Room-acoustic investigations of coprime linear microphone arrays

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Abstract

Coprime linear microphone arrays can obtain narrow beam patterns with significantly fewer, sparsely distributed microphones, exceeding the half wavelength limit. A coprime microphone array consists of two nested uniform linear subarrays with M and N microphones, where M and N are coprime with each other. When the subarray outputs are properly combined the two subarray outputs overlap with one another completely in just one direction retaining the shared beam while mostly canceling the other superfluous grating lobes. The narrow beampattern is on the order of that achieved using uniform linear arrays consisting of M times N microphones using delay-and-sum processing. Recently, the present authors have experimentally validated the coprime array theory [D. Bush & N. Xiang, *J. Acoust. Soc. Am.* **138**, 447-456 (2015)], confirming that broadband processing of array signals can achieve narrow beam patterns while suppressing unwanted sidelobes. After a brief introduction of coprime array theory and broadband implementation, this paper will discuss results obtained in extremely reverberant room-acoustic environments using the coprime microphone arrays. Potential applications of enhancing speech intelligibility will also be discussed.

Introduction

Microphone arrays have found many acoustics applications. Conventional microphone arrays employ a certain number of array elements through separate channels to process receiving acoustic signals in achieving desirable directional characteristics and array gain. The main goal is to enhance the spatial (directional) selectivity of the array. The array elements have to fulfil 'half-wavelength' distance governed by the spatial sampling theorem. [1, 2] If this condition cannot be met, spatial aliasing will occur in the directional characteristics, known as grating lobes [2].

This work investigates a line array design by combining two line-arrays of sparsely distributed microphone elements based on number-theoretical coprimality. So-combined corpime line arrays will achieve superdirectivity without significant grating lobes. The performance of coprime microphone arrays will be investigated both in anechoic and reverberant sound field. Achieved steerable superdirectivity of the coprime arrays hints at wide potential applications, including acoustic holographic techniques and enhancing speech intelligibility highly critical in communication acoustics, to name a few. This paper heavily replies on most recent publications, [3, 4, 5] and demonstrates the efficacy and efficiency of the coprime array technique in room-acoustic environments. The coprime array technique can conceivably be applied in enhancing the speech intelligibility as often needed in teleconferencing applications.

Coprime Line Arrays

Figure (Abbildung) 1 illustrates design concept and the theoretical directional characteristics of the coprime arrays nested in a line array arrangement. For convinient comparison, Fig. 1 (a) also illustrates a conventional line array. This work uses integer numbers M and N which are prime to each other to construct two line subarrays. Given an array length L, the conventional line array contains $M \cdot N$ microphones in meeting the Nyquist spatial sampling theorem on microphone distance $d \leq \lambda/2$, with λ being the design wavelength assocaited with the design frequency f_d [3]

$$f_d = \frac{c\, M\, N}{2\, L}, \tag{1}$$

where c is sound speed. Figure 1 (b) illustrates a line array with sparsely distributed microphones, it can be virtually viewed as nested two subarrays, separately illustrated in Fig. 1 (c) and (d). The subarray as shown in Fig. 1 (c), termed M-subarray, contains M microphone elements. Given the array length, it can only offer the element separation to be $N \cdot \lambda/2$. Its corresponding directional characteristics, at the design frequency, exhibit N grating lobes as shown in Fig. 1 (e), since it breaks the theoretical limit N times.[4] Figure 1 (d) shows N-subarray with element separation of microphone of $M \cdot \lambda/2$, it yields directional characteristics with M grating lobes (see Fig. 1 (e) as well). Note that due to coprimality of M and N, their grating lobes completely overlap only at one angular passband. When nesting these two subarrays into a line array as shown in Fig. 1 (b) with the first element shared by two subarrays, and processing their output signals in a combined way, the resulting line array (shown in Fig. 1 (b)) with sparely distributed $M + N - 1$ microphones yields directional characteristics as shown in Fig. 1 (f). Other than two small side lobes at the design frequency, the directional characteristics achieve the spatial selectivity which can only be obtained by a conventional line array of $M \cdot N$ microphones.

