Directivity as monopole decomposition for active noise reduction

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The framework

A noise reduction framework is introduced (as given in Figure 1) for low frequency directive sources. It is based on the decomposition of the source into weighted and spatially spread monopoles, which are used to calculate the sound field in the area where the noise needs to be reduced including the reflection of the ground and other objects by using a mirror image source model. This resulting sound field in the reduction area is taken as input to inversely calculate the driving functions of a loudspeaker array that generates the anti-noise sound field.

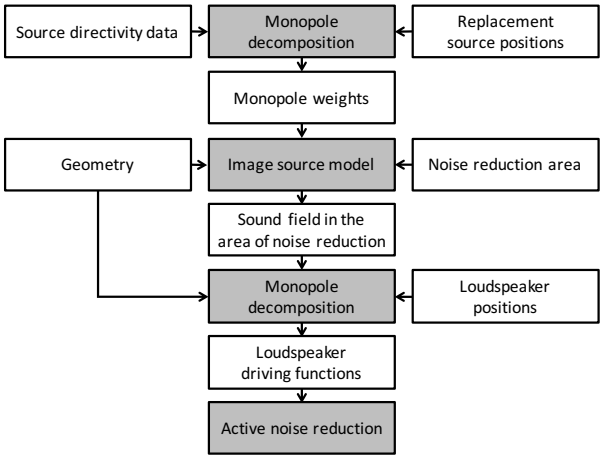

Figure 1: Flow chart of the noise reduction framework data flow as presented in this paper. The boxes with a grey background represent actions by algorithms on the provided or generated output data of the boxes with a white background.

Monopole decomposition

The principle of the monopole decomposition is to reproduce the sound field of a source that is sampled at a certain number of receiver points with a finite set of spatially spread monopoles [1, 2, 3].

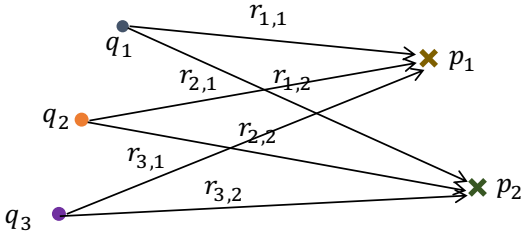

Figure 2: A representation of a monopole synthesis of a sound field measured at two receiver points (p_1, p_2), synthesized by three monopoles (q_1-q_3).

As input the number and position of the monopoles is chosen. The transfer paths between the monopoles q_i and the receiver points p_i are described by the free field Green's function and can be formulated by the matrix representation:

$$\underline{\boldsymbol{p}}_{\mathrm{P}} = \underline{\underline{\boldsymbol{C}}}\,\underline{\boldsymbol{q}}_{\mathrm{Q}}. \tag{1}$$

The $\underline{\underline{\boldsymbol{C}}}$-Matrix contains the Green's functions

$$\underline{\underline{\boldsymbol{C}}} = \begin{pmatrix} \frac{e^{-jkr_{1,1}}}{4\pi r_{1,1}} & \cdots & \frac{e^{-jkr_{N,1}}}{4\pi r_{N,1}} \\ \vdots & \ddots & \vdots \\ \frac{e^{-jkr_{1,M}}}{4\pi r_{1,M}} & \cdots & \frac{e^{-jkr_{N,M}}}{4\pi r_{N,M}} \end{pmatrix}. \tag{2}$$

The weighting factors q_i of the monopoles can be determined by minimizing the linear problem

$$\min_{\underline{\boldsymbol{q}}_{\mathrm{Q}}} = \left\| \underline{\underline{\boldsymbol{C}}}\,\underline{\boldsymbol{q}}_{\mathrm{Q}} - \underline{\boldsymbol{p}}_{\mathrm{P}} \right\|_2. \tag{3}$$

Using a singular value decomposition a pseudoinverse of the $\underline{\underline{\boldsymbol{C}}}$-Matrix ($\underline{\underline{\boldsymbol{C}}}^{+}$) can be determined. With a regularized least squares method the linear problem can be solved by the following matrix equation to find the weighting factors [4]:

$$\underline{\boldsymbol{q}}_{\mathrm{Q}} = \underline{\underline{\boldsymbol{C}}}^{+}\,\underline{\boldsymbol{p}}_{\mathrm{P}}. \tag{4}$$

Source description

As theoretical source, a large 3-core transformer with a 120° ($\frac{2}{3}\pi$ rad) phase difference between the cores is used, as given in Figure 3.

