Excitation Signals for Online Secondary Path Estimation in Active Noise Control

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Abstract

For active noise control in audio devices such as headphones or hearing aids, the knowledge of the secondary path between the noise-cancellation loudspeaker and the error microphone is crucial for the performance. For the identification of this time variant path a measurement or excitation signal is required. It should allow fast tracking, be robust as well as uncorrelated with respect to the ambient noise, and be subjectively not too annoying.

Most approaches proposed in the literature apply white Gaussian noise (WGN). So far, alternative options have largely been disregarded. In this contribution, we propose and investigate by example three different classes of signals, which are objectively optimal, common for acoustic measurements or subjectively less disturbing but still effective. The first two classes consist of unnatural synthetic signals. Two representatives are the perfect sweeps, characterized by an impulse-like autocorrelation function, and the logarithmic sweep. The third subjectively less disturbing class consists of signals derived from or shaped by natural sounds. One example is sea noise which exhibits a comparable spectral flatness as WGN but shows amplitude modulation over time. For the performance evaluation we use an adaptive feedforward noise cancellation algorithm.

Introduction

The principles of Active Noise Control (ANC) have found entry in various consumer products in the past years. The most prominent representative are ANC headphones, which have a wide variety of applications. The primary goal is to attenuate an existing outer disturbances $x(n)$. The headphone itself offers a passive insulation, which results in an attenuated inner disturbance $d(n)$ in the ear canal. A compensation signal $y(n)$ is calculated by digital signal processing, for reducing the inner disturbance $d(n)$ by destructive interference. The achievable ANC performance, manifesting in the residual error signal $\tilde{e}(n)$, is greatly influenced by the acoustic front-end. This is represented by a headphone containing one outer microphone, one inner microphone and a speaker. The primary path represented by the transfer function $G_d(z)$ describes the transfer between the outer and the inner microphone, corresponding to the passive insulation of the headphone. The secondary path $G(z)$ is the transmission between the inner loudspeaker and the inner microphone. It also covers the acoustic properties of the ear canal.

In general, both acoustic paths are time-variant with respect to ear-piece position, occlusion, etc. Furthermore, an accurate estimation of the real acoustic path is necessary for the ANC system. A false estimation especially of the secondary path $G(z)$ will result in instability. Therefore, we need to monitor the changes and estimate the current path online.

Several approaches for the secondary path estimation (SPE) have been proposed, based on the initial idea of Eriksson et.al [1]. He induced an additional excitation signal $v(n)$ to monitor the secondary path $G(z)$. However, the ANC and the SPE system interfere with each other. One of the major improvements has been proposed by Zhang et. al [2] with an additional adaptation system to reduce this interference. Various works introduced a variable step size for faster adaptation and lower steady state error, e.g., [3]. Furthermore, several suggestions have been made for excitation power scheduling, e.g., [4]. All of these publications are using WGN as an excitation signal. In this publication we are investigating alternative excitation signals.

Modelling and Simulation System

We are considering an adaptive feedforward ANC system with a reference and an error microphone. For the SPE the influence of different excitation signals may best be demonstrated with the approach by Eriksson [1]. The combined digital model is shown in Fig. 1.

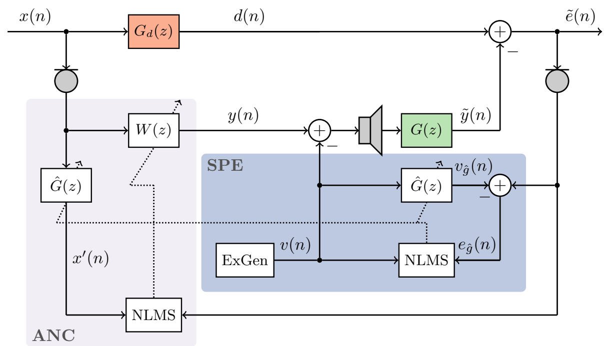

Figure 1: ANC system model including the SPE approach by Eriksson et.al. [1].

For illustration purposes the microphone and loudspeaker positions are symbolized. Their characteristics are represented within the measured acoustic paths $G_d(z)$ and $G(z)$. To assure a realistic simulation environment, we conducted acoustic measurements, illustrated in Fig. 2. The acoustic paths were measured with a real-time system (DS1005, dSPACE GmbH, Paderborn, Germany) with *Bose QC 20* headphone hardware (without the Bose ANC electronics) as the acoustic front-end [5]. The round trip delay of this system without the acoustics is 1 sample at a sampling rate of $f_s = 48\,\text{kHz}$.

