A compact physics-based model of the piano
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Introduction
This paper introduces a physical parameter-based piano model, where the main components of a piano as hammers, strings and the soundboard are modelled independently. All effects of parameter manipulation are observable in in the output signal of each subsystem, and finally in the radiated sound.

In general the main purpose of modelling can be twofold. In the first case the main output of the system is a virtual instrument. This makes it possible to reserve or recreate the sound of historical instruments and create numerous new instruments for musical experimentation. In the other case the virtual instrument is only an intermediate step towards creating new instruments. This means that the model could also play an important role in improving and perfectionating the instruments.

Our developed system is mainly considered as an educational and research tool, which allows examination and thorough comparison of different modelling approaches and parameter sets available from the literature. The modular setup makes it possible to easily modify or replace different subsystems of the whole instrument.

In this paper we describe possible modelling techniques of the piano and briefly introduce our recent modelling results.

Figure 1: Subsystems of implemented piano model and their connections.

Hammer-felt modelling
There are 88 notes in a conventional piano. A hammer belongs to each note which has a wooden core covered by 3-4 layers of felt. We do not deal with the hammer action, so the model input is the initial hammer velocity and the outputs are the hammer displacement, force and the actual felt compression.

The piano hammer is generally modelled as a mass-spring system in the literature. To describe the behaviour of the spring (felt) there are several possibilities. Among different models for spring behaviour the simplest one is the linear force model (s.a. eq. 1) where \( F_f \) is the hammer force, \( K_f \) is the felt stiffness and \( u_f \) is the felt compression. [1]

\[
F_f = K_f \cdot u_f^p \quad [N] \quad (2)
\]

Those models match best with the reality, which take into consideration, that the hammer felt shows hysteretic behaviour according to measurements. The first hysteretic closed formula was given by Stulov. [3] In his model the felt force is determined by two further parameters, the hysteresis constant (\( \varepsilon \)) and the time constant (\( \tau_0 \)). (s.a. eq. 3)

\[
F_f = K_f \left[ u_f^p(t) - \frac{\varepsilon}{\tau_0} \int_0^t u_f^p(\xi) e^{(t-\tau)/\tau_0} d\xi \right] \quad [N] \quad (3)
\]

Rocchesso and Avanzini describe the hysteretic behaviour using the felt loss coefficient (\( \beta \)) and the compression velocity. [4] (s.a. eq. 4)

\[
F_f = K_f \cdot u_f^p \left[ 1 + \beta \frac{du_f^p}{dt} \right] \quad [N] \quad (4)
\]

Later Stulov simplified his model. [5] His three-parameter model has the same form as the results by Brenon and Boutillon. [6] (s.a. eq. 5)

\[
F_f = K_0 \left[ u_f^p + \alpha \frac{du_f^p}{dt} \right] \quad [N] \quad (5)
\]

\[
K_0 = K_f (1 - \varepsilon)
\]

\[
\alpha = \frac{\tau_0}{(1 - \varepsilon)}
\]

We ran several simulations to compare these models. To have stable simulation data, the time resolution of hammer model should be very fine (\( 10^{-7} \)s). In our example simulation the initial velocity was chosen as 1 m/s, the hammer mass 8 g. For the felt the hysteretic model by Rocchesso and Avanzini was applied with parameters \( 3.2 \cdot 10^8 \) N/m\(^p\) for stiffness, 2.3 for nonlinearity exponent and 0.69 for loss coefficient. The hammer strikes the string at its quarter point and a second string was vibrated freely. In this simulation case the hysteretic behaviour of the felt is shown clearly. (s.a. fig. 3), and the resulting force function follows the expectations based on measurements published in the literature. (s.a. fig. 2)
String modelling

Nowadays the piano strings are made from drawn music wire. There are 1, 2 or 3 strings per each note. To model the numerous strings we implemented a multi-string subsystem. The model receives the output force of the hammer model as input and delivers the string displacements and the termination forces (bridge force) as outputs.

The modelling approach is to solve the wave equation numerically. To accomplish this task, we implemented a digital waveguide model. Digital waveguide models use the discretized form of d’Alambert’s solution (s.a. eq. 6), which describes the travelling waves as a sum of two waves moving in opposite direction.

\[ u(x,t) = u^+(ct-x) + u^-(ct+x) \quad [m] \quad (6) \]

All losses and reflections from string terminations are incorporated into a termination filter [7]. As the classical waveguide model behaves inappropriately if hammer-string interaction needs to be accounted for, Bank’s improved waveguide model has been implemented which uses an alternative force-input method. [8]. (s.a. Figure 4)

\[ V_{\text{out}} = C_{\text{out}} \cdot \left[ C_{\text{in}} \cdot V_{\text{in}} + z^{-1} \cdot Z_s \cdot (V_{\text{in}} - V_{\text{out}}) \right] \quad [m/s] \quad (7) \]

\[ C_{\text{in}} = Y_1 \cdot Z_s - I \]

\[ C_{\text{out}} = (Y_1 \cdot Z_s + I)^{-1} \]

The block diagram of the implemented multi-string system with \( N \) strings and the IIR-filter set is shown in Figure 5.

The coupled motion of the two strings in a chosen position is shown in Figure 7.
Soundboard modelling

The piano soundboard is a wooden resonator with ribs on one side and bridges on the other one. It has complex geometry and material properties, which vary from instrument to instrument.

The role of the soundboard in our piano model is twofold. Firstly, we use its modal description to parametrise the IIR filter set on string terminations (terminator). Secondly we calculate the sound radiation model from its motion (radiator). In the radiator case the soundboard model gets the string force on the termination as input and provides the soundboard displacement as output.

The soundboard is modelled by means of the FEM solving the inhomogeneous plate and beam equations. The plate is split up into triangular elements, while the ribs are modelled using beam elements. The wood is modelled as a transversally isotropic or orthotropic material. For FEM modelling, we also implemented our own solution.

In our example the soundboard has a shape as a modern grand piano. Its thickness is 1 cm. On the soundboard there are 13 ribs placed similar as on a real soundboard. The material damping of the soundboard has been assumed to take the constant value of 1 %. The soundboard edges are modelled as clamped. The material properties of the used transversally isotropic material are as generally used for pianos simulations: density is 392 kg/m³, Poisson-numbers are 0.3, Young-moduli are 11.5 GPa and 0.47 GPa, and shear moduli are 500 MPa and 42 MPa.

The first 186 soundboard modes were used for modelling, so the IIR-filter set contains the same number of filters. The calculated first eigenfrequency is at 32 Hz and the highest used at 2598 Hz.

Sound radiation modelling

We handle the sound radiation as an independent subsystem. The model inputs are the forces on string terminations and the modal description of the soundboard. We calculate as model outputs the sound pressure in selected room positions. This data represents the radiated sound which can be heard.

The sound radiation is modelled using the Rayleigh-integral. in the frequency domain. This modelling approach assumes that the soundboard is embedded in an infinite stiff plate. This hypothesis is valid only in nearfield. Although the model is very abstract, it has the advantage of small computational cost. The sound radiation model is implemented using a FIR-filter set, where the filter coefficients are obtained by inverse Fourier transforming the velocity-pressure transfer functions obtained from the Rayleigh integral.

In our example case, the observation point was placed 50 cm above the termination of the first string. (s.a. Figure 10.)
Conclusion
We implemented a complete piano model. Our system – implemented in MATLAB – is able to run on a desktop PC, enables to examine different parameter sets of a physical-model based piano. As a next step we would like to optimize the running time, simulate other parameter sets and implement some other models (e.g. for sound radiation). In the future we would like to validate the simulation results with own measurement data.

Literature