Solid-borne Sound Intensity: a Personal Perspective

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Introduction

It is now more than 30 years ago that I entered the field of acoustics. In fact, I came from theoretical solid-state physics [1, 2]. This might explain my inclination to solid-borne sound and periodic media. Some aspects of solid-borne sound intensity which became important to me are highlighted in the following. (Even though the term "structure-borne sound" is widely used, as a physicist I prefer the term "solid-borne sound", which better matches the fluid-borne and airborne counterparts and closely corresponds to the German "Körperschall".)

Fundamental Equations

The theoretical framework of solid-borne sound energy and its propagation – as considered here – is linearized elasto-dynamics – without fear of anisotropic solids [3, 4]. The definitions of the kinetic and potential energy densities

$$\begin{aligned} e_{\text{kin}} &= \tfrac{1}{2}\rho \dot{u}_i \dot{u}_i \\ e_{\text{pot}} &= \tfrac{1}{2} u_{i,j} C_{ijkl} u_{k,l} \\ e_{\text{tot}} &= e_{\text{kin}} + e_{\text{pot}} \end{aligned} \tag{1}$$

(summation convention etc. applied) lead to the Lagrangian density

$$L = e_{\text{kin}} - e_{\text{pot}} , \tag{2}$$

from which – in the absence of external forces, i.e. sound sources or sinks – the equations of motion follow:

$$\rho \ddot{u}_i = C_{ijkl} u_{k,lj} + C_{ijkl,j} u_{k,l} . \tag{3}$$

If dissipation is neglected, energy is conserved:

$$\dot{e}_{\text{tot}} + \nabla \cdot \vec{S} = 0 . \tag{4}$$

By analogy to the Poynting vector of electrodynamics it is suggested in the monograph [3] to name the energy flux density

$$\vec{S} = -\sigma_{ij} \cdot \dot{u}_j \tag{5}$$

Kirchhoff vector. This vector divided by the total energy density yields the energy velocity

$$\vec{v}_{\text{e}} = \frac{\vec{S}}{e_{\text{tot}}} . \tag{6}$$

Time Averages

The intensity equals the time-averaged Kirchhoff vector:

$$\vec{I} = \left\langle \vec{S} \right\rangle_{\text{t}} . \tag{7}$$

For harmonic time dependence in complex notation,

$$\vec{u}(\vec{r}, t) = \vec{U}(\vec{r}) \exp[-\mathrm{i}\,\omega t] , \tag{8}$$

one obtains the intensity

$$\vec{I} = -\tfrac{1}{2}\mathrm{Re}\left\{ \underline{\Sigma} \cdot \vec{V}^* \right\} \tag{9}$$

from the complex amplitudes of the stress tensor and the velocity vector.

Complex and reactive Intensity

Taking the imaginary part instead of the real part in eq. (9) defines the so-called reactive intensity $\vec{Q}$, which constitutes the imaginary part of the complex intensity

$$\vec{I}_{\text{c}} = \vec{I} + \mathrm{i}\vec{Q} = -\tfrac{1}{2}\underline{\Sigma} \cdot \vec{V}^* . \tag{10}$$

The divergence of the reactive intensity is related to the time-averaged Lagrangian density:

$$\begin{aligned} \nabla \cdot \vec{Q} &= 2\omega \langle L \rangle_{\text{t}} \\ &= 2\omega \left(w_{\text{kin}} - w_{\text{pot}} \right) . \end{aligned} \tag{11}$$

This equation (11) is commented by Fahy in his book on airborne sound intensity [5a, p. 67; 5b, p. 56]: 'One physical interpretation of eqn (4.48) is that a local difference between mean potential and kinetic energy densities can be likened to a 'source' of reactive intensity: I consider this interpretation to have little physical justification.' However, this remarkable equation is not only true for airborne sound, but also for sound in arbitrary, even anisotropic and inhomogeneous solids [6, 3, 7]! For airborne sound there are several general equations involving the reactive intensity, but these are not generally valid for solid-borne sound fields [3]. Thus I believe that there is a "deep meaning" of eq. (11) and that the reactive intensity is useful in some sense, since it contains fundamental physical information. There were lively discussions about this issue in the 1990s, but – to my knowledge – the matter is still not finally settled. (For divergence equations including dissi-pation see [8, p. 1144].)