The directional characteristics of M-subarray $H_M(z^N)$

$$H_M(z^N) = \sum_{m=0}^{M-1} h(m) z^{-Nm}, \tag{2}$$

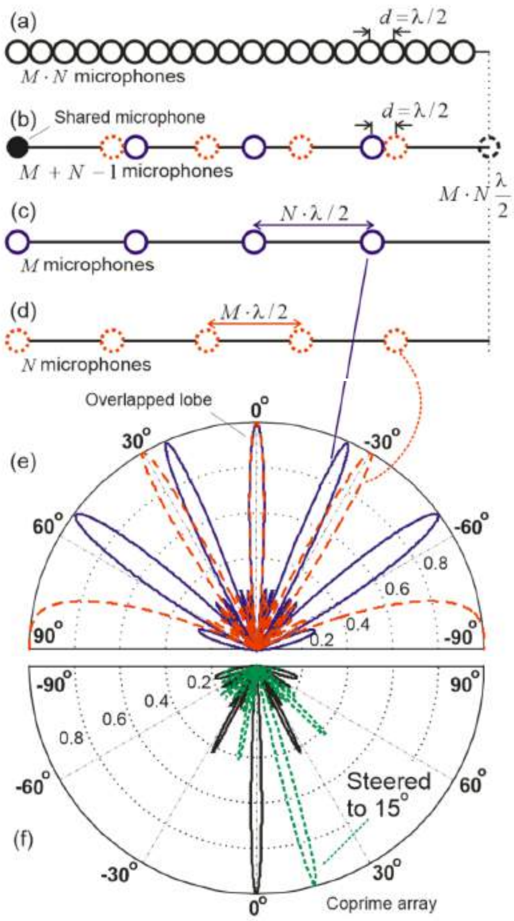

Abbildung 1: Conventional line microphone array and coprime microphone arrays with coprime numbers M and N elements. (a) Conventional microphone array containing $M \cdot N$ microphones fulfilling the half wavelength limit. (b) Coprime array combined from subarray (c) and (d) with total $M+N-1$ microphones. (c) Sparsely distributed line array with M microphones. (d) Similar to (c), with N microphones. (e) The grating lobes of two sparsely distributed microphone arrays with coprime numbers of M and N to each other, grating lobes are entirely overlapped at only one direction. (f) Directional characteristics of the combined coprime line arrays as shown in (d) in comparison with the conventional line array as shown in (a).

as shown in Fig. 1 (c) and that of N-subarray $H_N(z^M)$

$$H_N(z^M) = \sum_{n=0}^{N-1} h(n) z^{-Mm} \qquad (3)$$

as shown in Fig. 1 (d) can be combined as

$$H_C(z) = H_M(z^N) H_N^*(z^M), \qquad (4)$$

which is illustrated in Fig. 1 (f). Equations (2) and (3) represent well-known delay-and-sum algorithm. It can be further steered by a linearly delaying phase. Figure 1 (f) also illustrates one resulting beam steered 15 degree away.

Experimental Results

In order to validate the coprime theory, this research group has carried out systematic experimental investigations [3, 4]. Figure (Abbildung) 2 shows a photo of a one-meter long coprime array (with M =9, N =8) on a protractor so as to conduct experimental measurements in far field at one-degree angular resolution. In the following all the experimental results will be discussed in this resolution.

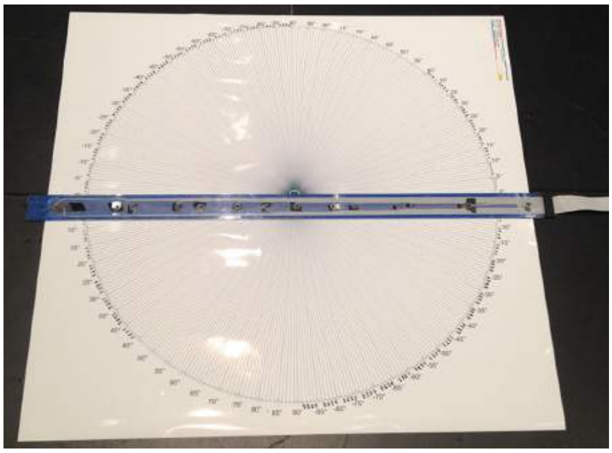

Abbildung 2: A one-meter long line coprime microphone array with $M = 9$ and $N = 8$ microphone elements on a protractor for experimental investigations. Far field measurements can be accomplished with 1-degree angular resolution.

Single frequency processing

For ease of comparison, Fig. (Abb.) 3 shows directional characteristics of a 1-meter long coprime (nested) line array consisting of total 16 microphones with $M = 9$ and $N = 8$. Its design frequency is determined using eq.(1) to be f_c =12.35 kHz. Fig. 3 (a) illustrates theoretical predicted directional characteristics of the two subarray, where M-subarray predicted using eq.(2) presents 8 grating lobes because the microphone distance is $8 \cdot \lambda/2$, while N-subarray predicted using eq.(3) presents 9 grating lobes due to the microphone distance $9 \cdot \lambda/2$. Due to coprimality, the grating lobes overlap in only one position, so that the combined directional characteristics from the two subarrays according to eq.(4) demonstrates a single, sharp main lobe. Its beam width can only be achieved when using a conventional line array with 72 microphones (channels)

