Hybrid Volterra and Hammerstein Modelling of Nonlinear Acoustic Systems

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Abstract

This paper studies suitable models for the identification of nonlinear acoustic systems. A cascaded structure of nonlinear filters is proposed that contains several parallel branches, consisting of polynomial functions followed by a linear filter for each order of nonlinearity. The second order of nonlinearity is additionally modelled with a parallel branch, containing a Volterra filter. These are followed by a long linear FIR filter that is able to model the room acoustics. The model is applied to the identification of a tube power amplifier feeding a guitar loudspeaker cabinet in an acoustic room. The adaptive identification is performed by the normalized least mean square (NLMS) algorithm. Compared with a generalized polynomial Hammerstein (GPH) model, the accuracy in modelling the dedicated real world system can be improved to a greater extend than increasing the order of nonlinearity in the GPH model.

Introduction

The nonlinear behaviour of a tube power amplifier connected to a guitar loudspeaker cabinet generates a characteristic tone that is quite pleasing to listen to for many guitarists and auditory listeners. The generated harmonic content and saturation of the guitar signal occurs when the tube power amplifier is driven at high volume levels. Loud listening volumes, high power consumption, generated heat and heavy and big sized guitar cabinets are the often unwanted side effects of achieving the desired guitar tone. Therefore, efforts are taken to simulate this typical sonic behaviour by a nonlinear model. For the identification of these nonlinear acoustic systems, there exist different models. The Hammerstein model, consisting of a static nonlinear function followed by a linear filter is capable of modelling higher order nonlinearities without high computational complexity [4], [5]. However, this model is not capable to identify every system with high accuracy, because often these systems contain nonlinear memory effects. The Volterra filter in comparison is capable of modelling a nonlinear system with higher accuracy [3], but with higher orders of nonlinearity and filter length, the computational demand grows fast. The generalized polynomial Hammerstein model [10] is capable of modelling certain terms of the Volterra filter. It consists of several parallel branches that contain a nonlinear function, followed by a linear filter. Often these functions are power series [9], [10], [6], but can also contain other functions like Chebyshev- or Legendre polynomials [11], [12]. For the identification of nonlinear systems, there exist several approaches in the literature. Deterministic methods, like the swept sine technique [9] [10], allow the identification of nonlinear systems that fit into the category of GPH models. Adaptive approaches try to fit a given model to the real world system by minimizing an error criterion. To reduce the computational calculations, a GPH model with power series filters followed by a long linear filter can be used [1] if the power series filter coefficients are highly correlated. The long linear filter is capable of modelling the room acoustics between the loudspeaker and a microphone which requires many filter tabs. The power filters can be modelled with shorter filter length, since the cascaded longer filter contains the room acoustics, that can be viewed as linear. Our approach, which is proposed in the following, combines the cascaded GPH model by Mossi et. al. [1] with a second order Volterra filter. In this hybrid approach, the measured error to the real system output can be further reduced compared to the GPH cascade. This is also the case when the order of the power filters in the GPH cascade is increased.

Nonlinear Model

In the following, the proposed model and the real world system are described. The model for the identification is shown in Figure 1. The nonlinear behaviour is taken in account by the generalized polynomial Hammerstein model with power series functions. Additionally, the second order of nonlinearity is modelled by a Volterra filter H_2 to consider further nonlinear memory effects. The required amount of filter tabs for the second order Volterra filter is still manageable. The structure is followed by a longer linear filter h, that is able to model the room acoustics.

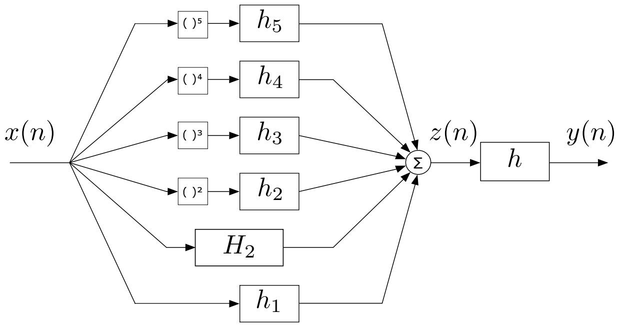

Figure 1: The proposed model to identify a nonlinear acoustic system

Figure 2 shows a block diagramm of the nonlinear acoustic system that describes a guitar amplifier, which is recorded with a microphone in an acoustic room. The amplifier feeds the incoming signal into the guitar loudspeaker cabinet. This signal is recorded with a microphone, which is a common setup in a recording studio scenario. The nonlinear behaviour is mostly created by the tube power amplifier and the loudspeaker chassis. The room can be viewed as a linear acoustic system, that can be modelled by an impulse response. The A/D and D/A converters of the recording interface, as well as the microphone preamplifier, are neglected in the block diagramm.

