Characterization of airborne sound sources using matrix inversion

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Introduction

The matrix inversion method (MIM) is a technique for the determination of sound source characteristics. The source is presented as a model of a few components, thus the characteristic of each of the components must be found. One requirement for input data for the calculation are transfer functions (TFs) from the locations of the components to the transducer positions. For the characterization of structure-borne sound sources, the procedure for measurements of inertance frequency response functions is described in detail in existing literature (e.g., in [1]). Although many authors mention applications of MIM for the characterization of airborne sound sources (e.g., in [2], [3]), there are no exact recommendations concerning the procedure of acoustic transfer function measurements. However, an inaccuracy in the measurements can lead to high uncertainties in the resulting volume velocity and wrong interpretation of the results.

In this paper, possible uncertainties are demonstrated by using the example of a simple model and by a real application. Brief recommendations on the application of different measurement methods and arrangements of measurement setups are presented as results.

Transfer function measurements as a step of the matrix inversion method

For airborne sound sources, the MIM aims to determine the volume velocity (*Q*) of the source.

The concept of the matrix inversion method is shown further through the example of its application in frequency domain. (More detailed information on MIM can be found in [2], an approach in time-domain is shown in [1]).

Airborne sound sources are represented as a superposition of simple components, such as monopoles, dipoles and so forth. In the current paper, monopoles are used as the most easily applicable type. Depending on the application, the source can be characterized more or less precisely in terms of its decomposition. The vector of volume velocities of each of the components is used to describe the source. The following measurements and calculations are required for the determination of the desired *Q*-vector.

1. Measurements of sound pressure produced by all the assumed components emitting together. These measurements are carried out by means of a microphone array in the near-field under operating conditions.
2. Transfer functions measured from the locations of each of the components to the microphone positions. These measurements allow to estimate the contribution of each component to the produced sound field. The measurements are done by means of a monopole source producing a defined *Q*. The consequent measurements are carried out for each component and all the receiver positions, resulting in a matrix of transfer functions.
3. Inversion of this matrix (or pseudo inversion in the case of unequal numbers of components and receivers) multiplied by the vector of operational measurements results in the vector of desired volume velocities.

Incorrect TF measurements cause wrong estimation of the components' contribution to the sound field and lead to wrong interpretation of the results.

Transfer function measurement procedure

In an ideal case of TF measurements, the monopole would be placed exactly at the position of the assumed components. Since this is not realizable, two approximated methods of transfer function measurements are used in a real application: direct and reciprocal, so-named according to the positioning of the monopole in regard to the position of the assumed component. In direct measurements, the monopole is placed very close to the assumed component position, while in reciprocal measurements, the monopole is located at the receiver position and the microphone is positioned next to the assumed component. The closer the monopole (for direct case) or the microphone (for indirect case) is located to the source surface and to the estimated component, the more accurate is the result. Precision of the assumption about the number and location of the components plays a very important role in the calculation of the *Q*-vector and as a consequence in the interpretation of the results as was shown in [4]. In the present paper, it is supposed, that the positions of the components are known and only the uncertainties caused by the measurement equipment and the distance between the monopole and the source surface are considered.

Uncertainties in transfer function measurements caused by the measurement equipment

Acoustic TFs (e.g., acoustic impedance by analogy with inertance for the case of structure-borne sound) are measured by means of a volume velocity source (*Q*-source) and microphones. Both of the devices are just an approximation of a point source and a point receiver. Deviations from omni-directivity of both devices as well as reflections from their structure cause uncertainties in the measurements. While measurement microphones are small and generally omnidirectional in a wide frequency range, the directivity of a *Q*-source is uniform only in a limited frequency range that is restricted by its size and form of a nozzle (as well as by the employed loudspeaker and its coupling to a tube). Another problem of using a Q-source as a monopole is its dimensions that cause a distance to the source surface in direct TF measurements. Stiffness of the tube also complicates the possibility to place the monopole in a proper way. To avoid these problems a reciprocal method can alternatively be used. However, it complicates the measurement procedure because the measurement setup must be completely rebuilt after operational measurements and changed for each component position. Thus, an engineer must find a compromise between the effort and the desired precision for each individual case. Some recommendations on the application of both methods are given below.

