Experimental Verification of the Distribution Function of the Cavitation Noise

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Introduction and statistical methods

A quantitative characterization of measured cavitation noise signals requires their analysis by means of descriptive statistics - e. g. by the average μ, the standard deviation σ, the excess E (with E = C-3 and C the curtosis) and by the skewness S /1/. Here we use tests suitable for checking samples whether they obey a Gaussian distribution or not: These are the Jarque-Bera-(JB-) and the Shapiro-Wilk-(SW-) test with their corresponding probabilities pJB and pSW. The significance level α for rejecting the assumption of a Gaussian distribution was set at α = 0.05 /2/.

Generation of samples of cavitation noise

In a small ultrasonic bath with dimension of 130 x 140 mm cavitation is generated with one transducer at the bottom with a radiation surface of 16 cm². The ultrasonic signal is sinusoidal with a frequency of 30 kHz and an intensity of 3 W/cm² at the transducer surface unless otherwise noted. The water surface is covered with a low-impedance reflector at a filling height of 36 mm. The hydrophone (Reson TC4034) is dived in 12 mm in the bath through a hole in the reflector. With an AD-converter the signal of the hydrophone is measured with a sample rate of 1 MHz. A data length of 512 values is weighted by a raised cosine function in time as shown in Figure 1 (at the bottom). The weighted data are transformed using a Fast Fourier Transformation (FFT) and the power density spectrum is calculated. In this spectrum the cavitation noise is calculated as an average between the spectral lines in a range of 2.15 to 2.35 times of the ultrasonic frequency. We perform this calculation 22 times with 512 data shifted by 256 data as shown in Figure 1 (top). The whole data length for 22 shifts is 5898 data corresponding to a duration of 5.898 msec. These 22 values form the sample for the statistic evaluation. All statistical calculations are performed with these samples of the power density spectrum. The average μ and the standard deviation σ are converted to dB after the statistical calculation.

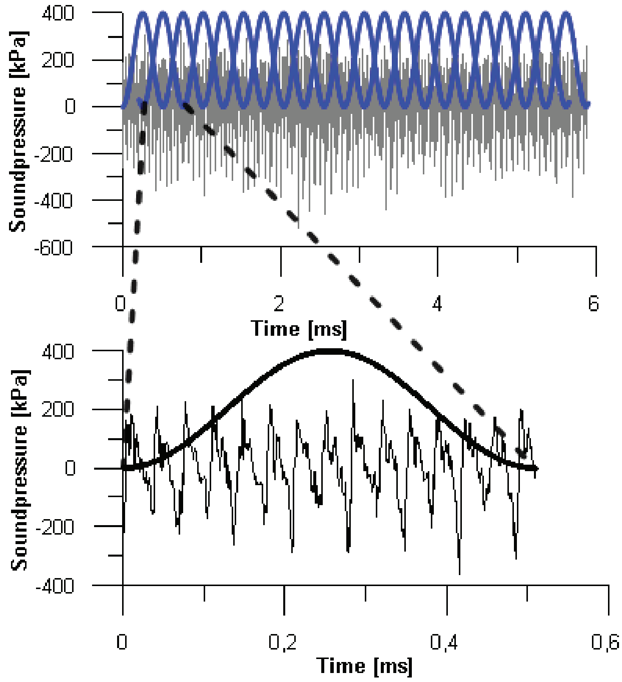

Figure 1: top: Time dependency of the sound pressure with shifted weighting functions. bottom: data length containing one FFT.

Statistics of samples of cavitation noise

Fig. 2 shows the statistics for 50 of the measured samples, all of them taken within 15 sec under continuously running ultrasound with an intensity of 3 W/cm². The mean noise level of μ = 37 dB is obtained with a standard deviation σ = 2.0 dB. The added boxplots for the 50 samples show deviations from a Gaussian distribution: S = 0.7 with outliers to higher values and E = -0.9 but also with outliers to higher positive values.