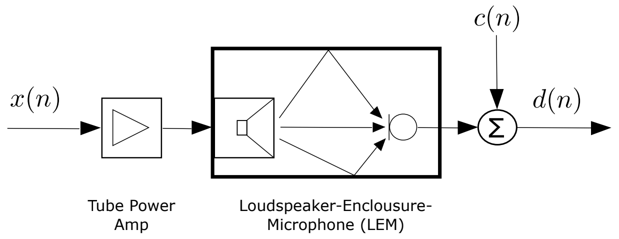

Figure 2: Block diagramm of the real world system

Adaptive System Identification

In the following section, the adaptive system identification is described. The aim of this approach is to minimize the error $e(n)$ between the measured output $d(n)$ of the device under test (DUT) and the estimated output $y(n)$ of the modelled system. Therefore a signal $x(n)$ is fed into both systems. A noise term $c(n)$ as shown in Figure 2 can disturb the identification process. As an input signal, an uniformly distributed white noise sequence was chosen as proposed in [13]. The error between the DUT and the model is reduced in a least mean square sense by the normalized least mean square algorithm (NLMS).

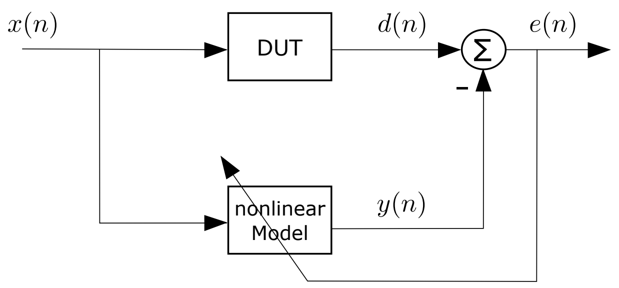

Figure 3: The adaptive system identification scheme

Model output signal

The linear filter $\mathbf{h}$, is calculated with the synchronized swept sine method after [10]. It is not adapted during the system identification process to avoid instability problems. The power filters are vectors of length N, the long linear cascaded filter h is a vector of length M. The following signals are required to calculate the estimated system output value $y(n)$ and are defined as:

$$\mathbf{x}_p(n) = [x^p(n), x^p(n-1), ..., x^p(n-N+1)] \quad (1)$$
$$\mathbf{h} = [h(0), h(1), ..., h(M-1)] \quad (2)$$
$$\mathbf{h}_p = [h_p(0), h_p(1), ..., h_p(N-1)] \quad (3)$$

$p \in \{1, 2, 3, \dots, P\}$ defines the exponent of the power series term, where P refers to the highest order to be modelled. $\mathbf{h}_p$ and $\mathbf{h}$ are the power filters and the linear filter of the model respectively.

The input of the second order Volterra filter can be rearranged in a row vector:

$$\begin{aligned}\mathbf{x}^{(2)}(n) =& [x^2(n), x(n)x(n-1), ...,\\ & x(n-1)x(n), x(n-1)^2, ...,\\ & x(n-N+1)x(n), ..., x(n-N+1)^2]\end{aligned} \quad (4)$$

and the corresponding Volterra filter tabs $\mathbf{H}_2$ as:

$$\mathbf{H}_2 = \left[H_2(0), H_2(1), ..., H_2\left(N^2-1\right)\right] \quad (5)$$

The input signals to the power filters can be written in matrix form:

$$\mathbf{X}_p(n) = [\mathbf{x}_p(n)^T, \mathbf{x}_p(n-1)^T, ..., \mathbf{x}_p(n-M+1)^T] \quad (6)$$

where T stands for the transpose of the vectors. The input of the Volterra filter is described as:

$$\mathbf{X}^{(2)}(n) = [\mathbf{x}^{(2)}(n)^T, \mathbf{x}^{(2)}(n-1)^T, ..., \mathbf{x}^{(2)}(n-M+1)^T] \quad (7)$$

