametric transductions based on the Westervelt wave equation [1]. Particular emphasis is on the inherent nonlinear signal distortions. Assuming a free space propagation scenario and neglecting linearly propagating wave components as well as higher order contributions, a block-oriented model consisting of a static nonlinearity followed by a dynamic linearity is proposed. The model is shown in Fig. 1(a), omitting a delay and a signal scaling for brevity. Worth noticing is that the signal is twice time-differentiated in the dynamic linearity.

A discrete-time representation of the model is shown in Fig. 1(b). The dynamic linearity can be described by a third-order moving-average (MA)-process [4].

Input/Output Relationship for a Parametric Transduction System

The model in Fig. 1(b) focusses purely on the nonlinear signal generation in fluid media, but lacks the linearly propagating wave components. Furthermore, neither distortions induced by the transmitter and/or the receiver nor the environmental propagation conditions are taken into consideration. Thus, a more realistic model describing a discrete-time input/output relationship of a parametric transduction system is discussed in the following.

First, the transfer function of a non ideal electro-acoustic resonance transducer is considered by means of an ARMA-process at the channel input, see Fig. 2. Let $q \in \mathbb{N}$ denote the model order of the ARMA-process and, with

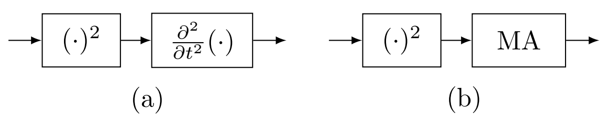

Fig. 1: Block-oriented model for the nonlinear signal generation in parametric transductions: (a) time-continuous model and (b) its discrete-time representation.

$N\in\mathbb{N}$, let further

$$\mathbf{u}\in\mathbb{R}^N\ ,\ \mathbf{u}=(u_0\ \ u_1\ \ \cdots\ \ u_{N-1})^{\mathrm{T}} \tag{1}$$

denote the input vector and

$$\mathbf{v}\in\mathbb{R}^{N-q+1}\quad \mathbf{v}=(v_q\ \ v_{q+1}\ \ \cdots\ v_N)^{\mathrm{T}} \tag{2}$$

the output vector of the ARMA-process. With the parameter vector $\boldsymbol{\phi}_{\mathrm{Tx}}\in\mathbb{R}^{2q}$,

$$\boldsymbol{\phi}_{\mathrm{Tx}}=(\underbrace{a_{\mathrm{Tx},1}\ \ldots\ a_{\mathrm{Tx},q}}_{\mathrm{AR_{Tx}}}\ \underbrace{b_{\mathrm{Tx},1}\ \ldots\ b_{\mathrm{Tx},q}}_{\mathrm{MA_{Tx}}})^{\mathrm{T}}\ , \tag{3}$$

which contains the polynomial coefficients of the ARMA-process, and with the observation matrix

$$\boldsymbol{\Psi}_{\mathrm{Tx}}=\begin{pmatrix} v_{q-1} & \cdots & v_0 & u_{q-1} & \cdots & u_0\\ \vdots & & \vdots & \vdots & & \vdots\\ v_{N-1} & \cdots & v_{N-q} & u_{N-1} & \cdots & u_{N-q}\end{pmatrix}\ , \tag{4}$$

the vector notation of the ARMA-process reads

$$\mathbf{v}=\boldsymbol{\Psi}_{\mathrm{Tx}}\,\boldsymbol{\phi}_{\mathrm{Tx}}\ . \tag{5}$$

Next, the acoustic wave propagation in the fluid is modelled by means of a generalized Hammerstein-Model [5]. A similar approach is used in [6] for modelling the underwater acoustic channel. In the generalized Hammerstein-Model, a K-th order static nonlinearity is followed by a dynamic linearity. The dynamic linearity is a MISO-ARMA-process, where individual MA-processes are considered for each input. Neglecting the contributions of higher order nonlinearities as it was done for the derivation of the model in Fig. 1(b), the order of the Hammerstein model is reduced to $K=2$ within the following analyses. As a result, the input/output-relationship shown in Fig. 2 is obtained.

The generalized Hammerstein-model takes both the linear wave propagation of the radiated (ultrasonic) wave and the propagation of the nonlinearly generated wave components into consideration. Due to the particular model structure, different propagation conditions for the individual wave components can be regarded. This way, the model of Fig. 1(b) can be directly incorporated in the $k=2$ path. For generality, an additional autoregressive (AR)-process can be incorporated in the model so that further environmental characteristics can be considered.