Historical Remarks

The first measurement of solid-borne sound intensity was performed by Noiseux in 1970: "Measurement of power flow in uniform beams and plates" [9]. (As an aside, the frequently used term “power flow” should be avoided, since it is the energy that flows.) The next three seminal papers were contributed by Pavić in 1976 ([10] on measurements of bending wave intensity including nearfield regions; by the way: there is no part II!), by Verheij in 1980 ([11] on cross-spectral density methods including quasi-longitudinal and torsional waves) and again by Pavić in 1987 ([12] on surface intensity capturing all wave types).

In the sequel two stimulating conferences on "Structural intensity and vibrational energy flow” took place in Senlis near Paris (1990 and 1993, [13, 14]) with a “First short course” on “Vibration Intensity” [15]. The only book – up to now – dedicated to the subject was published in 1994 [3, 4]; it addresses some theoretical aspects. Despite of numerous papers since then the field of solid-borne sound energy is still far less developed than its fluid-borne counterpart.

For comparison, the first measurement of airborne sound intensity dates back to the early 1930s (Olson [16]), i.e. four decades before Noiseux [9], and the comprehensive monograph “Sound Intensity” [5] by Fahy, first published in 1989, constitutes the reference of an already well established field. With respect to solid-borne sound energy Fahy remarks in 1995 [5b, p. 11]: ‘Readers may be disappointed to find no mention of “structural intensity” in this monograph. The measurement of vibrational energy flux in solid structures is considerably more difficult and complicated than the measurement of sound intensity in fluids. Consequently, it is still in the relatively early stages of research and development, and has yet to become established as a routine measurement tool: it will no doubt form the subject of a future monograph in its own right.’ Who will write it?

Surface Intensity

For the experimental determination of the solid-borne sound intensity at the surface of a solid in air it is normally assumed that no external forces act on this surface, i.e. that the component of the Kirchhoff vector normal to the surface vanishes [12]. Then only three stress components and two velocity components need to be measured. (Remarkably, the normal component, which is usually measured in case of bending waves, is not required!) The stresses are obtained from the strains via Hooke’s law. Three strain components can be measured at the surface, which is sufficient for calculating the stresses. This is true even for generally anisotropic solids [8, p. 1143f.; 7]. The system of six linear equations for six unknowns results in a simple solution in the isotropic case [12]. The surface intensity includes all wave types and nearfields.

Universal Sensor

If – in addition to the five quantities measured for the surface intensity – also the normal velocity component is measured, then the usual “vibrational level” and even the energy densities at the surface can be obtained as well. This is what I have in mind for a ‘universal sensor’ for solid-borne sound. It could be realized with three triaxial accelerometers around a ‘point of measurement’ (**Fig. 1**); the velocity vector results as an average over the three accelerometers and the strain components follow approximately from finite differences [7].

Appropriate graphical representation of the measured results, see e.g. **Fig. 2**, is essential for a detailed interpretation of the situation under study [7].

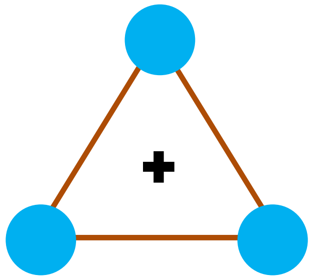

Fig. 1: Triangular arrangement of three triaxial accelerometers (+: nominal measurement position).

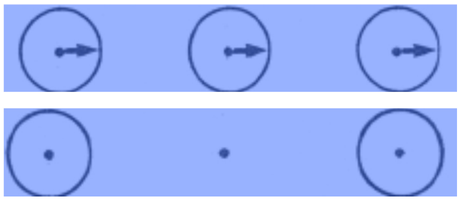

Fig. 2: Examples of circle-arrow plots [7]: propagating wave (above) and standing wave (below).

