FEM/FMBEM Modeling for a Coupled Acoustic Fluid-structure System with Damping Material

Nian Yang[1], Leilei Chen[2], Kheirollah Sepahvand[3], Hong Yi[1], Steffen Marburg[3]

[1] *Key Laboratory of Ocean Engineering, Shanghai Jiao Tong University, 200240 Shanghai, China E-Mail: yangnian@sjtu.edu.cn*

[2] *College of Civil Engineering, 464000 Xinyang, China E-Mail: chenleilei@mail.ustc.edu.cn*

[3] *Institute of Vibroacoustics of Vehicles and Machines, Department of Mechanical Engineering, Technical University of Munich E-Mail: k.sepahvand@tum.de*

Introduction

Passive noise control draws more and more attentions for designers. This structural-acoustic optimization shows high potential in minimization of radiated noise. An elaborate overview on developments in structural-acoustic optimization for passive noise control was presented by Marburg[1].

When light structures are immersed in the heavy fluid, the fluid influence has to be considered since it changes structure's vibrational and acoustical properties. However, most applications of fully coupled structural acoustic problems are time consuming. Accordingly, numerous methods have been developed to make this analysis as efficient as possible[2]. The Finite Element Method (FEM) is frequently used to model structural parts because of its high flexibility and applicability to large–scale complex problems. The Boundary Element Method (BEM) is frequently preferred to model the fluid domain since it avoids meshing the fluid domain. The coupled finite element/boundary element (FE/BE) approach has been used in many applications (e.g.[3, 4, 5, 6]). However, conventional BEM is known to be computationally costly because the system matrix is dense and non-symmetric. Fast BEMs, such as the Fast Multipole Boundary Element Method (FMBEM)[7, 8] and the adaptive cross approximation technique[9, 10], have been applied to accelerate the solution. Therefore, the coupling algorithm based on the FE/fast BE method can be effectively applied to solve large–scale fluid–structure interaction problems[11, 12, 13].

Adding damping material is the most common way to reduce structures' vibration and noise. Recently, FE-based commercial codes, such as ANSYS, ABAQUS, NASTRAN, etc., have been available to model the structure with damping material. Though it is convenient to model complex structural with general boundary conditions by FEM, the mesh has to be very dense to satisfy the requirement of element aspect ratio because of small thicknesses of viscoelastic and constraining layers, which results in extremely large model and very expensive computation, especially in optimization problems. Some efforts have been done to overcome the weakness of conventional FEM, such as Galerkin Element Method (GEM)[14], interface Finite Element Method (IFEM)[15] and so on.

In this paper, the structure with damping material is modeled by IFEM, and the coupling between the structure and the fluid is still considered. FMBEM is applied in the fluid part. In the end, some examples are given to validate the IFEM and FMBEM model.

IFEM modeling

Figure 1(a) shows an element with damping material. *h1*, *h2*, and *t* are the thicknesses of the base plate, constraining layer and viscoelastic layer, respectively. In conventional FE model, the base layer and constraining layer are modeled with shell elements and the viscoelastic layer is modeled with solid elements. The coupling between the shell elements and the solid elements are handled by kinematic constraints. The procedure of the solution is very time consuming. However, for IFEM, the viscoelastic layer is modeled using a special interface element with eight nodes (shown in Fig. 1(b)) that can couples the two stiff layers together directly, which means the dynamic equations don't have to modified and we can add the damping material on the original structure directly. It can help to reduce the modeling cost dramatically, which is very attractive to the large scale structure's optimization problem.

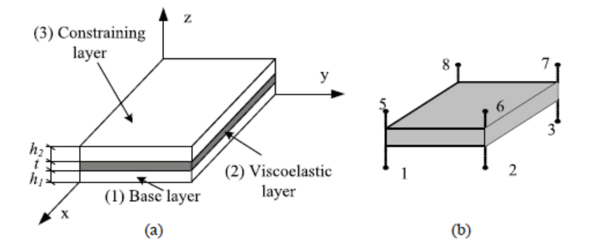

Figure 1: (a) element with damping material; (b) the IFE for the damping material

For IFEM, the modeling of base layer and constraining layer are as the same as conventional FEM plate modeling. But the damping material is constructed by the nodes in the locations as shown in Fig. 1(b), rather than at the corner of the solid element. The complex constant shear modulus model is used here:

$$G_v^* = G_v\left(1 + i\eta_v\right) \quad (1)$$

Where G_v is the storage modulus and η_v is the material loss factor. All the displacements are small compared to the structural dimensions thus linear theories of elasticity, viscoelasticity are adopted. Then the displacement field of the damping material can be expressed by the displacement field of its surfaces by linear interpolation as:

$$u^{(dm)} = [T_1 \quad T_2]\{u_b^{(dm)} \quad u_t^{(dm)}\}^T \tag{2}$$

Where:

$$T_1 = \begin{bmatrix} \frac{1}{2}-\frac{z}{t} & 0 & 0 \\ 0 & \frac{1}{2}-\frac{z}{t} & 0 \\ 0 & 0 & \frac{1}{2}-\frac{z}{t} \end{bmatrix}$$

$$T_2 = \begin{bmatrix} \frac{1}{2}+\frac{z}{t} & 0 & 0 \\ 0 & \frac{1}{2}+\frac{z}{t} & 0 \\ 0 & 0 & \frac{1}{2}+\frac{z}{t} \end{bmatrix}$$

And $u_b^{(dm)}$, $u_t^{(dm)}$ are the bottom and top surface displacement of damping material, respectively. We assume no slip happens at the interfaces of the layers. So:

$$\{u_b^{(dm)} \quad u_t^{(dm)}\} = \{u_t^{(bl)} \quad u_b^{(cl)}\} \tag{3}$$

Where $u_t^{(bl)}$ and $u_b^{(cl)}$ are the top surface displacement of base layer and bottom surface displacement of constraining layer, respectively. Therefore, in this way the damping material displacement can also be represend by shell element nodes:

$$u^{(dm)} = [N_v]\{u_e^{(bl)} \quad u_e^{(cl)}\}^T \tag{4}$$

In the end, By applying the Hamilton principle, the mass matrix and stiffness matrix of the element can be readily obtained. By using virtual displacement principle, the dynamic equation can be constructed.

