Hybrid modelling of transmission loss with acoustic treatment

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Introduction

The hybrid method combining finite element (FEM) and statistical energy analysis (SEA) is an efficient approach to determine the transmission loss (TL) of complex structures. In order to simulate the effect of acoustic treatment using for example fibre material, foams and heavy layers the hybrid theory needs some modification to stay efficient. This paper deals with the modal implementation of hybrid theory and the adaptations that are required to implement the modal approach for acoustic treatments. The treatment will be modelled as infinite layer using the transfer matrix method (TMM) under consideration of possible simplifications to reduce the calculation time tremendously. This requires the mode shape mapping to regular grids. The method is implemented as locally and non-locally reacting trim showing that the non local approach is too strict for valid results.

Theory

The transmission loss can be calculated using the hybrid FEM/SEA theory of Shorter and Langley [1] or the discrete implementation of Grahams theory [2] by Peiffer [3]. In the view of hybrid theory the twin chamber arrangement shown in figure 1 consists of the panel as deterministic system and the two cavities as random subsystems.

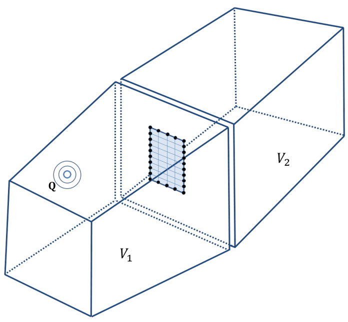

Figure 1: Twin chamber arrangement for TL tests of panels

In the following, random subsystems are called SEA subsystems and deterministic subsystems are denoted with FEM. The coupling loss factor (CLF) describing the energy flow between the two SEA systems is given by:

$$\eta_{21} = \frac{2}{\pi n_2 \omega} \sum_{i,j} \operatorname{Im} \mathbf{D}^{(1)}_{dir,ij} \left(\mathbf{D}^{-1}_{tot} \operatorname{Im} \mathbf{D}^{(2)}_{dir} \mathbf{D}^{-H}_{tot} \right)_{ij} \quad (1)$$

with n_m, $\mathbf{D}^{(m)}_{dir}$ being the modal density and the free field radiation stiffness of the m-th SEA subsystem, respectively. The free field radiation stiffness is the stiffness which is *seen* by the plate due to the connection to the fluid of the cavities and it is calculated using the wavelet approach from Langley [4].

The total stiffness matrix $\mathbf{D}_{tot}$ follows from the dynamic stiffness matrix of the structural system, here the plate

$$\mathbf{D}_s(\omega) u_s = f_s \quad (2)$$

and the radiation stiffness of both connected cavities:

$$\mathbf{D}_{tot} = \mathbf{D}_s + \mathbf{D}^{(1)}_{dir} + \mathbf{D}^{(2)}_{dir} \quad (3)$$

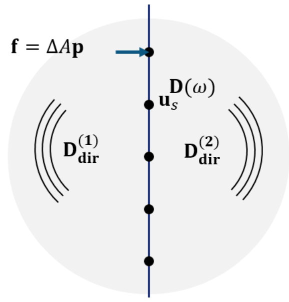

Figure 2: 2D sketch of plate with nodal DOFs and the free field radiation stiffness of both fluids

The TL of the plate is directly linked to the coupling loss factor by:

$$\tau \approx \frac{8\pi^2 \omega n_2}{k_2^2 A} \eta_{21} \qquad TL = 10 \log_{10} \frac{1}{\tau} \quad (4)$$

A is the area of the plate and and k_2 the wavenumber of fluid 2. The above formula eliminates the modal density, so the TL can be directly derived from the CLF without any knowledge of the geometry of the connected cavities. The arrangement is shown in figure 2. All degrees of freedom (DOFs) are shared by all matrices in equation (3).

The treatment model

The acoustic treatment can also be modelled by a stiffness matrix formulation, this is not usual as for example FE implementations are using structural degrees for the structure side and pressure degrees of freedom for the fluid side. For simplicity the trim will be modelled in this paper using a dynamic stiffness matrix, hence:

$$\begin{bmatrix} \mathbf{D}^{SP}_{11} & \mathbf{D}^{SP}_{12} \\ \mathbf{D}^{SP}_{21} & \mathbf{D}^{SP}_{22} \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} f_1 \\ f_2 \end{Bmatrix} \quad (5)$$

In figure 4 such a treatment connected to a structure is shown. With the local approximation it is assumed, that only the opposite nodes are connected, this assumption is valid for very thin layers and the block matrices in equation (5) are diagonal in that case. However, it will be shown that the local approximation is not sufficient and a non-local approach is required. Modern FEM packages allow the determination of stiffness matrices of complex combinations of treatment material but the complicated material laws require costly calculations. In order to provide a fast method the transfer-matrix-method (TMM) is used that is well known in the context of SEA [6].

$$\begin{Bmatrix} p_1 \\ v_1 \end{Bmatrix} = \begin{bmatrix} T_{11}(k_x) & T_{12}(k_x) \\ T_{21}(k_x) & T_{22}(k_x) \end{bmatrix} \begin{Bmatrix} p_2 \\ v_1 \end{Bmatrix} \tag{6}$$