Uncertainties in transfer function measurements caused by the displacement of a monopole

The distance between the monopole and the source surface causes reflections and consequently a change of the spectral pattern. Estimation of this effect can be shown through the example of the following simple model. The sound field produced by a monopole of a radius a placed at a distance d in front of an infinite rigid wall (eq.(1)) is considered and compared to the case where the monopole is placed on the wall (eq.(2)) (see Figure 1).

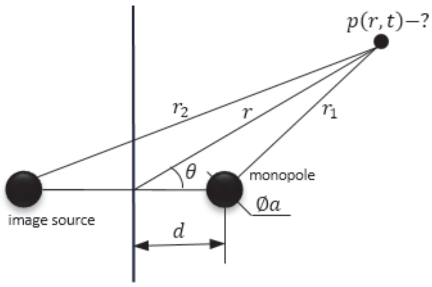

Figure 1: Illustration to calculate the sound field produced by the monopole of radius a placed at a distance d in front of an infinite rigid wall.

$$p(r,t) = p(r_1,t) + p(r_2,t)$$
$$p(r_1,t) = \frac{Q(t)}{4\pi}\cdot\frac{jk\rho_0 c}{1+jka}\cdot\frac{e^{-jk(r_1-a)}}{r_1}$$
$$p(r_2,t) = \frac{Q(t)}{4\pi}\cdot\frac{jk\rho_0 c}{1+jka}\cdot\frac{e^{-jk(r_2-a)}}{r_2} \qquad (1)$$
$$r_1 = \sqrt{r^2 - d^2 + 2rd\cos\theta},$$
$$r_2 = \sqrt{r^2 + d^2 + 2rd\cos\theta};$$

$$p(r,t)_{d=0} = \frac{Q(t)}{2\pi}\cdot\frac{jk\rho_0 c}{1+jka}\cdot\frac{e^{-jk(r-a)}}{r} \qquad (2)$$

$Q(t)$ is considered to be constant and the transfer functions in a perpendicular direction to the wall plane for different non-dimensional frequencies dk are considered (see Figure 2).

It can be seen that the monopole shifted away from the plane produces an omnidirectional sound field only in a frequency range that corresponds to $dk \ll 1$.

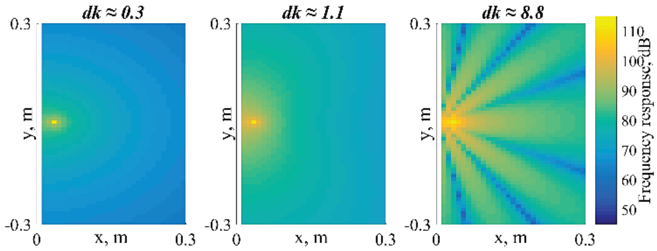

Figure 2: Transfer functions between sound pressure and volume velocity of a monopole placed at distance $\boldsymbol{d} = \mathbf{3\ cm}$ in front of an infinite rigid wall placed in the plane $\boldsymbol{x} = \mathbf{0}$ for different non-dimensional frequencies $\boldsymbol{dk}$, where $\boldsymbol{k}$ is a wavenumber.

At higher frequencies, the reflections lead to a significant change of the spectral pattern. The effect of the deviations of the TFs with changing d at different r along the x-axis is shown in Figure 3.

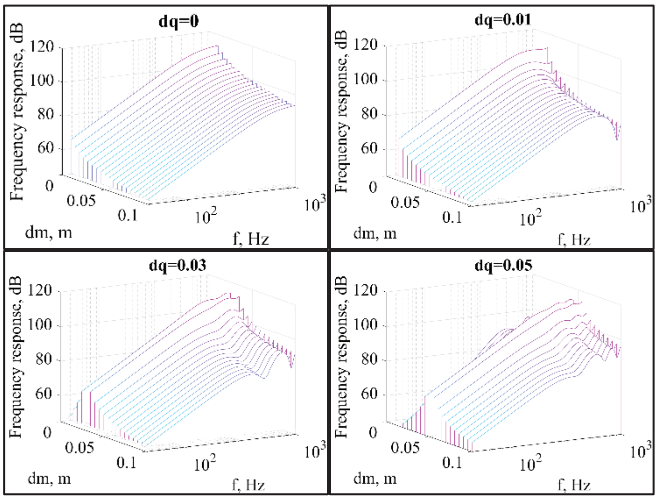

Figure 3: Transfer functions between sound pressure at distance d_m along the x-axis and volume velocity of a monopole positioned at distance d_q in front of an infinite rigid wall placed in the plane $x = 0$.

