Assessing the Effect of Laminate Soundboard Characteristics in the Physics-based Model of the Piano

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Introduction

Piano sound quality depends on many parameters. As the soundboard is responsible for inter-string coupling and sound radiation, one of the most important factor is the soundboard quality, mainly depending on geometry and material properties.

Traditional soundboards are built from solid high quality music wood (in general spruce), but in many low-priced instruments wood-laminates are used instead. Laminates are also getting more important because of the increasing demand for composite soundboards in piano manufacturing. In both cases, parameter variability – resulting either from natural wood characteristics or manufacturing tolerances – plays an important role in physics based modelling. In composite laminates both the fibre direction and layer thickness vary, that introduces uncertainty in the soundboard mechanics.

We present a stochastic finite element model of the laminated piano soundboard that is capable to assess the uncertainty of its lower eigenfrequencies and mode shapes resulting from uncertain fibre directions and layer thickness. The stochastic soundboard model is integrated in a piano simulation tool, so the effects of chosen models and parameters are examinable in detailed simulation.

In the next sections we briefly introduce all used models from the initial hammer velocity to the resulting sound pressure and explain the soundboard model in details.

The Piano Model

The piano is one of the most complex string instruments. Only the mechanical transmission path from the touch to the hammer-string contact contains hundreds of moving elements. After the first impact the sound is created and modified by the coupled system of felt dampers, steal strings, rim, wooden plates and the air. Surprisingly using just a few of these elements results in a quite good synthesised sound quality in an abstract piano model. In our simplified model we deal with hammer, string and soundboard behaviour examined through a nearfield radiation model.

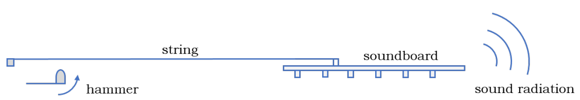

Figure 1: The abstract piano model.

In most pianos there are 88 hammers. Each of them consists of a wooden core covered by 3-4 layers of felt. One hammer strikes up to three strings. The strings are made from drawn music wire (steal core coated in copper wire). The silent vibration of strings is transferred through the bridge to the wooden soundboard. The ideal soundboard would be a perfect amplifier, but a real soundboard not only radiates the sound but filters, damps and couples the vibration of the strings.

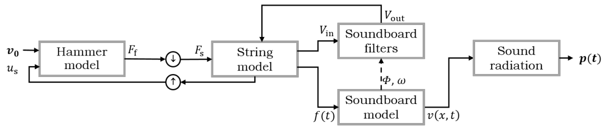

Figure 2: The block diagram of the piano model.

To model the force caused by the **hammer** we should write up a hysteretic mass-spring system with multiple springs coupled through the mass (e.g. [12, 3]);

The **strings** are model as a multi-string digital waveguide-model. The effect of the soundboard as well as the losses and damping caused by material properties and air contact is handled using IIR filter sets. (s.a [11, 1])

The soundboard displacement is converted into the sound pressure by a FIR filter set based on the Rayleigh-integral. This very abstract **sound radiation** model is valid only in the nearfield.

For a more detailed model description s.a. [9]. (In the literature further physics-based piano models are found e.g. [2, 4]).

The Soundboard Model

The traditional soundboards are made from high quality solid music wood (mostly spruce). They are stiffened and slightly bowed with ribs on the one side, while on the opposite side the bridges are placed. Since the 1960s wood laminated are also installed. Although the idea was to improve the soundboard quality and duration by homogenising the wood properties by using plies, wood laminates are exclusively installed in lower prised instruments. However laminated piano elements are getting more and more important in the last few years, because of the technical improvement of composite materials. Leading piano manufacturers have also came out with futuristic instruments design and solutions as a result of experimentation. These instruments have implied new manufacturing ways.

The geometry (shape and size) and material properties vary from instrument to instrument. To model the behaviour of such a complex element we should write up model for the material parameters and for the geometry. (Numerical soundboard models are described e.g. in [4, 5] and some interesting analytical results in [13].)