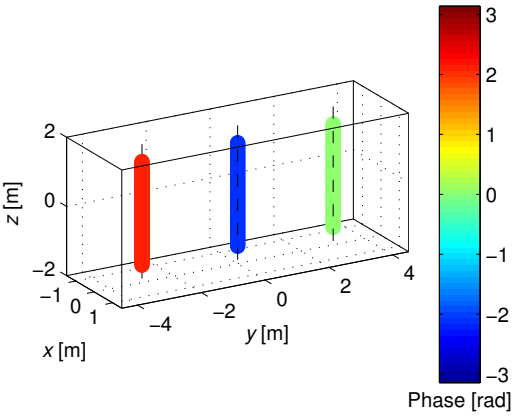

Figure 3: Theoretical transformer model with 3 cores with a 120° phase difference between them. Green: 0° (0 rad), red: 120° ($\frac{2}{3}\pi$ rad), blue: -120° ($-\frac{2}{3}\pi$ rad).

The radiated sound field from this source for 100 Hz and 800 Hz is shown in Figure 4.

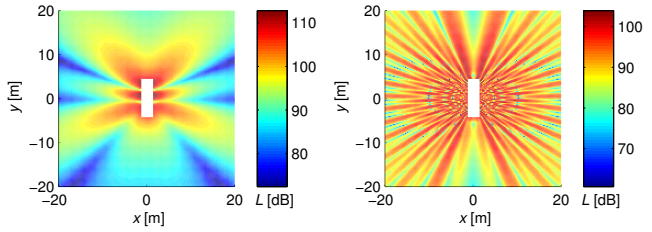

Figure 4: Radiating sound field of the theoretical transformer at a hight of 1m above the ground level for 100 Hz (left) and 800 Hz (right).

The original source will be approximated by a cloud of 2000 monopoles arranged as three double conical Archimedean spirals (10 windings) in each of the three cartesian dimensions, as shown in Figure 5. Among the several tested arrangements this is the most efficient one.

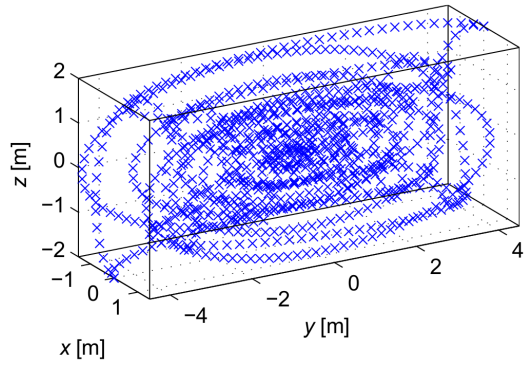

Figure 5: Three double conical Archimedean spirals with 10 windings (with their axes in the three cartesian coordinates) built up by 2000 monpoles, representing the original transformator source.

Driving function calculation

After the definition of the source it is necessary to define the noise reduction area (represented by the black dots in Figure 6) and the arrangement of the loudspeaker array. Two arrangements are compared: A 12 meter long 1D line-array (purple) and a 12x6.5 meter 2D plane-array (light blue), both with 527 loudspeakers.

The same principle as for the decomposition of the source by monopoles is used to calculate the driving functions of the loudspeakers. In the monopole decomposition the positions of the loudspeakers (purple and light blue crosses in Figure 6) are used as input and the weighting factors should be chosen, such that the same original sound field is generated by the secondary sources. The driving functions for the loudspeakers are found by inverting the phase by 180 degrees with respect to these weighting factors.