In order to find a vector formulation for the Hammerstein-model of Fig. 2, the model orders of the AR-process and the MA-processes are set to $p\in\mathbb{N}$. Furthermore, a process delay $d\in\mathbb{N}$ is considered in the model. This enables a reduction of the required model order p. Then, the input vector of the generalized Hammerstein-model is defined as

$$\mathbf{v}\in\mathbb{R}^N\ ,\ \mathbf{v}=(v_0\ \ v_1\ \ \cdots\ \ v_{N-1})^{\mathrm{T}} \tag{6}$$

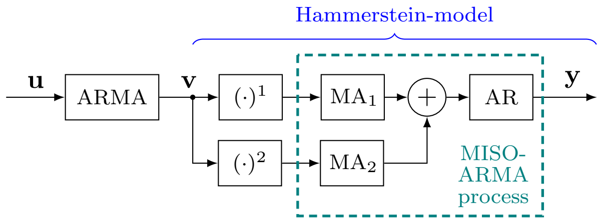

Fig. 2: Considered input/output-relationship.

and the output vector is defined as

$$\mathbf{y}\in\mathbb{R}^{N-p+1}\quad \mathbf{y}=(y_{d+p}\ \ y_{d+p-1}\ \ \cdots\ \ y_{d+N})^{\mathrm{T}}\ . \tag{7}$$

Further, with the polynomial coefficients $a_1,\ldots,a_p$ of the AR-process and the polynomial coefficients $b_{k,1},\ldots,b_{k,p}$ of the k-th MA-process, the parameter vector $\boldsymbol{\phi}_{\mathrm{H}}\in\mathbb{R}^{(K+1)p}$ is defined as

$$\boldsymbol{\phi}_{\mathrm{H}}=(\underbrace{a_1\ \ldots\ a_p}_{\mathrm{AR}}\ \underbrace{b_{10}\ \ldots\ b_{1p}}_{\mathrm{MA_1}}\ \underbrace{b_{20}\ \ldots\ b_{2p}}_{\mathrm{MA_2}})^{\mathrm{T}}\ . \tag{8}$$

Introducing the substitutions

$$\boldsymbol{\psi}_{\mathrm{y}}(m)=\left(-y_{m-1}\quad -y_{m-2}\quad \cdots\quad -y_{m-p}\ \right) \tag{9}$$

and

$$\boldsymbol{\psi}_n(m)=\left(v^k_{m-d-1}\quad v^k_{m-d-2}\quad\cdots\quad v^k_{m-d-p}\ \right) \tag{10}$$

for convenience and defining the observation matrix $\boldsymbol{\Psi}_{\mathrm{H}}\in\mathbb{R}^{(N-p+1)\times(K+1)p}$ as

$$\boldsymbol{\Psi}_{\mathrm{H}}=\begin{pmatrix}\boldsymbol{\psi}_{\mathrm{y}}(d+p) & \boldsymbol{\psi}_1(d+p) & \boldsymbol{\psi}_2(d+p)\\ \boldsymbol{\psi}_{\mathrm{y}}(d+p+1) & \boldsymbol{\psi}_1(d+p+1) & \boldsymbol{\psi}_2(d+p+1)\\ \vdots & \vdots & \vdots\\ \boldsymbol{\psi}_{\mathrm{y}}(d+N) & \boldsymbol{\psi}_1(d+N) & \boldsymbol{\psi}_2(d+N)\end{pmatrix}, \tag{11}$$

the vector notation of the generalized Hammerstein-model reads

$$\mathbf{y}=\boldsymbol{\Psi}_{\mathrm{H}}\,\boldsymbol{\phi}_{\mathrm{H}}\ . \tag{12}$$

Regarding the transfer function of a non ideal acoustic-electro receiver, an ARMA-process can be considered analogously. However, a corresponding ARMA-process is not shown in Fig. 2 because the process can be incorporated in the dynamic linearity of the Hammerstein-model.

Identification Method

Fortunately, both the input ARMA-process as well as the generalized Hammerstein-model are linear in their parameters. Thus, the identification can be done by using the method of least-squares. Following the vector notation of the equations (3) and (12), respectively, the least-squares approaches read

$$\widehat{\boldsymbol{\phi}}_{\mathrm{Tx}}=\left(\boldsymbol{\Psi}_{\mathrm{Tx}}^{\mathrm{T}}\,\boldsymbol{\Psi}_{\mathrm{Tx}}\right)^{-1}\boldsymbol{\Psi}_{\mathrm{Tx}}^{\mathrm{T}}\,\mathbf{v}\ , \tag{13}$$

$$\widehat{\boldsymbol{\phi}}_{\mathrm{H}}=\left(\boldsymbol{\Psi}_{\mathrm{H}}^{\mathrm{T}}\,\boldsymbol{\Psi}_{\mathrm{H}}\right)^{-1}\boldsymbol{\Psi}_{\mathrm{H}}^{\mathrm{T}}\,\mathbf{y}\ . \tag{14}$$

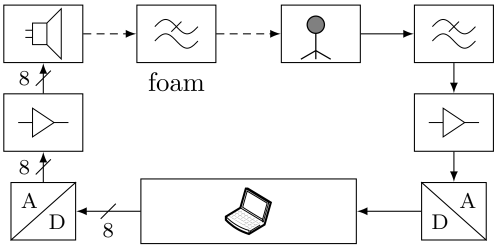

Fig. 3: Measurement setup.