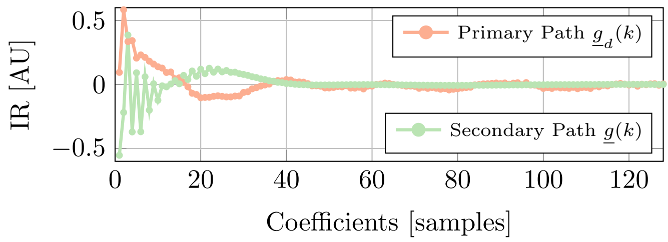

Figure 2: Measured impulse responses (IR) of the primary and secondary path ($\underline{g}_d(k)$ and $\underline{g}(k)$).

The filter $W(z)$, which processes the reference signal $x(n)$ and creates the compensation signal $y(n)$, is adjusted by an adaptive algorithm. In our system, we are using a filtered-x normalized least-mean-squares (FxNLMS) algorithm, which updates the filter coefficients of $W(z)$ in every timestep by the following equation

$$\underline{w}(n+1) = \underline{w}(n) + \frac{\mu_w}{||\underline{x}'(n)||^2} \cdot \underline{x}'(n) \cdot \tilde{e}(n), \qquad (1)$$

with the stepsize μ_w, and $\underline{w}(n)$ being the vector with the filter coefficients in the timestep n [6].

One may observe that the secondary path $G(z)$ alters the compensation signal before the acoustic subtraction is performed. Due to its influence, it is necessary to provide a filtered reference signal $x'(n)$ for the adaptive algorithm. To perform this prefiltering we need a precise estimation $\hat{G}(z)$.

The secondary path estimation is done by a second adaptive approach following

$$\hat{\underline{g}}(n+1) = \hat{\underline{g}}(n) + \frac{\mu_{\hat{g}}}{||\underline{v}(n)||^2} \cdot \underline{v}(n) \cdot e_{\hat{g}}(n) \qquad (2)$$

with

$$e_{\hat{g}}(n) = [\underline{g}(n)*\underline{v}(n) - \hat{\underline{g}}(n)*\underline{v}(n)] + [d(n) - \underline{g}(n)*\underline{y}(n)]. \quad (3)$$

The $*$-operator describes the convolution in the time domain. The first squared bracket represents the desired error signal for the adaptation. Whereas the second squared bracket contains the disturbing influence of the outer disturbance $d(n)$ and the compensation signal $y(n)$.

The ANC adaptation is also disturbed by the SPE, as the modified error signal $\tilde{e}(n)$ contains the influence of the excitation signal $v(n)$

$$\tilde{e}(n) = [d(n) - \underline{g}(n) * \underline{y}(n)] + [\underline{g}(n) * \underline{v}(n)]. \qquad (4)$$

Excitation Signals

We are focusing on the influence of the excitation signal $v(n)$ on the secondary path estimation.

Commonly this signal is WGN, as it provides adequate adaptation and is tolerably disturbing. However, it is possible to improve on these two properties. We are examining three groups of excitation signals, namely objectively optimal signals for the adaptation, typical acoustical measurement signals and subjectively less disturbing signals.

The periodic autocorrelation function (PACF) gives an indication of the suitability for the adaptation [7]

$$r_{vv}(\lambda) = \frac{1}{E_v} \sum_{i=0}^{N_v-1} v(i) \cdot v(i + \lambda \bmod N_v) \qquad (5)$$

with the signal energy $E_v = \sum_{i=0}^{N_v-1} v^2(i)$ for normalization and the signal (period) length N_v. The autocorrelation function of an infinitely long WGN signal with zero-mean is

$$r_{vv}(\lambda) = E[v(n)v^*(n-\lambda)] = \begin{cases} \sigma_v^2, & \lambda = 0 \\ 0, & \lambda \neq 0 \end{cases} \qquad (6)$$

with E denoting the statistical expectation operator and σ_v^2 being the variance of the noise [6]. However, as we are only considering a finite window of $N_v = 128$ samples, the WGN loses its perfect autocorrelation properties. This may also be observed in the PACF in Fig. 3. It leaves room for improvement on the side of excitation signals.