Figure (Abbildung) 3 also shows directional characteristics of a 1-meter long coprime (nested) line array measured in far field with a source 12.25 m away from the coprime microphone array, the measurements were carried out at 1-degree angular resolution in anechoic environment. At the design frequency, all grating lobes processed from individual subarrays as shown in Fig. 3 (c) correspond to the theoretical predicated ones. Since they are not significantly overlapped, expect in one postion/direction, the combination of the two subarrays according to eq.(4) exhibits a narrow beam. There are two side-lobes both in predicted (Fig. 3 (b)) and experimentally measured results (Fig. 3 (d)) which may be slightly larger than the ones from conventional line arrays. However this is the array processing within a narrow frequency range centered around the design frequency. The authors' team has

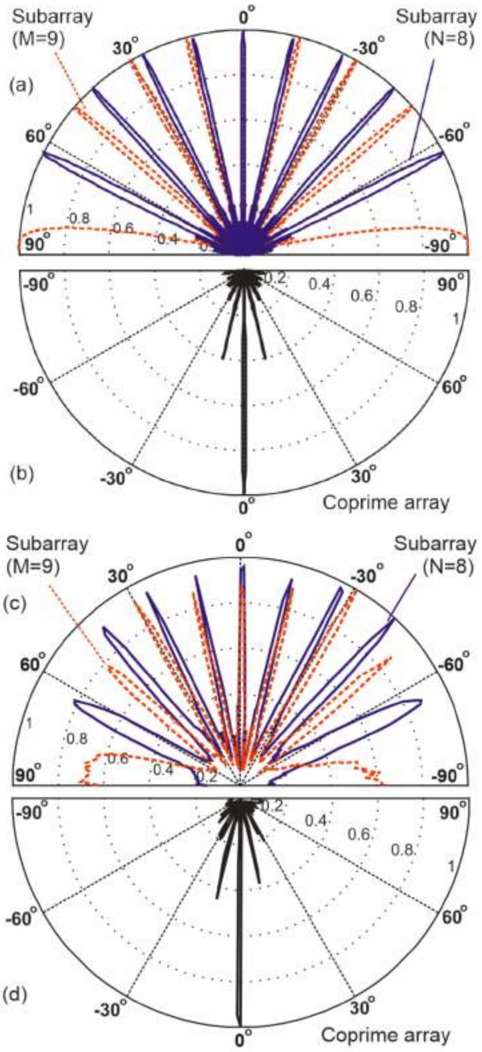

Abbildung 3: Theoretically predicted and epxerimentally measured directional characteristics of a coprime microphone array with $M = 9$ and $N = 8$ microphone elements. (a) Directional characteristics of two subarrays, predicted using eqs.(2) and (3). (b) Directional characteristics when combining the two subarrays using eq.(4). (c) Experimentally measured results when processing M-subarray and N-subarray separately. (d) Directional characteristics of the combined result from experimentally measured results (c) using eq.(4).

furthered the simulation and experimental investigations to process broadband data [4].

Broadband processing

Figure (Abbildung) 4 shows both theoretical prediction and experimental results processed over one-third octave band around the design frequency for the corpime 9 by 8. The simulation is carried out first in single frequencies , then averaged over the same number of frequency bins as those of measured data over the same frequency band. The averaging effort of beam-forming over certain bandwith can effectively reduce the side-lobes. As pointed out in Ref. [4], the grating lobes at each single frequency will change their angles, yet the coprimality still make the varied grating lobes largely not-overlapped except in one position/direction, so the averaging effectively

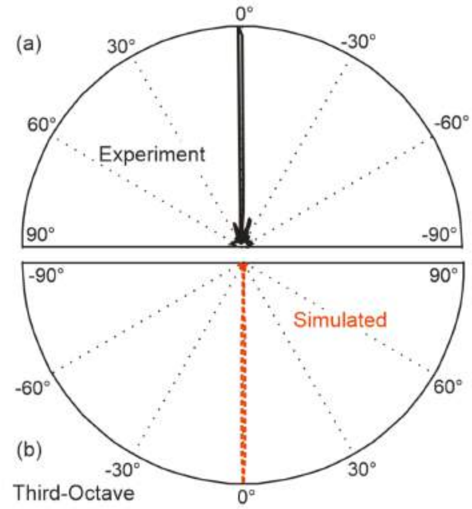

Abbildung 4: Experimental directional characteristics of the coprime microphone array with $M = 9$ and $N = 8$ microphone elements. Processed over 3rd-octave band around the design frequency. (a) Experimental result. (b) Theoretical prediction.

reduces the side-lobes when combing the two subarrays. The wider the bandwith, the stronger the side-lobe reduction will be. [4]. Unlike the conventional line arrays, the corpime microphone array will have insignificant side lobes over a certain frequency bandwidth. This result is of practical significance for many real-life applications.