It can be observed that the deviations from the omni-directional pattern increase with increasing distance to the receiver point. The distance between the monopole and the surface causes more structured transfer functions but with less pronounced level changes.

It can be concluded that the distance d must be minimized as well as the distance r. However, it is obvious that r must be much greater than d. Other parameters that must be considered for making a decision about the placement of the receivers are the sound source dimensions, the number of components and their distribution. It can be estimated from [2], that an optimal distance r is comparable to the distance between the components as well as comparable to the distance between the receivers. This relation minimizes the condition number of the TF matrix.

In such a circumstance, the reciprocal method seems to be more accurate since the distance between the wall and the microphone (that is placed close to the component position) is easier to minimize. However, the displacement of the Q-source causes a change of the directivity pattern in a wider frequency range, compared to the direct method. While the measurements very close to the component area are more accurate, a slight shift of the measurement points can cause high uncertainties. This effect can be visualized by considering the change of the spectral pattern of transfer functions obtained directly and reciprocally varying the microphone positions around the initial measurement points (see Figure 4).

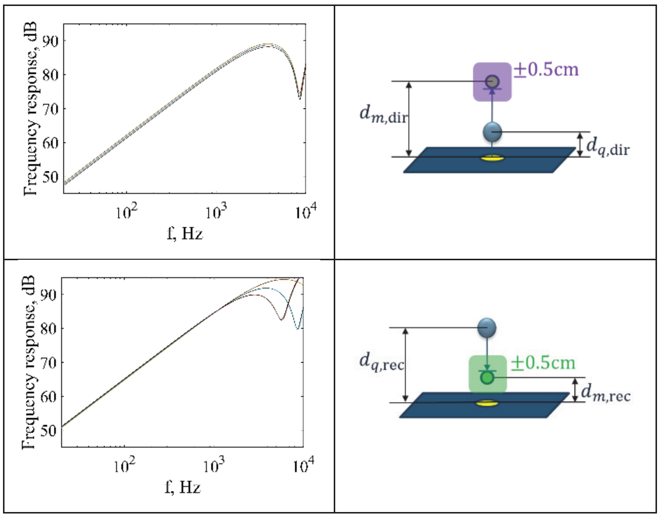

Figure 4: First row: change of the direct transfer functions in the small area around the measurement point placed at $\boldsymbol{d_{m,\text{dir}} = 10}$ **cm**. The monopole is located at $\boldsymbol{d_{q,\text{dir}} = 1}$ **cm** in front of an infinite rigid wall.
Second row: change of the reciprocal transfer functions in the small area around the measurement point placed at $\boldsymbol{d_{m,\text{rec}} = 1}$ **cm**. The monopole is located at $\boldsymbol{d_{q,\text{rec}} = 10}$ **cm** in front of an infinite rigid wall.

It can be seen that a slight shift of the microphone position in the direct TF measurements leads to a minor change of the TF magnitude but to almost no change of the spectral pattern, while a slight movement of the microphone in the reciprocal TF measurements results in a strong change of the spectral pattern at high frequencies.

Although the consideration presented above was done for the ideal model with an infinite rigid wall, the achieved results are applicable to the case when a characteristic dimension of a real source surface is comparable to interesting wavelengths.

Application example

The above-described effects were observed in a real scenario. A vacuum cleaner was chosen as an example of a source with only one assumed sound component – the motor. This assumption was proved by means of beamforming techniques. The location of the component can be seen in Figure 5.

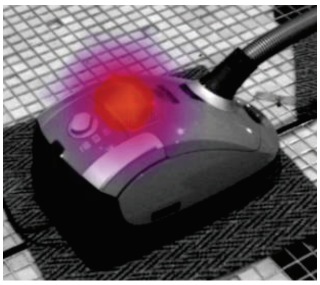

Figure 5: Location of the main sound component of the vacuum cleaner defined by means of beamforming techniques.