The signal $\mathbf{z}(n)$ is the output of the parallel paths:

$$\mathbf{z}(n) = \sum_{p=1}^{P} \mathbf{h}_p \cdot \mathbf{X}_p(n) + \mathbf{H}_2 \cdot \mathbf{X}^{(2)}(n) \quad (8)$$

Then the output $y(n)$ can be written as

$$y(n) = \mathbf{h} \cdot \mathbf{z}(n)^T \quad (9)$$

and the error signal is:

$$e(n) = d(n) - y(n) \quad (10)$$

Parameter update

After each iteration, the filter coefficient update for each order of nonlinearity $p \in \{1, 2, 3, \dots, P\}$ is calculated based on the error signal $e(n)$. The needed update vectors are calculated as:

$$\begin{aligned}\mathbf{x}_p^{up}(n) &= \mathbf{X}_p(n) \cdot \mathbf{h}^T \\ \mathbf{x}_{up}^{(2)}(n) &= \mathbf{X}^{(2)}(n) \cdot \mathbf{h}^T\end{aligned} \quad (11)$$

and the update of the filter coefficients by the NLMS algorithm:

$$\mathbf{h}_p(n+1) = \mathbf{h}_p(n) + \mu \cdot \frac{\mathbf{x}_p^{up}(n)^T}{|\mathbf{x}_p^{up}(n)|^2} \cdot e(n) \qquad (12)$$

$$\mathbf{H}_2(n+1) = \mathbf{H}_2(n) + \mu \cdot \frac{\mathbf{x}_{up}^{(2)}(n)^T}{|\mathbf{x}_{up}^{(2)}(n)|^2} \cdot e(n)$$

where μ is the adaptation step size.

Simulation Results

The proposed algorithm was applied to identify an nonlinear acoustic system, as shown in Figure 2. The amplifier is the tube power section of a Mesa Triple Rectifier guitar head, the loudspeaker is a Mesa Traditional cabinet loaded with four twelve inch Vintage Celestion V30 loudspeaker chassis. The microphone is a Horch RM2J large diaphragm condenser microphone. The input and output signals are recorded with a Mbox 2 USB recording interface from Digidesign/Avid and the sampling rate was set to $f_s = 44.1\ kHz$. For the filters of the power series, a length of 64 tabs was chosen. After the identification of the linear part of the system with the synchronized swept sine technique, the output filter **h** was truncated to a length of 1024 coefficients. The length of the Volterra filter was chosen to 4096 filter tabs and the adaptation step size was set to $\mu = 0.1$.

To verify the improvement in using the hybrid approach of the generalized polynomial Hammerstein model combined with the second order Volterra filter, several simulations were calculated. The adaptation was performed with and without the Volterra part and the system was further modelled in a linear manner by just taking the first order of the GPH model. The output filter **h** is the same in all simulations.

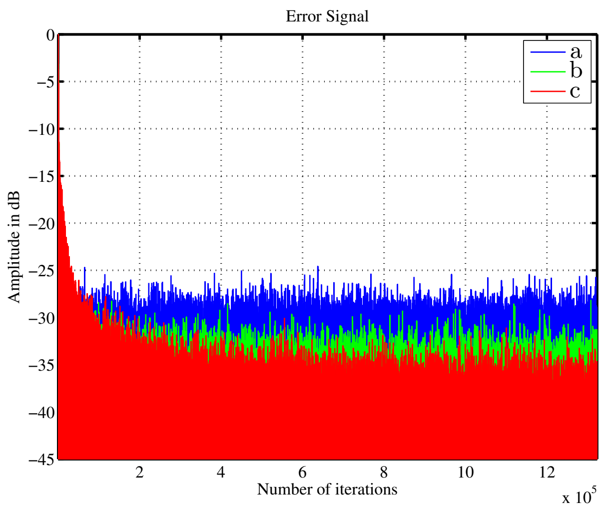

Figure 4: Error signal during iteration process for (a) linear, (b) 5th order GPH cascade and (c), the proposed approach of combining the 5th order GPH cascade with the second order Volterra filter

Figure 4 shows the error signal during the adaptation process. It can be observed, that the linear model is capable of reducing the error signal significantly due to the only slightly nonlinear behaviour of the system, but it can also be observed, that the GPH model of 5th order reduces the error further. The best performance is achieved by combining the cascaded GPH approach with the Volterra filter.