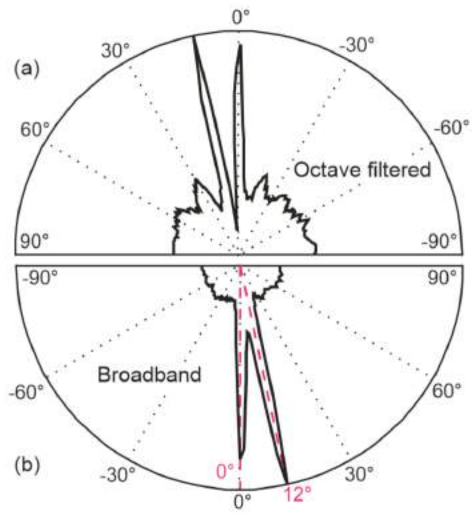

Abbildung 5: Experimental beam-forming results of the coprime microphone array with $M = 9$ and $N = 8$ microphone elements. The measurement was carried out in a reverberant enclosure with two concurrent sound sources. (a) One octave band experimental result. (b) Broadband experimental result.

The directional characteristics of coprime microphone arrays derived from experimental investigation discussed so far substantiated the coprime theory which can be exploited to design sparsely distributed microphone arrays for highly selective sensing of sound events. [5] The experiments discussed above were all carried out in simulated free-field environment by gating out only the direct sound portion of the measurements.

Room-acoustic investigation

A group of experimental results carried out in a reverberant enclosure (L: 24.5 m, W: 7.6 m, H: 4.5 m) will be discussed. The mid-frequency reverberation times range between 1.93 s and 2.0 s. Such a reverberant enclosure is particularly unsuitable to obtain satisfied speech intelligibility. Using traditional teleconferencing devices, e.g. the speech communication will strongly suffer from the degradation. A sparse sensing technique based on the coprime microhpne arrays discussed above will enhance the speech transmission significantly, high angular selectivity of the corpime microphone array when pointed at a speech source of interest will strongly suppress undesirable reverberation, receive only the speech signals at specific narrow angular range. Figure (Abbildung) 5 illustrates the measurement result in this extremely reverberant enclosure. In order to test this angular selectivity in reverberant enclosures, Fig.5 (a) shows beam-forming results processed over one octave band when two concurrent sound sources are at 0 degree and 12 degree incident directions, while Fig.5 (b) shows the results processed broadbandly. Both results indicate that the broadband processing will suppress possible side-lobes. The broader the bandwidth, the stronger the side-lobe suppression. The sharp angular selectivity obtained from a 9 by 8 corpime microphone array containing total of 16 microphones can resolve concurrent sound sources with a high angular resolution of $4/(M\,N)$. [3]. The excessive reverberation in the enclosed space will slightly increase the noise level outside the two main signal lobes. The broader the bandwidth is, the lower the side-lobe noise will become.

Summarying Remarks

Number-theoretical coprimality can be exploited for designing sparsely distributed microphones in a line array arrangement to achieve highly directional selectivity. The channel processing can be significantly reduced from conventional line array of $M \cdot N$ microphone channels down to $M + N - 1$ channels with comparable angular selectivity when virtually grouping the sparsely deployed microphones in two subarrays, each with corpime number of M and N microphone elements. The channel processing saving arrives at maximum when the two coprime numbers M and N can be selected as close together as possible. This work uses $M = 9$ and $N = 8$ to demonstrate its effectiveness in terms of coprime-theory predictions and experimentally measured results. Using sparsely distributed microphone coprime arrays, a superdirectional characteristics of the coprime microphone arrays will find broad applications, including teleconferencing technique to enhance speech transmission index/intelligibility in excessive reverberant enclosures.

Future research effort includes retaining speech signals using the coprime microphone arrays for distortion-free speech processing for real-time applications. Another possible direction of research will be detection and localization of multiple concurrent speech sources when the number of sources and their incident directions are unknown prior to associated processing. Research attention should also be given to near field deployment of coprime arrays with adoptive algorithms where the source distance is also unknown prior to the processing.

Literatur

[1] Blauert, J. and Xiang, N.: *Acoustics for Engineers.* 2^{nd} Ed., Springer Verlag, Berlin, Heidelberg, 2009

[2] Bai, M. R., Ih, J.-G. and Benesty, J.: *Acoustic Array Systems: Theory, Implementation, and Application.* John Wiley & Sons, Singapore, 2013

[3] Xiang, N., Bush, D. and Summers, J.: Experimental validation of a coprime linear microphone array for high-resolution direction-of-arrival measurements, *J. Acoust. Soc. Am. Express Letters*, **137** (2015), EL261-EL266

[4] Bush, D. and Xiang, N.: Broadband implementation of coprime linear microphone arrays for direction of arrival estimation *J. Acoust. Soc. Am.* **138** (2015), 447- 456

[5] Vaidyanathan, P. P. and Pal, P.: Sparse sensing with co-prime samplers and arrays, *IEEE Trans. Signal Process.* **59** (2011), 573–586