The Material Model

To model a general elastic material first we should start from the generalised Hooke's law

$$\sigma = D \cdot \varepsilon, \tag{1}$$

in which σ and ε are the 3×3 stress and strain tensors and D is the $3 \times 3 \times 3 \times 3$ material tensor.

Fortunately in most cases the tensor equation can be simplified in vector-matrix form. For modelled orthotropic, transversely isotropic and isotropic materials the material matrix takes the form

$$D = \begin{pmatrix} D_{11} & 0 \\ 0 & D_{22} \end{pmatrix}, \tag{2}$$

where D_{11} is a symmetric and D_{22} is a diagonal matrix of size 3×3. The elements of the matrix are determined by the Young's moduli, the shear moduli and the Poisson's ratios. [7]

Beside of the Hooke's equation we need the displacement equation as used in the Kirchoff-Love thin plate theory

$$u_i = \sum_{k=0}^{\infty} z^{\mathrm{k}} \cdot \mathrm{f}_{\mathrm{i}}^{\mathrm{k}}. \tag{3}$$

In case of thin plates it is sufficient to keep the first two terms of the summation, where the first term is the displacement of the mid-plane, and the second one is describes the rotation. In tangential direction we assume the displacement field as constant. After some algebraic manipulations starting out from equation (3) and using the listed assumptions, the strain vector can be written as

$$\varepsilon = \varepsilon_0 + \gamma + z \cdot \kappa, \tag{4}$$

where the ε_0 is the reduced strain vector, the γ is the engineering strain vector and the κ is the curvature vector of the plate. As the engineering strain vector is independent from the to other ones the Hooke's law can be reformulated as

$$\begin{aligned} \sigma &= \sigma^{\mathrm{b}} + \sigma^{\mathrm{s}}, \\ \sigma^{\mathrm{b}} &= D^{\mathrm{b}} \cdot (\varepsilon_0 + z \cdot \kappa), \\ \sigma^{\mathrm{s}} &= D^{\mathrm{s}} \cdot \gamma, \end{aligned} \tag{5}$$

so the problem is split up into separately bending and shear parts. We can write up the general connection for material forces ($F = \begin{bmatrix} N & Q \end{bmatrix}^{\mathrm{T}}$) and moments ($M = \begin{bmatrix} M^{\mathrm{b}} & M^{\mathrm{s}} \end{bmatrix}$) assuming that the material is equally distributed in tangential direction:

$$\begin{aligned} F &= \int_{-t/2}^{t/2} \sigma \cdot dz, \\ M &= \int_{-t/2}^{t/2} z \cdot \sigma \cdot dz. \end{aligned} \tag{6}$$

We can show that the general result has the form

$$\begin{pmatrix} N \\ M^{\mathrm{b}} \\ Q \\ M^{\mathrm{s}} \end{pmatrix} = \begin{pmatrix} A & B & 0 & 0 \\ B & D^{\mathrm{b}} & 0 & 0 \\ 0 & 0 & D_1^{\mathrm{s}} & 0 \\ 0 & 0 & 0 & D_2^{\mathrm{s}} \end{pmatrix} \begin{pmatrix} \varepsilon_0 \\ \kappa \\ \gamma \\ \gamma \end{pmatrix}, \tag{7}$$

where the submatrices are extensional (A), coupling (B), bending (D^{b}) and shear (D^{s}) stiffness matrices.

In case of solid materials the coupling stiffness matrix is zero. For laminated materials the same formula has to be used but the integration should be done plywise. Because of the thin plate assumption the shear force and moment can be neglected. The fibre direction is taken into consideration as tensor-rotation.

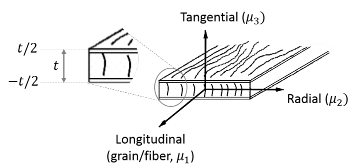

Figure 3: The laminated material model.

