

# Parameter Variations for a Porous Layer Simulation

Ferina Saati, Lennart Moheit, Steffen Marburg

*Lehrstuhl für Akustik mobiler Systeme, 85748 Garching b. München, E-Mail: Ferina.Saati@tum.de*

## Introduction

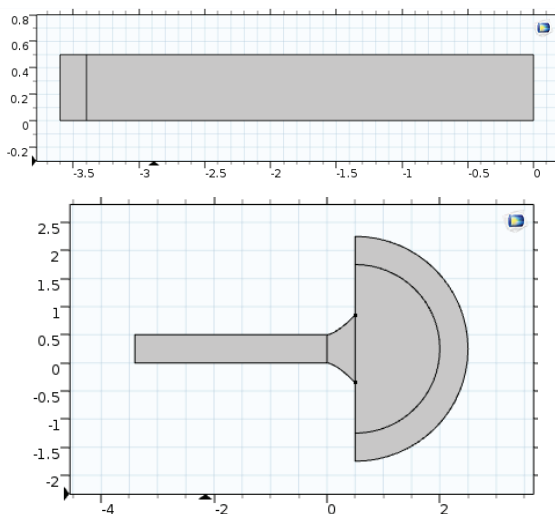
The impedance tube is a well-known test rig for measurement of absorption coefficient and impedance which provides a suitable approach for validating measurements at frequencies below cut-off frequency. An assumption is made for achieving reliable results in modeling the tube: using three particular test cases, if they work well using finite element method, more complex porous layers can be simulated and reflection coefficient  $r$ , absorption coefficient  $\alpha$  and the impedance  $Z$  can be determined. In this study, modelling a layer of porous medium in impedance tube using predefined rigid frame models and expressing dependence upon porosity and other parameters as well as the thickness of the material are described.

## Test cases

The three particular test cases are as follows:

- Sound hard boundary wall ( $r=1$ ,  $\alpha=0$  and  $Z \rightarrow \infty$ )
- Open end of the tube ( $r=0$ ,  $\alpha=1$  and  $Z=\rho c$ )
- Horn shape ( $r=-1$ ,  $\alpha=0$  and  $Z \rightarrow 0$ )

As can be seen in Figure 1, upper figure, the two first test cases are modeled as a basic academic example of a 2D rectangular with a perfectly matched layer on the left end. In the third test case (the lower figure), the perfectly matched layer is modeled in circular coordinates on the right end of the geometry after the open exit at which the wave expands and results in a negative reflection coefficient. The PML is required for the application of a background pressure field inside the tube. This background pressure field is equivalent to a structural surface velocity, but a more general formulation. Both ways are applied individually and work in the same way.



**Figure 1:** Geometry in 2D for testing the three cases: upper figure represents the geometry for the two first cases and

lower figure for the third case. Figures are derived using commercial FE software [1].

The derived results for reflection and absorption coefficient and the impedance were exactly as expected and therefore, the test cases confirm the next step for modeling the porous layer.

## Modelling a porous layer in impedance tube

The porous layer is added to the geometry at the right end with porous matrix properties set as “rigid approximation” with user-defined properties and air as fluid material. It is also possible to model the porous layer without modeling the layer but applying impedance condition with effective thickness at the right end of the tube. The poroacoustics fluid models are equivalent fluid models that mimic the behaviors of a full poroelastic material model, which is defined by Biot’s theory. A poroacoustics fluid model is based on describing the frequency-dependent effective fluid density  $\rho(\omega)$  and the effective fluid bulk modulus  $K(\omega)$  of the combined equivalent fluid-solid system using different number of parameters depending on the model [2]. One of them is open porosity, commonly known as porosity and is defined as the ratio of the air volume to the total volume of porous material. The air bubbles locked within the frame are considered filled. Starting from analytical solution by Zwikker and Kosten with rigid frame assumption for cylindrical pores, we move on to other models and later on, compare the analytical with a thermoviscous model.

The parameters involved in each of the rigid frame or limp porous models are as follows:  $H_r$  as the hydraulic radius in Delany Bazley Miki, the fully empirical model,  $R_f$  as flow resistivity,  $\tau$  as tortuosity parameter and  $b$  as fitting parameter as a measure of pore geometry in Attenborough’s model,  $K_\infty$  as bulk modulus infinite frequency limit,  $\rho_\infty$  as density infinite frequency limit,  $\tau_{ent}$  as entropy-mode relaxation time,  $\tau_{vor}$  as vorticity-mode relaxation time in Wilson model,  $L_v$  as viscous characteristic length and  $L_{th}$  as thermal characteristic length in Johnson Champoux Allard model,  $\kappa'$  as static thermal permeability in Johnson Champoux Allard Lafarge,  $\rho_{gr}$  as grain density,  $K_{gr}$  as grain bulk modulus and  $\kappa$  as permeability in Williams EDFM model. The description of these models includes the losses associated with the propagation of acoustic waves in porous materials. An equivalent fluid model is computationally less demanding than the full poroelastic model. However, it is only physically correct for certain choices of material parameters. Most poroacoustic models are only valid in the rigid or limp porous matrix approximations. In the rigid porous matrix limit, the matrix is assumed to be so stiff that it does not move. In this case it is assumed that the structural velocity is zero which yields a wave equation with complex density and complex bulk modulus. The parameters for each

of the models mentioned above are swept in the range at which the common absorber materials are found.