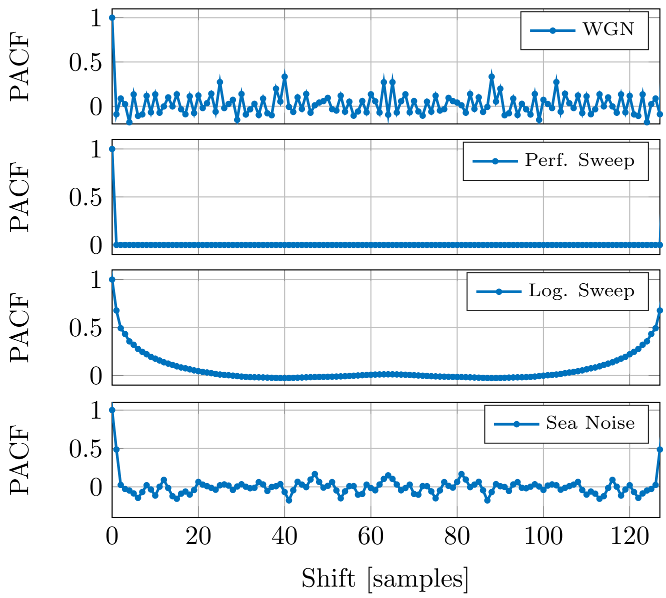

Figure 3: Periodic autocorrelation functions with period length $N_v = 128$ samples.

Within the group of objectively optimal signals for the adaption, we are considering the perfect sweep [7]. In general a signal is described as perfect, when for a periodic repetition of all time-shifted versions of this signal are orthogonal to each other. This manifests in a PACF of

$$r_{vv}(\lambda) = \begin{cases} E_v, & \lambda \bmod N_v = 0 \\ 0, & \lambda \bmod N_v \neq 0 \end{cases}. \qquad (7)$$

For the second group of typical acoustical measurement signals, we are further investigating the logarithmic sweep [8]. Periodically repeated logarithmic sweeps are incorporating a considerable amount of correlation as visible in Fig. 3. They are expected to show weaker performance than the other prospects. Another wide-spread signal is the Maximum Length Sequence (MLS), also known as m-sequences. As shown in [9], they only provide quasi orthogonality and are therefore not optimal for the adaptation process.

The third group regarded in this paper are subjectively less disturbing signals derived from or shaped by natural sounds. As the measurement signal is played via the loudspeaker and is audible for the user, it is eligible to utilize a pleasant measurement signal. One example is sea noise. It exhibits a comparable spectral flatness as WGN but shows amplitude modulation over time. The PACF of sea noise in Fig. 3 is comparable to WGN.

Overall, with respect to the SPE, the perfect sweep to outperform WGN, followed by sea noise and the logarithmic sweep.

Evaluation

For judging the performance of the system identification we are considering the relative system error $D(n)$. It describes the relative distance between the measured secondary path coefficients $\underline{g}(n)$ and the estimation $\underline{\hat{g}}(n)$ and is defined as

$$D(n) = 10\,\log_{10}\frac{||\underline{g}(n) - \underline{\hat{g}}(n)||^2}{||\underline{g}(n)||^2}. \qquad (8)$$

For evaluating the potential of the different excitation signals we are first considering the isolated secondary path estimation without disturbance ($x(n) = 0$) and with deactivated ANC system ($y(n) = 0$). Following Eq. 3 the SPE therefore has the best possible adaptation conditions with $e_{\hat{g}}(n) = [\underline{g}(n) * \underline{v}(n) - \underline{\hat{g}}(n) * \underline{v}(n)]$. The filter length of $\underline{\hat{g}}(n)$ as well as the signal period length N_v of the perfect sweep the logarithmic sweep are chosen to 128 coefficients. The filter coefficients $\underline{\hat{g}}(n)$ are initalized to zero. Fig. 4 shows the system error $D(n)$ for the four different excitation signals with a stepsize of $\mu_{\hat{g}} = 1$. This stepsize exhibits the fastest convergence, however, is very susceptible to disturbance. The WGN signal is showing a

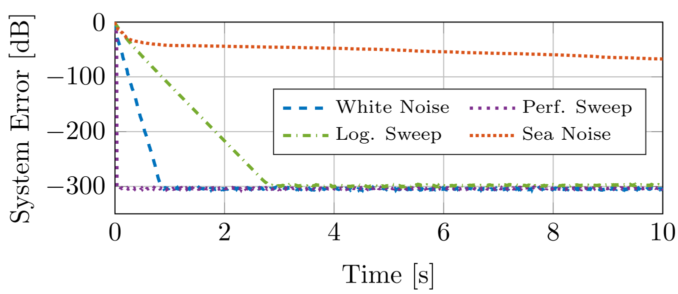

Figure 4: Relative system error $D(n)$ under optimal conditions with stepsize $\mu_{\hat{g}} = 1$ for different excitation signals ($N_{\hat{g}} = N_v = 128$, $f_s = 8000$ Hz, $\underline{\hat{g}}(0) = 0$).