To observe the above-discussed uncertainties the following measurements were carried out using the vacuum cleaner in an anechoic chamber:

- Reciprocal measurements with the microphone placed at the distance $d_{m,\text{ref}} = 1$ cm above the component. These measurements with the minimized distance between the source surface and the measurement point at the component position are considered as *reference measurements* for further comparison: distance to the monopole above the component $d_{q,\text{ref}} = 7, 10, 15, 20$ cm.
- *Direct measurements* with $d_{q,\text{dir}} = 2.7$ cm, and $d_{m,\text{dir}} = 7, 10, 15, 20$ cm.
- The operational sound pressure measurements were carried out at the points $d_m = d_{m,\text{dir}}$.
- As a comparison to the direct measurements, measurements for the same $d_{m,\text{dir}}$ and $d_{q,\text{dir}}$ were carried out reciprocally ($d_{q,\text{rec}} = 7, 10, 15,$ 20 cm, $d_{m,\text{rec}} = 2.7$ cm). They are called here *reciprocal measurements*.

The measurements were carried out using white pseudo-noise with a 200-8000 Hz frequency range.

Since the source has only one component with a known position, the change of the real transfer functions with varying distance d_m can be estimated from the operational measurements. The smoothed FFT of the operational sound pressure (see Figure 6) does not show strong deviations in the spectral pattern with changing distance. It can be noticed as well that doubling the distance yields approximately half of the sound pressure. This leads to the conclusion that the component has a behavior close to a monopole.

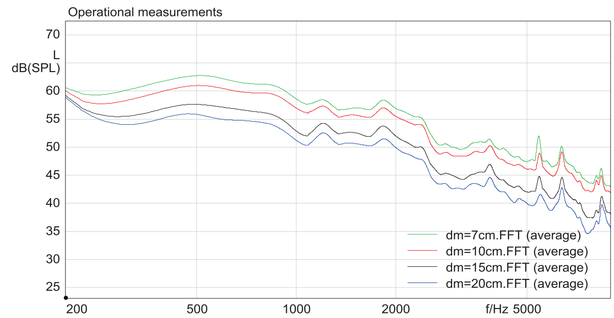

Figure 6: Smoothed FFT of sound pressure measured at distances $\boldsymbol{d_m = 7, 10, 15, 20}$ **cm** under operating conditions.

The results of the *reference* and the *direct TF measurements* are presented in Figure 7 and 8.

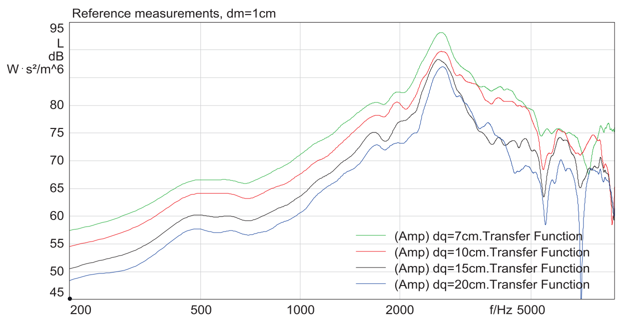

Figure 7: *Reference* transfer functions.

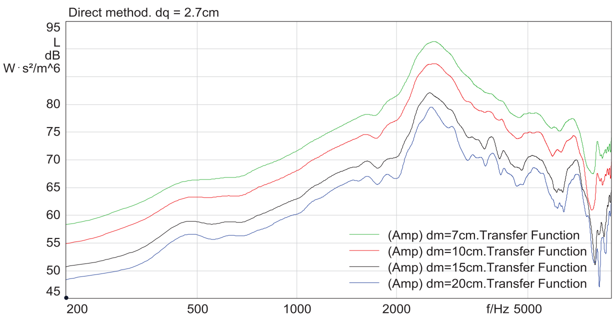

Figure 8: *Direct* transfer functions.