Coupling with FMBEM

We do not consider the incident wave here. So the control equations for the structure domain by FEM and fluid domain by BEM are:

$$Au = f_s + C_{sf}\,p \tag{5}$$

$$Hp = Gv_f \tag{6}$$

Where A is the structural matrix, f_s is the excitation vector, H and G are the BEM matrix, C_{sf} is the transform matrix, p is the fluid pressure vector and u is displacement vector. Based on the interface continuity condition, the general equation can be obtained by link the above two equations together:

$$\begin{bmatrix} A & -C_{sf} \\ -GS^{-1}C_{fs} & H \end{bmatrix}\begin{Bmatrix} u \\ p \end{Bmatrix} = \begin{Bmatrix} f_s \\ 0 \end{Bmatrix} \tag{7}$$

It can be reduced into:

$$(H - GY_c)p = GS^{-1}C_{fs}A^{-1}f_s \tag{8}$$

Where $Y_c = S^{-1}C_{fs}A^{-1}$.

The iterative solver GMRES and FMBEM can be used to accelerate the eq. (8) solving.

For the right hand side, S^{-1} is easy to obtain when discontinues boundary element is applied, because S is diagonal. $A^{-1}f_s$ can be obtained by solving $Ax=f_s$. For each GMRES iteration, A and f_s are unchanged, so this equation only needs to be solved once. For the left hand side, $A^{-1}C_{sf}p(k)$ can be obtained by solving $Ay=C_{sf}p(k)$. However, for each GMRES iteration, p(k) is different, so this equation needs to be solved during each iteration, and it takes most part of the computation cost. FMBEM can be applied to accelerate the matrix-vector product Hp(k) and $G(Y_c p(k))$. So in this way, the general fluid structure coupling problem solving can be accelerated.

Numerical examples

The IFEM model is validated by a simply supported plate with damping material. The plate's length is 0.348m, the width is 0.3048m, the base layer and constraining layer are both 0.762mm and possess the same physical parameters Young modulus 68.9GPa, Poisson ratio is 0.3, density 2740kg/m^3. The damping material thickness is 0.254mm, shear modulus is 0.896MPa, Poisson ratio is 0.49, density is 999kg/m^3 and the material loss factor is 0.5. The natural frequency results obtained by IFEM and analytical solution are compared as follows:

Table 1: The comparison of natural frequency and loss factor

Mode	Analytical solution		IFEM	
	Frequency (Hz)	Loss factor	Frequency (Hz)	Loss factor
1	60.3	0.190	60.7	0.232
2	115.4	0.203	113.5	0.213
3	130.6	0.199	128.7	0.206
4	178.7	0.181	175.4	0.187
5	195.7	0.174	193.6	0.180

From the comparison, we can see that IFEM model is validated and it can be used in the modeling for the structure with damping material without modifying original dynamic equation.

The FMBEM model is validated by a submerged sphere with point excitation. Its density is 7860 kg/m3, radius is 5m, thickness is 0.05m, Young's modulus is 210GPa, Poisson ratio is 0.3, the water density is 1000 kg/m3 and the sound velocity in the water is 1482m/s. The analytical solution for this example can be found in Junger's book[16]. The sound power comparison result can be obtained as follows:

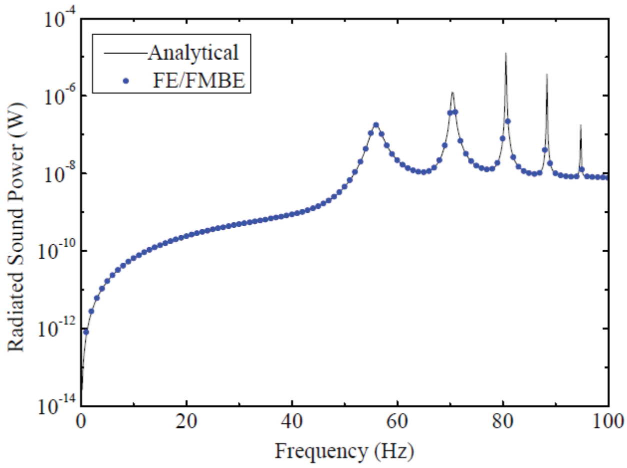

Figure 2: sound power comparison result

The results obtained from Analytical solution and FEM/FMBEM match well. So it can be validate the FEM/FMBEM modeling.

Conclusions

In this paper, the IFEM and its coupling with FMBEM are introduced. The IFEM can help to reduce the modelling cost for the structure with damping material, and its coupling with FMBEM can accelerate the fluid structure coupling system solving.

Therefore, this method can be applied in the optimization problem of the underwater large scale structure by adding damping material, which is very time consuming by the traditional methods. In the future, this modeling strategy will be connected with different optimization algorithms to find out the most efficient optimization way for the underwater structures.

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