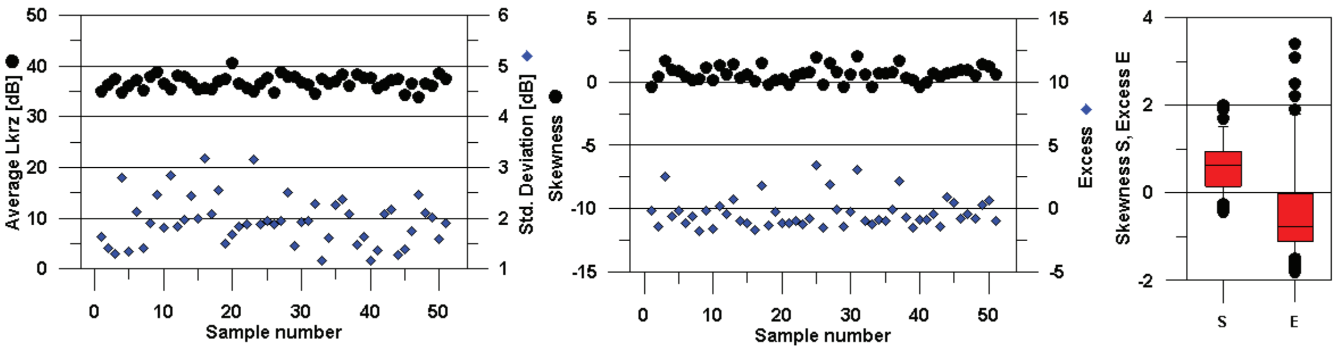

Figure 2: Statistics for 50 measured samples (see text) and boxplot for their corresponding 50 distributions.

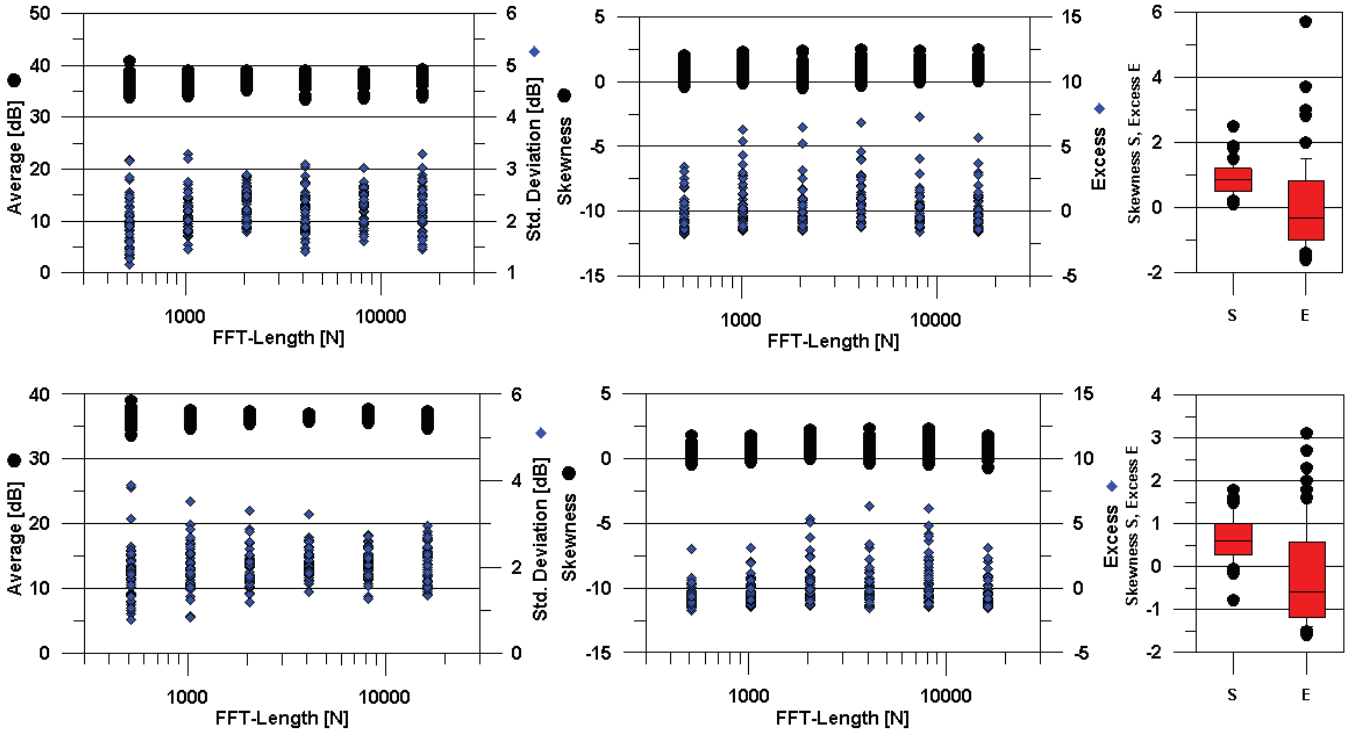

Figure 3: Influence of the data lengths (512, 1024, 2048, 4096, 8192, 16384 data) used for the FFT.
Top: Statistics of measured samples. Bottom: Statistics of white noise.