There is a good agreement to the calculations shown for the simple model. A steep gap in the spectral pattern of the *reference TFs* at high frequencies can be observed. This gap rises significantly with increasing distance of the receiver point ($d_{q,\mathrm{ref}}$ in this case). The same tendency was shown for the calculation of the simple model (see Figure 3, case $d_q = 0.01$). In the *direct measurements,* the deviations of the spectral pattern are not as steep as those observed in the *reference measurements*. It can be seen as well that the increase of the distortions with increasing $d_{m,\mathrm{dir}}$ is not so crucial. The *direct TFs* can be compared to the calculated TFs for the case $d_q = 0.03$ (Figure 3). As an additional observation, it can be concluded that the change of the *direct TFs* looks similar to the change of the SPL for the operational measurements, even if there are slight fluctuations at high frequencies.

The *reciprocal TFs* are presented in Figure 9. Stronger deviations between the direct and reciprocal measurements for the corresponding positions can be observed at higher frequencies. There seems to be no tendency of changing the spectral pattern with increasing $d_{q,\mathrm{rec}}$. That was probably caused by the uncertainties in the positioning of the measurement equipment, that lead to crucial changes in the spectral pattern as shown above.

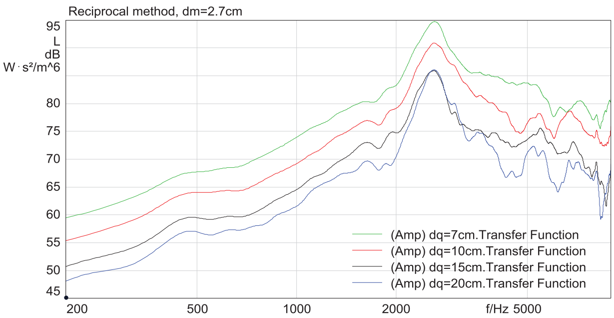

Figure 9: *Direct* transfer functions.

Comparison of the transfer functions measured *directly* and *reciprocally* to the *reference TF* for the receiver point $d_{q,\mathrm{ref}} = d_{m,\mathrm{dir}} = d_{q,\mathrm{rec}} = 7$ cm is shown in Figure 10. It can be seen that the *directly measured TFs* are more similar to the *reference TF*, even if the *Q*-source was placed further away from the source. The *reciprocal TF* deviates from the *reference measurement* much more strongly in the wider frequency range (most probably due to some slight shift of the position of the *Q*-source compared to the *reference measurements*).

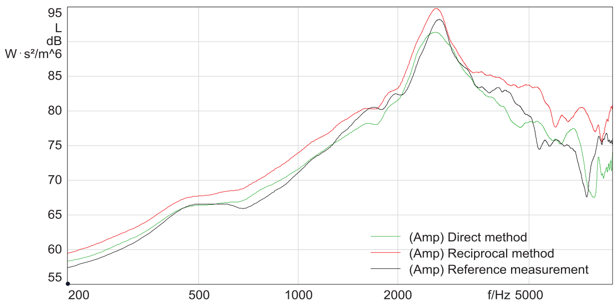

Figure 10: Comparison of the *direct, reciprocal and reference* TFs for $\boldsymbol{d_{m,\mathrm{dir}} = d_{q,\mathrm{rec}} = d_{q,\mathrm{ref}} = 7}$ cm.

Conclusion

It was shown by calculations for a simple model and proved by real measurements, that the necessary distance of the *Q*-source from the source surface leads to uncertainties in transfer functions measurements. However, the inaccuracy appears only in the high frequency range. The uncertainties can be estimated by means of a number of measurements at different distances from the source and the optimal measurement location can be found.

The direct method should be preferred for cases where the position of the component is not precisely defined since the deviation of the measurement equipment positioning does not lead to a strong change of the transfer functions. This method is recommended as well for the case when a high frequency sound source must be analyzed. The great advantage of the direct method is its easier realization: the microphones remain at the same positions for operational measurements and the measurement of transfer functions: only the *Q*-source position needs to be changed.

The reciprocal method can provide more precise results due to the ability to place a measurement microphone closer to the position of an assumed component, than a *Q*-source in the case of direct measurements. (Reciprocal measurements very close to the assumed component are considered as *reference measurements* in this paper). However, the reciprocal method needs additional measurement efforts and is more sensitive to changes of the microphone locations.

The choice of the transfer function measurement method should be made depending on the application. Combination of both methods can be considered for more accurate results.

Literature

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