See [7] for further details. As a consequence, the identification problem can be decomposed in two steps: First, an identification of the input ARMA-process, using equation (13), and subsequently an identification of the generalized Hammerstein-model, using equation (14). This enables an individual adjustment of each identification step, e.g. the choice of an appropriate transmit signal and measurement setup.

A challenge arises from equation (13) because the vector $\mathbf{v}$ is not directly observable. However, as will be discussed in Sec. , the identification can be done straightforwardly using an approriate measurements setup together with identification signals of low amplitudes.

Experimental Validation

Measurement Setup

Measurements are conducted using an air parametric transduction system in order to test the identification approach. The measurement setup is depicted in Fig. 3. The signal is transduced by an acoustic transducer array consisting of 8×16 PROWAVE 400ST100 array elements. Each row, i.e. 16 array elements, can transmit a different signal, which is generated by a desktop PC, subsequently D/A-converted and amplified. The maximum achievable sound pressure level (SPL) is $p \approx 150\,\mathrm{dB}(1\,\mathrm{m}, \mathrm{re}\ p_0)$. Unless otherwise mentioned, the maximum achievable SPL is used for all experiments in order to generate sufficient intermodulation products in the air.

At the receiver side, a condenser microphone of the type Microtech Gefell MK301 and a corresponding microphone preamplifier of the type Microtech Gefell MV210 is used. This way, acoustic waves can be captured up to $f = 100\,\mathrm{kHz}$. The microphone capsule is muffled in a foam in order to attenuate the ultrasonic wave. Subsequently, a lowpass filter with an integrated amplifier of the type Alligator Technologies USBPGF-S1 is implemented for anti-aliasing filtering and signal conditioning. The D/A- and the A/D-converter sample with $f_\mathrm{s} = 250\,\mathrm{kHz}$ and are synchronised so that the method of least-squares can be employed. Absorber panels are positioned behind the microphone to reduce the influence of the room.

Identification of the Input ARMA-Process

First, the input ARMA-process describing the transfer function of the electro-acoustic transducer array is identified in a 4 kHz bandwidth with a centre frequency equal to $f = 40\,\mathrm{kHz}$, which equals the centre frequency of the transducers. Transmit signals consisting of a sum of sinusoids spread equally over the frequency band of interests are employed, yielding an observation matrix $\mathbf{\Psi}_{\mathrm{Tx}}$ of full rank up to $q = 50$.

Low transmit magnitudes are used for the experiment so that a linear relation of $\mathbf{v}$ and $\mathbf{y}$ can be reasonably approximated. The ultrasonic wave is captured in 1 m distance. The receiver is adjusted so that mainly a frequency-independent gain is induced, i.e. the foam and the anti-aliasing filter are omitted. As a consequence, the Hammerstein-model of Fig. 2 only represents a delay and a scaling. The delay is estimated by the impulse response of the measurement system and is afterwards removed from the identification results.

The identification is done separately for each array row. The obtained transfer functions show the characteristic of a pair of conjugate-complex poles in the considered frequency range. Thus, a low approximation error between the process output and the model output can be obtained using a small model order q. The deviations of the individual transfer functions are negligibly small.

Hammerstein-Model: Preliminary Considerations

A challenge arises from the high transmit power required for generating sufficient intermodulation products in the air. For this reason, preliminary considerations towards an adjustment of the measurement setup are discussed.

In order to reduce nonlinear contributions originating from the transmitter, only sinusoids are transmitted via each array row. This way, no intermodulations can take place in the transducer.

Next, the SPL of the difference frequency component at $f = 1\,\mathrm{kHz}$ is conducted in the acoustical axis of the transducer array. The maximum SPL is achieved at a distance $r \approx 2{,}3\,\mathrm{m}$. The microphone is positioned at this distance in the following.

The microphone capsule is embedded in a foam to protect the microphone against the high intensity of the ultrasonic wave. Experiments with a horizontally rotated microphone reveal that a 30 dB-attenuation mitigates the nonlinear contributions from the receiver below the nonlinear contributions originating from the air. The foam is correspondingly dimensioned. This way, the microphone capsule and the microphone preamplifier are exited very much below their specified 3% total harmonic distortions points. An attenuation less than 5 dB results for frequencies $f < 4\,\mathrm{kHz}$.