In Fig. 3, upper part, the influence of the data length used for the FFT of the measured samples - from 512 up to 16384 data - is presented. The unchanged mean noise level of μ = 37 dB is obtained with a nearly unchanged standard deviation σ = 2.2 dB. The boxplots for the 50 samples with the longest data length of 16384 data show still deviations from a Gaussian distribution: S = 0.9 again with outliers to higher values and E = -0.4, again with outliers to higher positive values. The expected improvement with smaller σ and S-, E-values closer to zero was not obtained with increased data lengths of the FFT.

Comparison of the statistical results for samples of cavitation and of white noise

The statistics of the samples of cavitation noise as shown above in Fig. 3 are compared with the statistics of a white noise signal (lower part of Fig. 3). The digital white noise signal is mathematically generated. It consists of a discrete uniform distribution of amplitudes (μ = 37 dB) and frequencies, plus an exact sinus of the same frequency as for the ultrasonically generated measured cavitation noise for triggering. In Fig. 3 the comparison is shown for the same different 6 data lengths of the samples of digital white noise. The boxplots for the 50 samples with the longest data length of 16384 data show again similar deviations from a Gaussian distribution: S = 0.6 again with outliers to higher values and E = -0.7, again with outliers to higher positive values.

The conclusion is that the statistics of cavitation noise and of white noise, both measured by the same kind of digital signal processing described above, behave quite similarly.

Significance tests on Gaussian distribution for the samples of cavitation and of white noise

Because of the comparable deviations from the Gaussian distribution shown for both samples by their 2^{nd} to 4^{th} moments, the JB- and SW-tests of high power for the assumption of both samples (of data length equal to 512) as Gauss-distributed samples are applied. The results are presented in Fig. 4, were the red bars mark the chosen significance level for α = 0.05. Below the significance level another stretched scaling is used at y-axes. It turns out that according to the JB-tests for the cavitation noise 6 of the 50 samples and for the white noise only 1 of the 50 samples reject the hypothesis of being Gaussian distributed. Using the SW-tests for 18 of the 50 cavitation noise samples and for 11 of the 50 white noise samples the hypothesis of being Gaussian distributed is rejected.

The final conclusion is that for the cavitation noise at 30 kHz at an intensity of 3W/cm^2 as well as for the mathematically generated white noise the hypothesis of a Gaussian distribution is rejected for a significant number of samples.

Dependence of the cavitation noise on the intensity and application of statistics for ultrasonic devices

Furthermore the dependence of the average μ of the cavitation noise level and of the standard deviation σ on the intensity of the ultrasound is measured. The cavitation is still generated from a 30 kHz pure sinusoidal working frequency. The statistic is taken from 50 samples, each averaged from 30 power density spectra and each spectrum with 8192 data.