Moreover, the measurement should be accompanied by simulations of the situation, starting from plausible assumptions about the sound field [7]. By updating these assumptions in order to match the measured results the actual sound field may be approached iteratively. In simple cases the sound field inside the solid can be extrapolated from measurements at the surface and one can integrate over the thickness, e.g. with bending waves on thin plates. Possibly waves on thick plates [17] or thin shells are amenable to such a procedure, too. The theoretical problem of such reconstructions is addressed by Bobrovnitskii in [18].

Soundbridge Localization

Sound bridges in double walls and floating floors can be identified by solid-borne sound intensity measurements fairly efficiently. In the laboratory a sound bridge (diameter 4 cm) in a double wall (4 x 3 m²) could be localized within 10 cm by means of only seven intensity measurements [19], the first four of which are shown in **Fig. 3**. Back then in the late 1980s a stationary excitation and heavy equipment was used.

An attempt to make the method more practical was undertaken some 15 years later [20] with impulsive excitation and portable devices. The possibly larger uncertainty of the measured intensity at a point can be easily compensated for by increasing the number of measurements at additional points. Unfortunately, this updated method didn’t make it to field tests either.

Fluid-filled Porous Solids

In fluid-filled porous solids the sound energy can propagate both in the solid frame and the pore fluid. The energy exchange between solid phase and fluid phase is illustrated by a 1D calculation of the sound transmission through a porous plate in air [21]. In a 'thin' plate the intensity in the pores dominates throughout (not shown here), however, in a 'thicker' plate the intensity in the frame dominates in the right half (**Fig. 4**). Note that the intensity in one of the phases can become negative, which means that energy is transported backwards to the left.

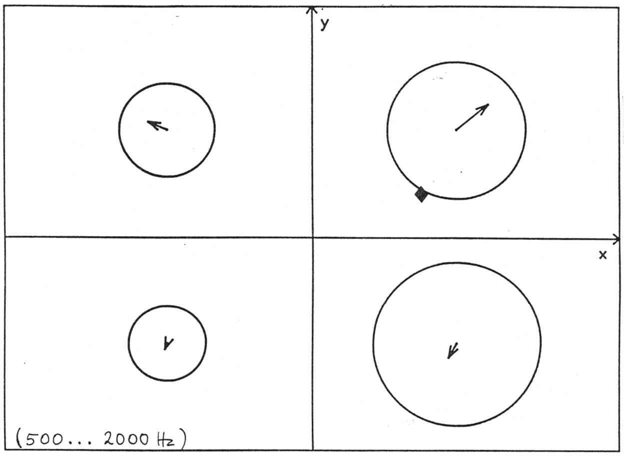

Fig. 3: Circle-arrow plot of four intensity measurements on a double wall (frequency averages from 500 Hz to 2 kHz; ♦: position of the sound bridge).

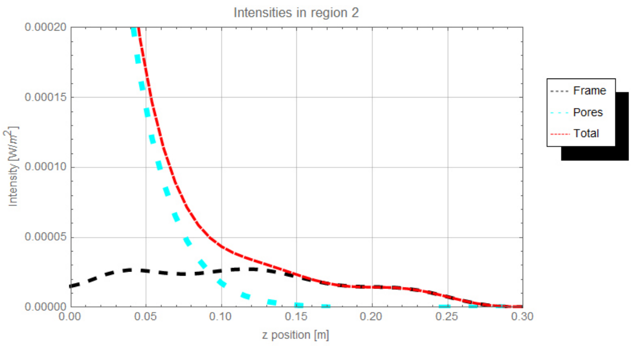

Fig. 4: Calculated sound intensities in a 30 cm thick porous plate in air due to a normally incident sound wave on the left. The 1D calculation [21] is based on Biot's theory in the formulation of Gurevich and Schoenberg [22] combined with Wilson's relaxation model "Simplified" [23].