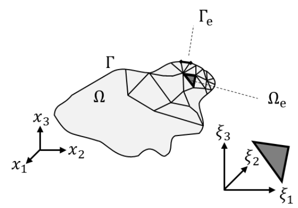

Figure 4: The geometry (FEM) model.

The Finite Element Model

The dynamics of an elastic volume (Ω) given by arbitrary boundary (Γ) is given by the displacement equation

$$S^{\mathrm{T}} \cdot \sigma + b = \rho \cdot \frac{\partial^2 u}{\partial t^2}, \tag{8}$$

where $S^{\mathrm{T}} \cdot \sigma$ is the divergence of the stress tensor, b is the vector of body forces, ρ is the density and u is the displacement. [14] We substitute equation (1) into (8), apply the theory of the virtual work and the integration theory of Gauss on it and neglect the body forces.

To compute the numerical solution of the resulting equation we use the Finite Element Method (FEM).

The idea of this method is to discretise the problem by splitting the geometry into elements. Over the elements defined polynomial functions form a basis ($\mathcal{N}$). The solution is given as a linear combination of this functions.

After several steps the displacement equation (8) can be written in form

$$\int_\Omega \delta u^{\mathrm{T}} \cdot M_{\mathrm{e}} \cdot \frac{\partial^2 u}{\partial t^2} \cdot d\Omega + \int_\Omega \delta u^{\mathrm{T}} \cdot K_{\mathrm{e}} \cdot u \cdot d\Omega = \int_\Gamma \delta u \mathrm{T} \cdot \mathcal{N}^{\mathrm{T}} f \cdot d\Gamma, \tag{9}$$

where M_{e} and K_{e} are the element mass and stiffness matrices given by

$$M_{\mathrm{e}} = \int_{\Omega_{\mathrm{e}}} \mathcal{N}^{\mathrm{T}} \cdot \rho \cdot \mathcal{N} \cdot d\Omega_{\mathrm{e}}$$
$$K_{\mathrm{e}} = \int_{\Omega_{\mathrm{e}}} \mathcal{B}^{\mathrm{T}} \cdot D \cdot \mathcal{B} \cdot d\Omega_{\mathrm{e}}. \tag{10}$$

The equation (10) can be reformulates in an equivalent form

$$K \cdot u + M \cdot \frac{\partial^2 u}{\partial t^2} = f, \tag{11}$$

that in case of no external excitation ($f = 0$) forms a standard eigenvalue problem. As solution we get the mode shapes ($\Phi(\omega) \sim \varphi(t)$) and eigenvalues ($\omega$). Choosing the mode shapes as basis function, we can write up the soundboard displacement as a linear combination

$$u = \sum_{k=1}^{n} \alpha_{\mathrm{k}} \cdot \varphi_{\mathrm{k}}, \tag{12}$$

where α represent the modal weights.

To reduce the number of used nodes in the numerical model, we apply the master-slave multi-freedom constrain theory. We handle the plate as master model, the ribs and bridges as slave models. It means that after the mass and stiffness matrices are independently written up for each model, the nodes of slave models are interpolated to the master nodes.

The Stochastic Approach

Karhunen–Loève decomposition

Some input parameter of the soundboard are representeted as stochastic processes ($r(x, \theta)$) over the given geometry (FEM) model (x). A stochastic process can be approximated as a linear combination of orthonormal functions. To select a proper set of solutions we use the Karhunen–Loève decomposition (s.a. equation (13))

$$r(x, \theta) = \sum_{k=1}^{\infty} \xi_{\mathrm{k}}(\theta) \cdot \mathrm{g}_{\mathrm{k}}(x), \tag{13}$$

where ξ_{k}-s represent pairwise uncorrelated random variable, and $\mathrm{g}_{\mathrm{k}}(x)$-s are the orthonormal functions. [6, 10] For practical solutions the problem is reduced to the m-dimensional stochastic space. In multi-variate case we apply the decomposition independently to the stochastic variables and evaluate the model in the 'full dimensional' (summing up the dimensions for each variable) space.