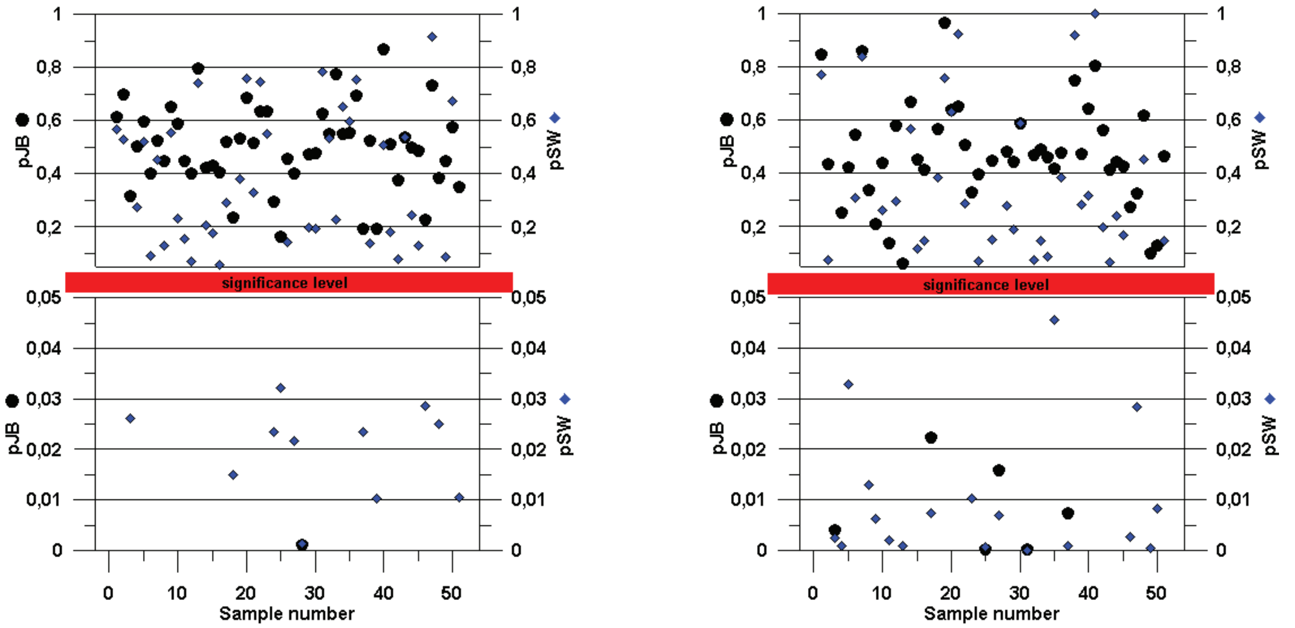

Figure 4: Probabilities of the 50 samples for the hypothesis of a Gaussian distribution, calculated according to Jarque-Bera (left axis) and Shapiro-Wilk (right axis) with a significance level set at α=0.05.

In the upper part of Fig. 5 it is recognized that at the marked kink in the shape of the curve of the average μ, which can be identified with the threshold of the transient cavitation /3/, σ reaches its maximum values, whereas σ reaches its lowest values in the whole range only at higher intensities above the threshold. This is the well-known range of proportionality with a slope equal to 1 between the average of the cavitation noise in dB and the intensity on a logarithmic scale /3/.

In the lower part of Fig. 5 the averages μ for the same 50 samples are calculated by simply summing up all squared amplitude values within each of 30 power density spectra. Each spectrum is taken with 8192 data and the spectral lines of the working frequency and its harmonics are eliminated by using a running median within the averaged power density spectra.

It is seen that this kind of averaging (applied in the lower part of Fig. 5 only) gives a slightly changed picture for the average of the cavitation noise level μ and its standard deviation σ for intensities at and below the threshold of transient cavitation /3/.

In most commercially devices the intensity of the ultrasound can be controlled (“power” regulation) and they are driven by amplitude-modulated (e. g. double half-wave AM) as well as frequency-modulated (“sweep”, FM) signal. To achieve a robust statistical characterization throughout these variations, each sample consists of 50 values. Each value calculated from 30 averaged power density spectra and each spectrum taken with a length of 8192 data.

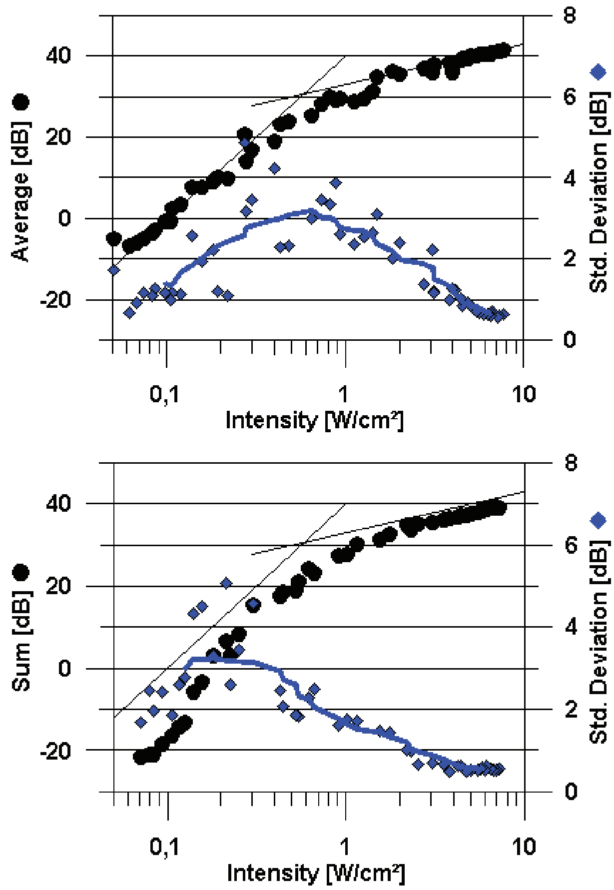

Figure 5: Dependence of the average of the cavitation noise level and of the standard deviation σ on the intensity of the ultrasound. Bottom: special different signal processing applied for the averaging of μ (“Sum”, see text).