Periodic Structures

It took a while until knowledge and methods accomplished in solid state physics became familiar in acoustics, in particular with respect to two- and three-dimensional periodicity. Nowadays – with the recent advent of metamaterials – periodic structures receive considerable attention regarding both fluid- and solid-borne sound propagation. Although the matter is intrinsically complicated, analytical results are still feasible and can be very useful like the two theorems for Bloch waves proved in [24]: Rayleigh's principle (the space-time averages of kinetic and potential energy are equal) and the equivalence of group velocity and energy velocity, where the latter is defined by averages over a unit cell. In addition, low-frequency approximations were derived including closed-form expressions for energy densities and intensities [25, 3].

Meanwhile numerical results are conveniently generated by a suitable Finite-Element software like COMSOL. It was used to compute Bloch waves in the two-dimensional structure which was already the subject of the (semi)analytical studies described in [3]; a periodically perforated sheet with quadratic symmetry [26]. **Figs. 5** and **6** show intensity and reactive intensity of a Bloch wave running backwards. COMSOL animations of the wave motion reveal intriguing features of the waves and greatly help to better understand the dynamics of apparently simple periodic structures with yet astonishingly rich behavior.

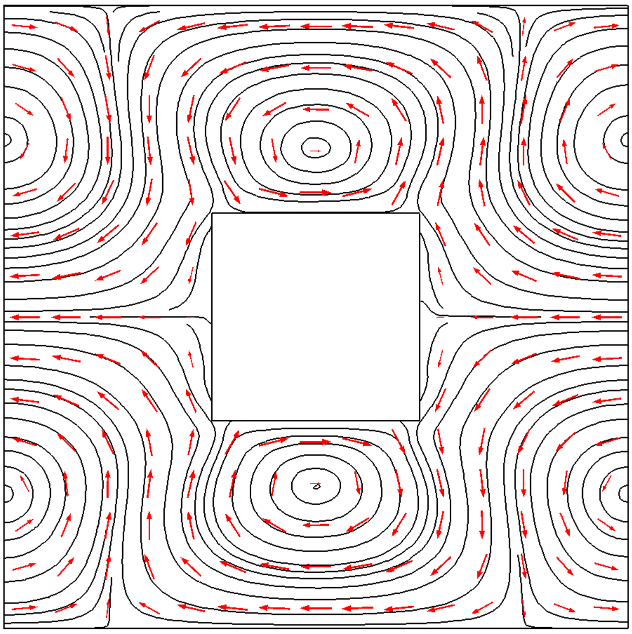

Fig. 5: Intensity (arrows and streamlines) of a Bloch wave on the second optical branch with wave vector pointing to the right. The group velocity is negative. Therefore, on average the arrows point to the left, i.e. backwards.

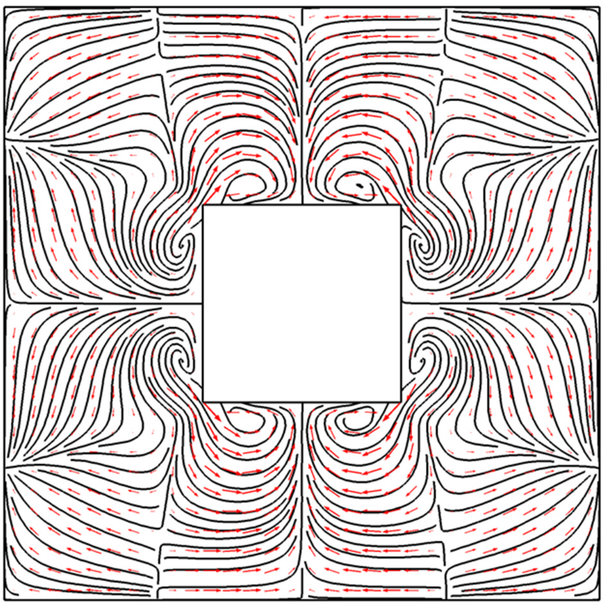

Fig. 6: As Fig. 5, but reactive intensity.

Conclusion

I would like to finish with several imperatives:

- Resume the reactive-intensity discussion and find precise and useful interpretations!
- Realize the "Universal Sensor" and assisting software!
- Reanimate localization method!
- Explore intensity vortices and energy redirection!
- Apply findings to practice!

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