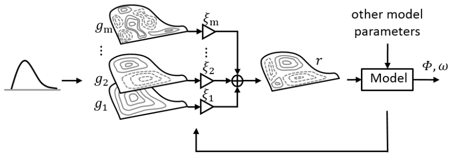

Figure 5: KL decomposition of stochastic processes.

Collocation point selection

To select the collocation points in the stochastic space we have basically to options: running a Monte Carlo simulation or using some kind of quadrature method. In the first case the points are selected randomly, as in second the one we use a strategised sampling. Using a quadrature based grid can be beneficial in higher dimensional space, because the number of selected points (P) can be reduced radically applying some sparse grid method (e.g. Smolyak's sparse grid method extends any kind of one dimensional quadrature rules to higher dimensional space [8]).

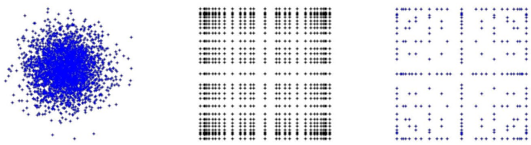

Figure 6: Collocation point selection strategies in 2D space: Monte Carlo simulation (left), full tensor grid using Clenshaw-Curtis quadrature (middle), Smolyak's sparse grid using Clenshaw-Curtis quadrature (right).

Model evaluation

For each realisation we can run our non-linear model deterministically. As a result we will have in all P points n mode shapes and eigenfrequencies. From these we can compute some statistics (e.g. expected value, variance) and apply these values in the original piano model.

An Example

As example we shortly describe a computation of a laminated soundboard with stochastic thickness and fibre direction parameters. This construction is simplified to have a quick test of the described models. The example soundboard consist of three spruce layers (plies). Only the middle layer is considered to have stochastic parameters. The prescribed expected values are 8 mm and 60°, the standard deviations are 0.5 mm and 2°. The KL-decomposition was computed over a grand piano soundboard mesh with a correlation length of 10 cm.

Figure 7: Used soundboard model: 1.50 m, 13 ribs, 1 curved bridge.

Our goal was to determine the first mode of the soundboard (case $n = 1$). We used a Clenshaw-Curtis quadrature based Smolyak's sparse grid (full dimensional case). After running the simulation we found out that the used number of KL-modes was $10 - 10$ (case $m = 20$), and that the computed first mode ($\omega_1 = 86 \pm 1.5$ Hz) slightly differs from the deterministic solution ($\omega_1 = 88$ Hz). The mode shapes as expected has the same form in both cases (s.a. figure 9).

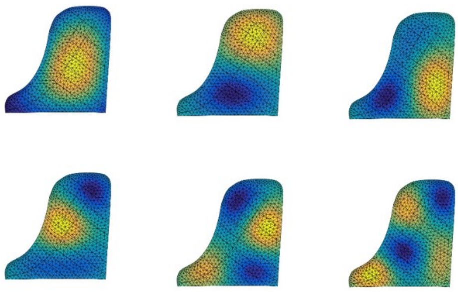

Figure 8: The first some KL-modes of the example.

The presented numerical results should be treated with a critical point of view. Analysing the results we can assume, that in the future we should adjust the tolerance of the KL-decomposition and take more KL-modes into account for more realistic simulation.

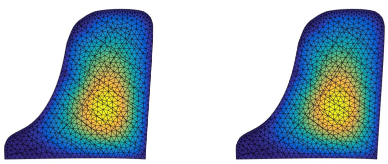

Figure 9: The first modes of the soundboard in deterministic (left) and stochastic (right) cases.

Conclusion and future work

The described soundboard model is able to handle laminated soundboards with stochastic parameters as thickness and fibre direction. It fits into the previous implemented MATLAB-based piano model, so enable to examine the effect of parameter uncertainties. In the future first we would like to run some simulations with more KL-modes and incorporate further parameters as moduli and material density. Both of them cause a radical increasing of the problem space dimension, so we have to reformulate our solution to deal with this situation. In the future we would also like to validate the parameter usage by own measurements.

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