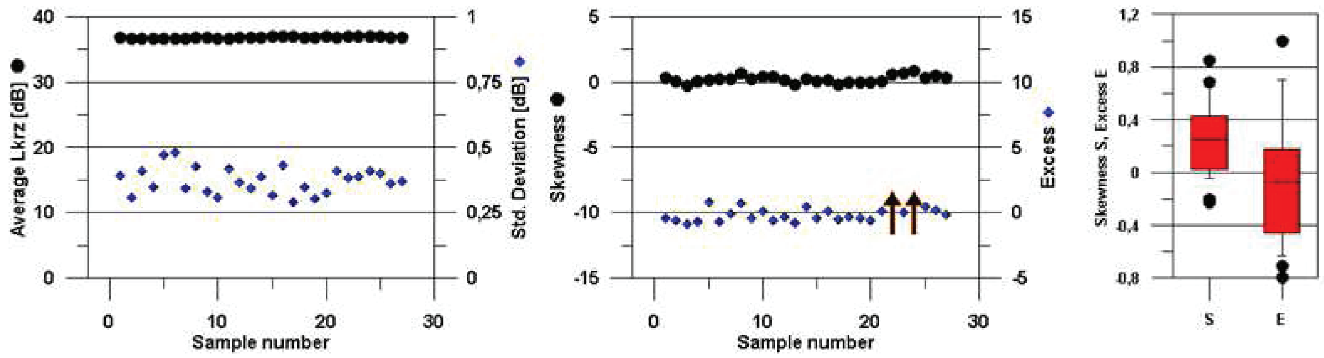

Fig. 6: Statistics for 27 measured samples and boxplot for the corresponding 27 distributions for an intensity of 3 W/cm², AM and FM.

Fig. 6 shows the results of the statistics for an intensity of 3 W/cm^2. Again a mean noise level of μ = 37 dB is obtained but because of the extended accumulation of samples with a much smaller standard deviation σ = 0.4 dB. The boxplots for the 50 samples show only small deviations from a Gaussian distribution: S = 0.2 and E = -0.1, both with 3 outliers to positive and negative values. The arrows mark the two outlying samples below the significance level α = 0.05 according to the SW-test. This result suggests that Gaussian-like distributions of cavitation noise may be obtained with a higher number of averaged power density spectra.

Summary and outlook

It is possible to calculate the first 4 moments even for the samples of the cavitation noise within a short sampling time of 6 msec only. The skewness results in S = 0.7 and the excess in E = -0.9, i.e. the distribution is right-skew and flatter than a Gaussian distribution. But there are steep-edged outlying distributions with a positive excess of E=3. The relation between the standard deviation on one side and the skewness as well as the excess on the other side requires further investigations. The change of the FFT-length from 0.5 k to 16 kByte does not change this characteristic of the distribution.
Furthermore the moments of mathematically calculated values of digital white noise are given. In this investigation done at the first time the statistics of the measured samples appear to be still comparable with the statistics of the samples of the white noise.

The tests of significance based on Jarque-Bera and on Shapiro-Wilk show that the measured distribution cannot be assumed always to be Gaussian-like. The same applies to the statistics of the results with white noise. This kind of evaluation should be considered as a first trial only.

The dependence of the average of the cavitation noise level on the intensity of the ultrasound shows their well-known proportionality and the cavitation threshold as a kink of the shape of the corresponding curve. The standard deviation has a maximum near the threshold and declines to higher intensities.
If samples are taken from several averaged power density spectra instead of only one spectrum, the samples become Gauss-distributed approximately.

Literature

[1] https://en.wikipedia.org/wiki/Moment_(mathematics), downloaded 2016-03-18.

[2] H. Lohninger: Fundamentals of Statistics, taken from: http://www.statistics4u.info/fundstat_eng/ee_shapiro_wilk_test.html, downloaded 2016-03.18.

[3] Sobotta R, Jung Ch.: Messung der Kavitationsrauschzahl, Fortschritte der Akustik, p. 581, 2005.