Hammerstein-Model: Identification Results

Linear chirp signals of a bandwidth of $B = 4\,\mathrm{kHz}$ are used for the identification of the Hammerstein-model. Chirps with an upward frequency sweep are transmitted via four array rows and chirps with a downward frequency sweep are transmitted via the remaining rows. Due to the intermodulations, an downward-upward chirp is generated at the low frequency regime from $f = 0\,\mathrm{kHz}$ up to $f = 4\,\mathrm{kHz}$. The process delay d was estimated from the impulse re-

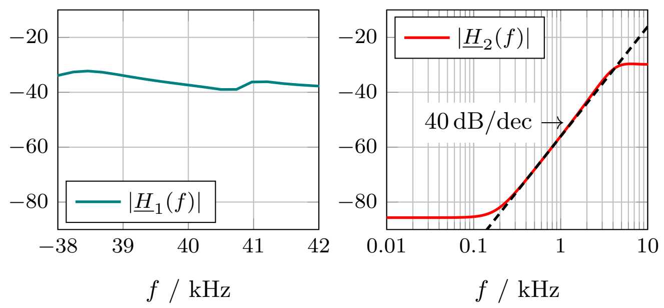

Fig. 4: Identified transfer functions $\underline{H}_n(f)$ of the Hammerstein-model in the corresponding frequency range.

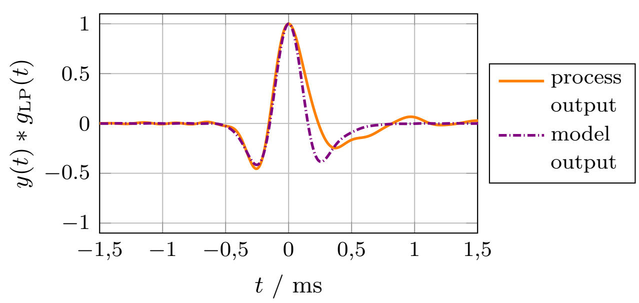

Fig. 5: Parametric transduction of a Gaussian pulse. Shown are the output signals after lowpass filtering.

sponse of the transduction system. Due to the particular measurement setup, the impulse response reveals mainly the presence of a direct propagation path. Thus, the room characteristics contribute insignificantly in this experiment.

Different model orders p are tested. The observation matrix $\mathbf{\Psi}_{\mathrm{H}}$ is of full rank in each case. Setting $p \geq 12$ does not change the characteristics of identification results basically. The identified transfer functions of the Hammerstein-model are shown in Fig. 4 for the model order $p = 12$. The transfer function $\underline{H}_1(f)$ indicates an attenuation of the linearly propagating waves above 30 dB, which is mainly due to the foam. In the transfer function $\underline{H}_2(f)$, a highpass characteristic of 40 dB/dec can be clearly seen. This is due to the double time differentiation of the nonlinearly generated components, which is predicted by the block-oriented model of Fig. 1. This characteristic is only observable up to $f = 4$ kHz because the used excitation stops at this frequency. As a consequence of the highpass characteristic, the identification can be hardly done in the very low frequency regime. This can be seen for example at $\underline{H}_2(f)$ below $f = 200$ Hz.

The identification results for the high frequency components at the sum frequencies and the harmonics are not discussed at this point because these components suffer a severe attenuation induced by the channel (and the foam) and are of minor interests for the parametric transduction.

Exemplary, the output of the identified model is compared with the process output by means of the parametric transduction of a Gaussian pulse. The used Gaussian pulse has a 3 dB-bandwidth of $B = 2$ kHz and is modulated on the carrier frequency $f_{\mathrm{c}} = 40$ kHz.

For the purpose of noise reduction, the process outputs of 250 transmitted pulses are averaged. The results are shown in Fig. 5. For reasons of comparison, the output signals are scaled, time shifted and ideal lowpass filtered in order to suppress high frequency components. It can be seen that the obtained signals are in agreement, which indicate the validity of the model and its identification. Slightly different behaviour can be observed in the decaying of the pulses, which may originate from the low signal to noise ratio of the measurements.

Conclusions

In this paper, an input/output relationship of a parametric transduction system is discussed. The model consists of an ARMA-process followed by a generalized Hammerstein-model. The identification of the model is done in two steps. First, an identification of the input ARMA process and, subsequently, an identification of the Hammerstein-model is carried out. A least-squares approach is applied in both steps.

First experiments using parametric transductions in air are discussed. The highpass characteristic inherent to the parametric transductions can be clearly seen from the identification results. Further experiments have to be conducted in order to fully assess the validity of the model.

In this connection, the influence of higher order nonlinearities on the parameter identification has to be investigated. This can be done straightforwardly by increasing the model order K in the proposed model and the identification approach.

Furthermore, an improvement in the model identification may be achieved by applying the total least squares algorithm, which allows additionally errors in the observation matrix $\mathbf{\Psi}_{\mathrm{H}}$. However, a challenge is here in the realisation of computationally efficient solution methods required for processing a very large number of samples N.

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