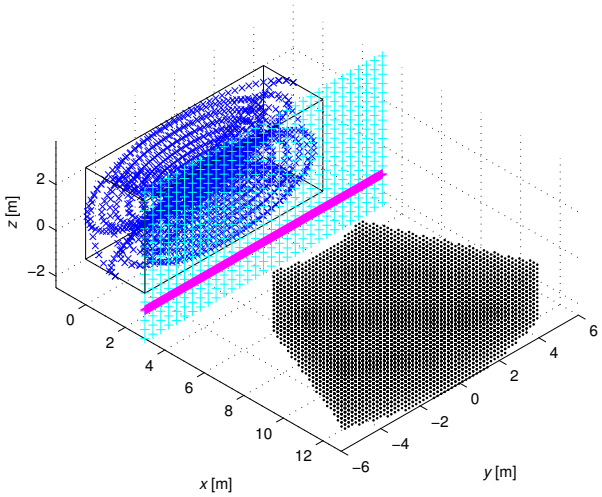

Figure 6: The modeled Active Noise Reduction (ANR) system with the area to reduce (black dots), the linear loudspeaker array (purple crosses), the planar loudspeaker array (light blue crosses) and the substitute source (rectangular cuboid with blue dots).

Results

The resulting sound field in the noise reduction area is a superposition of the noise source and the loudspeaker array. The resulting sound levels are shown in Figure 7 for this area in case of a free field radiation and the usage of a loudspeaker array as a 1D line-array and a 2D plane-array as described above. With the 1D line-array a mean noise reduction of about 10 dB is achieved and about 70 dB with the 2D plane-array at 400 Hz.

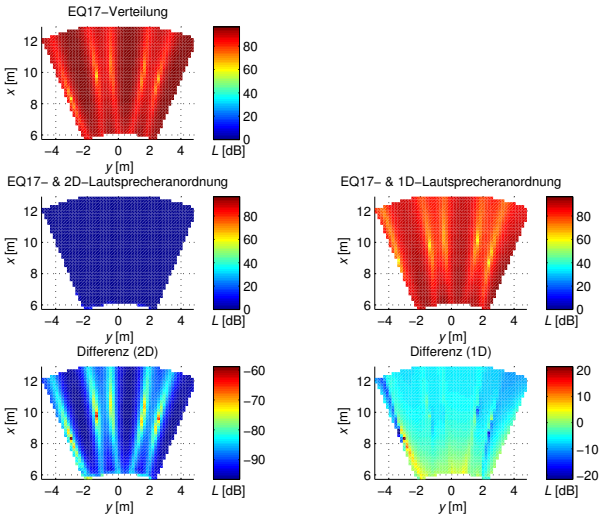

Figure 7: Resulting sound pressure levels by 400 Hz at 1m height in the noise reduction area in free field (no ground reflection) for the direct sound (ANR off) of the source (top left), the source sound field added with the secondary sound field (ANR on) by the 2D loudspeaker array (mid left), the difference between the original and reduced sound field for the 2D loudspeaker array case (lower left), the source sound field added with the secondary sound field (ANR on) by the 1D loudspeaker line-array (top right) and the difference between the original and reduced sound field for the 1D loudspeaker line-array case (lower right)

The next set of results shows the difference between two cases of loudspeaker driving functions for the 2D plane-array, with and without the ground reflection included by a first oder image source method (Figure 8). Including the ground reflection results on average in a 30 dB lower noise level at 400 Hz.

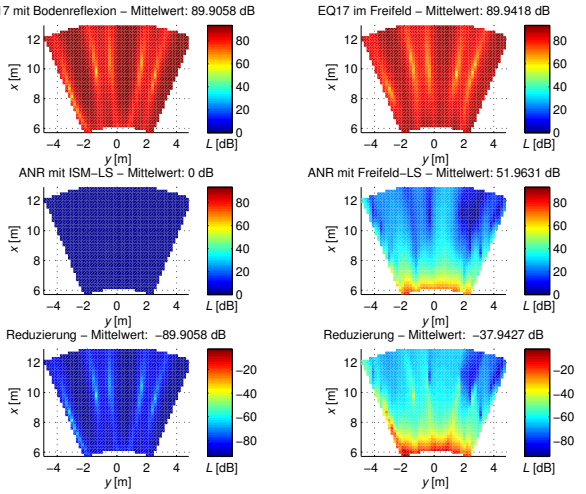

Figure 8: Resulting sound pressure levels by 400 Hz at 1m height in the noise reduction area for the direct sound in free field (top left) and with floor reflection (top right). The resulting sound pressures for the system (ANR on) with the 2D loudspeaker array with ground reflection correction for the driving functions (mid left) and without correction (mid right) and their respective differences with respect to the original source sound field (lower left and lower right).

References

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