Table 1 shows the calculated system distance E_{sys} to the real world system, that is defined as:

$$E_{sys} = 10 \cdot \log_{10} \left(\frac{\sum_{n=1}^{T} |(d(n) - y(n))|^2}{\sum_{n=1}^{T} |x(n)|^2} \right) \qquad (13)$$

The system distance was calculated after the adaptation has reached a minimum, to ensure that the results are independent of the adaptation time.

Model	E_{sys}
Linear	-34.00 dB
GPH cascade 5th Order	-38.53 dB
GPH cascade 10th Order	-37.85 dB
Linear + 2nd Order Volterra	-40.26 dB
GPH cascade 5th Order + 2nd Order Volterra	-41.01 dB

Table 1: System distance to real world device

It can be seen, that the GPH cascade of 5th order improves the system distance by over 4 dB. The GPH cascade of 10th order shows also an improvement, but is slightly less accurate than the 5th order model, where the task of estimating more coefficients than necessary could be the issue of this degradation. The Volterra model in parallel to a first order GPH cascade (which equals a linear modelling) gives good results, which can be improved by increasing the GPH model up to the 5th order. For the given real world system, the proposed hybrid model gives the most accurate results, regarding the system distance.

Figure 5 shows the result of exciting the proposed hybrid model (c) and the Linear + 2nd order Volterra model (b) with a summed sine signal of $200Hz$ and $1.5kHz$. The generated frequency content is shown in the Fourier domain and compared with the real world system output (a). All output signals are weighted with a Hanning window to prevent spectral smearing. To allow a better comparison between the signals, the frequency plot of the linear branch + second order Volterra filter output is shifted by a few Hz to the right and the hybrid model output is shifted slightly to the left. As can be seen, the Volterra filter is only able to model the dominant frequency content, despite its significant improvement of the system distance. Further frequencies are only modelled by the 5th order GPH cascade. This indicates, that the system distance calculated by the white noise input is not a sufficient measure for the accurate reproduction of the real frequency content. The perceptional impression by the human ear can also vary significantly from

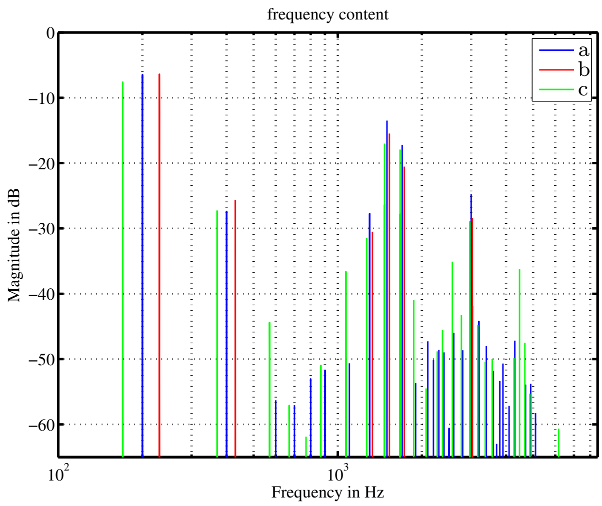

Figure 5: frequency content for excitation with a summed sine signal of $200Hz$ and $1.5kHz$ (a) real world system output (b) modelled system with linear part and second order Volterra filter (c) modelled system with proposed hybrid approach

the quantitative measure of the system distance which is already mentioned in the work of Geddes and Lee [8] [7], where a perceptional impression weighted measure of generated harmonic content was proposed.

Conclusions

In this paper a suitable model to identify a nonlinear acoustic system is proposed. Therefore the error signal between the real system output and the output of the model is minimized in the least mean square sense by the NLMS algorithm. It can be shown, that by combining the generalized polynomial Hammerstein model with the second order Volterra filter, the system distance can be significantly improved compared with a linear modelling approach. However, the results indicate that the system distance is not the best measure to verify the accuracy of harmonic overtone and intermodulation distortion modelling. Further research should be addressing this issue to improve the accuracy of the model regarding the harmonic content and a measure to improve the listening perception.

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