## Results

The range for the parameter sweep of each parameter is as follows: Porosity in 0.1 to 1.0, flow resistivity in 1000 to 10000 N s/m<sup>4</sup>, hydraulic radius in 1 to 10 mm, tortuosity in 1 to 10, permeability in m<sup>2</sup>, layer thickness in 0.1 to 1 m, fitting parameter in 0.5 to 5,  $K_\infty$  in  $3 \times 10^9$  to  $3 \times 10^{10}$  Pa,  $\rho_\infty$  in 0.5 to 5 kg/m<sup>3</sup>,  $\tau_{\text{ent}}$  in 100 to 1000,  $\tau_{\text{vor}}$  in 1 to 10,  $\kappa'$  in  $9 \times 10^{-10}$  to  $9 \times 10^{-9}$  m<sup>2</sup>, grain density in 1000 to 10000 kg/m<sup>3</sup> and grain bulk modulus in  $10^{10}$  to  $10^{11}$  Pa. Figure 2 shows a sample sweep result of absorption coefficient for one model over one parameter in frequency range up to 3000 Hz. It is clear that there is high absorption around the resonance frequencies of the pores when considered as cylindrical tubes and an abrupt drop as the porosity increases.

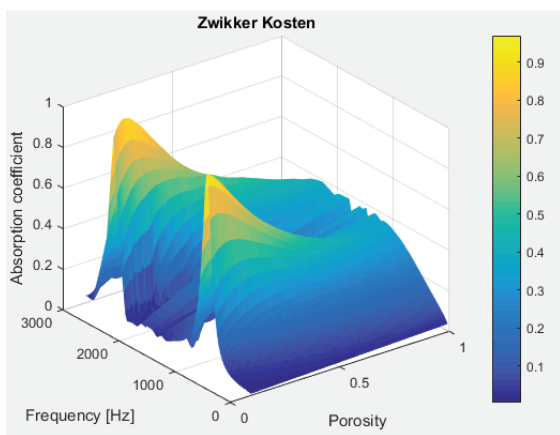


Figure 2: Porosity sweep and Zwikker Kosten.

## Outlook: Porous thermoviscous modeling

The idea to create a model with certain geometry of pores is to include the viscothermal losses in the fluid inside the pores and depending on the degree of complexity, comparing with the fluid equivalent models.

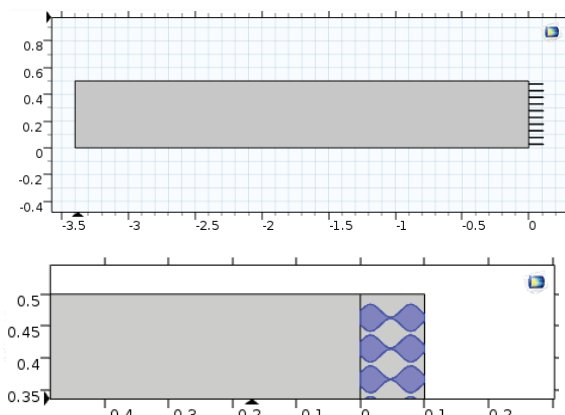


Figure 3: (Up) Thermoviscous model with cylindrical pores- without solid blocks in between (Bottom) Higher degree of complexity of pore geometries with thermoviscous modeling and structural coupling.

Figure 3 shows how the simplest pore types can be modeled. In this figure, the boundary conditions for the upper case is

sound hard boundary walls which is considered to be comparable with the rigid frame models. It could also be that the model includes the fluid structure interaction with structural mechanics and adds two pore coupling phenomena to the solution. In such a case, the results are expected to be comparable with corresponding poroelastic models. In the lower figure, a degree of complexity is added to the pores. Further work requires adding the third dimension to the models and only in such a case will the comparison make sense. As is shown in Figure 4, the absorption coefficient plot for comparing an analytical model and a viscothermal model of the same characteristics have levels of similarities. The analytical model however gives a much higher coefficient in resonance frequencies of the pores. One possible reason is that the 2D configuration is not properly representing the cylindrical tubes but rather infinite slits that has to further be checked. Furthermore, periodic boundary condition at the two sides of the tube is being tested for the idea of modifying the lateral impedance or deriving an angular-dependent admittance in an infinite area of continuous medium. The idea is to include directionality of impedance despite what is commonly used which is only the normal component.

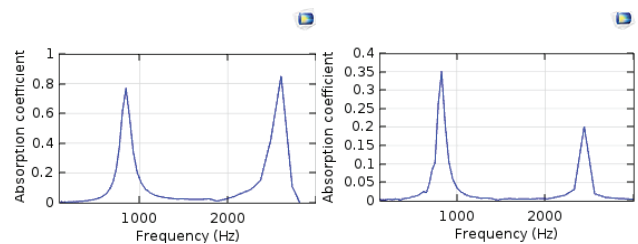


Figure 4: (Left) Zwikker Kosten with hydraulic radius of 2.5 mm (Right) thermoviscous model of narrow tubes with the same radius in 2D

## Conclusions

Poroacoustics of absorber materials are studied in this paper in terms of parameter sweep and surf plots with the aim of understanding the physical parameters in the equivalent fluid models. As examples, some of the most significant conclusions are as follows: wall acceleration or background pressure field showed no sensitivity to the sweep; porosity parameter sweep showed an increase in a smooth manner and above 0.3, almost linear behavior is observed. Hydraulic radius sweep gives sharp peaks which shift according to the pore resonances when considered as cylindrical; static thermal permeability is nearly insensitive with minor increase and tortuosity sweep shows similar behavior to layer thickness sweep and gives sharp peaks with shifts. Flow resistivity sweep shows interesting behavior at the resonances but overall, has a smooth increase.

## References

- [1] COMSOL Multiphysics® 5.2a
- [2] Allard, Jean, and Nouredine Atalla. Propagation of sound in porous media: modelling sound absorbing materials 2e. John Wiley & Sons, 2009.