fast adaptation and reaches the computational accuracy of $2^{-52} = -313$ dB in less than 1 s at a sampling rate of $f_s = 8000$ Hz. However, with the perfect sweep, the filter converges within two periods of $2 \cdot N_v = 256$ samples [9]. Exciting the system with a logarithmic sweep results in a slower convergence, but still reaches the desired accuracy. The sea noise shows a quick convergence at the beginning. Its convergence degrades at roughly −30 dB and continues with a smaller slope. This degradation may be explained by the non-uniform energy distribution over all frequencies.

Within a disturbed scenario, one would have to lower the stepsize $\mu_{\hat{g}}$, in order to avoid instability. Therefore, we are also looking a stepsize of $\mu_{\hat{g}} = 0.005$, realistic for this application, in Fig. 5. This reveals that the performance

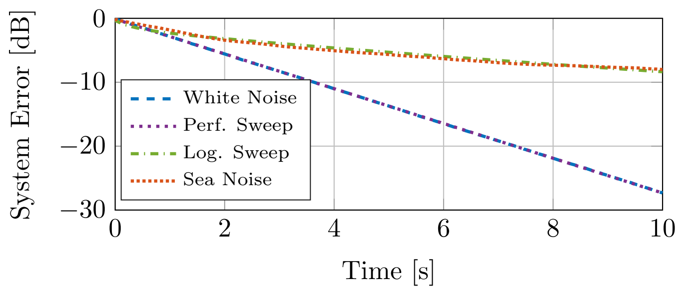

Figure 5: Relative system error $D(n)$ under optimal conditions with stepsize $\mu_{\hat{g}} = 0.005$ for different excitation signals ($N_{\hat{g}} = N_v = 128$, $f_s = 8000$ Hz, $\underline{\hat{g}}(0) = 0$).

of WGN and the perfect sweep are similar for this stepsize.

In the following, we are looking at the performance of the complete system. The primary goal is now to attenuate the disturbance $d(n)$, besides the secondary path estimation regarded previously. For this we are evaluating the energy ratio between the inner disturbance $d(n)$ and the residual error $\tilde{e}(n)$ smoothed by a mean filter of length N_a. The attenuation $A(n)$ is given by

$$A(n) = 10\,\log_{10}\frac{\frac{1}{N_a}\sum_{i=1}^{N_a} d^2(n-i)}{\frac{1}{N_a}\sum_{i=1}^{N_a} e^2(n-i)}. \qquad (9)$$

For a clearer illustration the smoothing has been chosen to $N_a = 1024$ for all following investigations. We have chosen bandpass filtered WGN (300 − 1000 Hz) for the outer disturbance $x(n)$, to simulate a realistic noise scenario. We are scaling the energy of the excitation signal $v(n)$ according to a predefined signal-to-noise ratio from the perspective of the SPE as

$$\mathrm{SNR}_{vd} = 10\,\log_{10}\frac{\sum_i v^2(i)}{\sum_i d^2(i)}. \qquad (10)$$

As the NLMS algorithm needs some initialization, we start the adaptation w.r.t. the repeatability of the experiments with a disturbed version of the true (measured) impulse response of the secondary path. This is realized by additive WGN disturbance according to a system distance of −5 dB.

First we are setting $\mathrm{SNR}_{vd} = -10$ dB, which means the excitation signal is 10 dB below the inner disturbance. Fig. 6 shows the attenuation by the ANC system in the upper and the system error of the SPE in the lower plot. As a reference the ANC performance with an optimal SPE without an excitation signal is shown in the upper plot ($\underline{\hat{g}}(n) = \underline{g}(n)$ and $v(n) = 0$). The attenuation saturates at $\mathrm{SNR}_{vd} = -10$ dB for WGN and the perfect

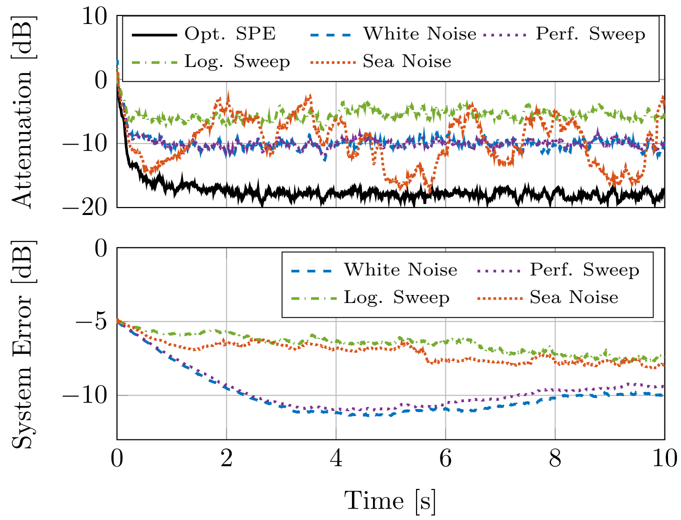

Figure 6: Results for secondary path estimation under realistic conditions with Eriksson's approach with $\text{SNR}_{vd} = -10$ dB ($\mu_{\hat{g}} = 0.005$, $\mu_w = 0.05$).

sweep. The ANC performance is limited by the additional excitation signal. WGN and the perfect sweep are showing similar performance in attenuation and system error. The logarithmic sweep not only suffers from slow adaptation concerning the SPE, but the ANC performance is also degraded. The sea noise signal also exhibits slow SPE adaptation, however, the ANC performance is less degraded. The attenuation reveals the amplitude modulation of the sea noise signal.

We are now reducing the power of the excitation signal to $\text{SNR}_{vd} = -20$ dB. The results are shown in Fig. 7. The

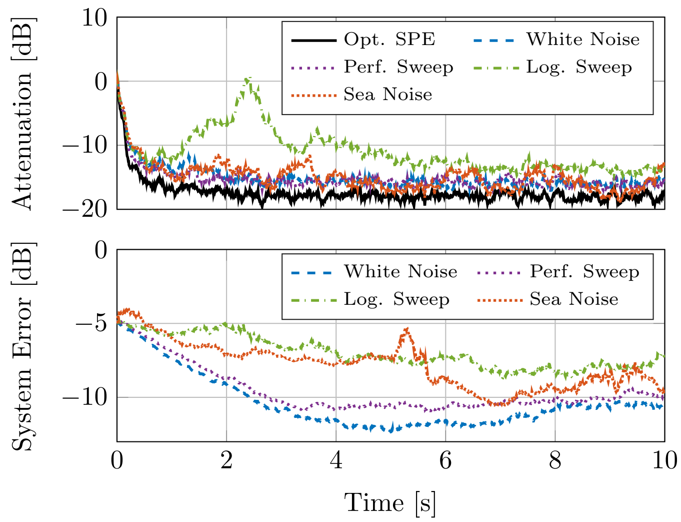

Figure 7: Results for secondary path estimation under realistic conditions with Eriksson's approach with $\text{SNR}_{vd} = -20$ dB ($\mu_{\hat{g}} = 0.005$, $\mu_w = 0.05$).

ANC system can profit from the reduced interference by the SPE and may almost reach the optimal performance ($\underline{\hat{g}}(n) = \underline{g}(n)$, $v(n) = 0$) with the SPE. Surprisingly, the system error does not deteriorate even though the energy of the excitation signal is reduced. The performance is roughly equal. Here the SPE profits from the better ANC performance following Eq. 3 ($d(n) - \underline{g}(n) * \underline{y}(n)$ gets smaller).

Conclusion

The influence of different excitation signals on online secondary path estimation has been investigated. We considered the approach of Eriksson [1] within an adaptive feedforward ANC system, to demonstrate the principle limitations. Signals that have been further investigated are the common WGN, perfect sweeps [7] as optimal adaptation signals, logarithmic sweeps [8] as a common acoustic measurement signals and sea noise, as an example for subjectively less disturbing signals.

For the convergence speed of the adaptive normalized least-mean-squares (NLMS) filter, we considered the periodic autocorrelation function of the excitation signal [9]. In idealized scenarios the perfect sweep may exploit its optimal autocorrelation properties. However, the examinations have shown that its performance is limited by the stepsize $\mu_{\hat{g}}$ under realistic circumstances. The sea noise signal still showed an adequate adaption for the SPE and only minor interference with the ANC system. We thereby raise the question, if more weight should be put on psychoacoustic characterstics for audible excitation signals. It would for instance be possible to utilize sound design for creating a subjectively pleasant measurement signal for quick calibration, e.g., using a welcoming signal.

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