The coordinate in x-direction is given in wavenumber space by k_x. In case of infinite layers - and only then - k_x remains constant in each layer and the in-depth wavenumber $k_z = \sqrt{k^2 - k_x^2}$ is changing. When N layers are connected the total transfer matrix (TM) is:

$$\mathbf{T}_{tot} = \mathbf{T}_1 \, \mathbf{T}_2 \cdots \mathbf{T}_N \tag{7}$$

The TM elements used in this paper are the mass layer

$$\mathbf{T}_{mass} = \begin{bmatrix} 1 & j\omega\rho_s \\ 0 & 1 \end{bmatrix} \tag{8}$$

and the fluid layer

$$\mathbf{T}_{fluid}(k_x) = \begin{bmatrix} \cos(k_z d) & jz\sin(k_z d) \\ j\frac{\sin(k_z d)}{z} & \cos(k_z d) \end{bmatrix} \tag{9}$$

The TM is converted to a stiffness matrix by the following relationship under consideration of the convention as shown in figure 3

$$\mathbf{D}^{SP}(k_x) = \frac{j\omega\Delta A}{T_{tot,21}} \begin{bmatrix} T_{tot,21} & -1 \\ -1 & T_{tot,22} \end{bmatrix} \tag{10}$$

assuming a regular mesh with constant nodal area ΔA. This matrix must be converted into the space domain by inverse fourrier transform. Using Langleys *jinc*-function approach [4] and assuming isotropy in the exciting layer this transformation is simple and implies a finite wavenumber integration.

$$D_{ab}^{SP}(x_{ij}, \omega) = \frac{\Delta A}{2\pi} \int_0^{k_s} D_{ab}^{SP}(k) J_0(k x_{ij}) k dk \tag{11}$$

with

$$a, b = 1, 2 \qquad x_{ij} = \sqrt{x_i^2 - x_j^2} \text{ and } k_s = \sqrt{\frac{2\pi^2}{\Delta x \Delta y}} \tag{12}$$

being the projected distance between nodes and the maximum wavenumer that can be represented be the spacial sampling of the mesh, respectively. The integral in (**??**) is solved numerically. To conclude, using this approach the stiffness matrix of the trim is available for further considerations.

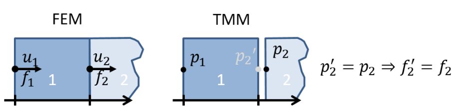

Figure 3: Convention for stiffness matrix and TMM

Integration of trim in hybrid theory

The sound package or trim is an additional deterministic layer that must be considered in the total stiffness matrix. When a layer of acoustic treatment is applied to one surface of the plate the situation is different (figure 4).

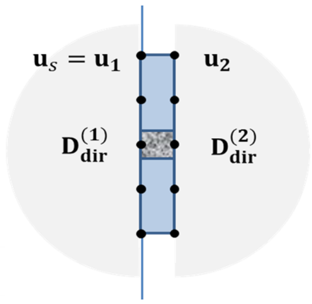

Figure 4: 2D sketch of plate with trim

The degrees of freedom are now separated and DOFs u_s of the structure are supposed to coincide with the left side of the trim, thus $u_s = u_1$. The total matrix reads as:

$$\left(\begin{bmatrix} \mathbf{D}_s & \\ & \end{bmatrix} + \begin{bmatrix} \mathbf{D}_{dir}^{(1)} & \\ & \mathbf{D}_{dir}^{(2)} \end{bmatrix} + \begin{bmatrix} \mathbf{D}_{ss}^{SP} & \mathbf{D}_{s2}^{SP} \\ \mathbf{D}_{2s}^{SP} & \mathbf{D}_{22}^{SP} \end{bmatrix} \right) \begin{Bmatrix} u_s \\ u_2 \end{Bmatrix} = \begin{Bmatrix} f_s \\ f_2 \end{Bmatrix} \tag{13}$$

Writing the equations using the upper and lower block matrix gives

$$\left[\mathbf{D}_s + \mathbf{D}_{ss}^{SP} + \mathbf{D}_{dir}^{(1)} \right] u_s + \mathbf{D}_{s2}^{SP} \, u_s = f_s \tag{14}$$

$$\mathbf{D}_{2s}^{SP} \, u_2 + \left[\mathbf{D}_{22}^{SP} + \mathbf{D}_{dir}^{(2)} \right] u_2 = f_2 \tag{15}$$

The task is to eliminate u_2 in order to express the total stiffness in structural coordinates. Doing this with equations (14) and (15) leads to

$$\left(\left[\mathbf{D}_s + \mathbf{D}_{ss}^{SP} + \mathbf{D}_{dir}^{(1)} \right] - \mathbf{D}_{s2}^{SP} \left[\mathbf{D}_{22}^{SP} + \mathbf{D}_{dir}^{(2)} \right]^{-1} \mathbf{D}_{2s}^{SP} \right) u_s = f_s - \mathbf{D}_{s2}^{SP} \left[\mathbf{D}_{22}^{SP} + \mathbf{D}_{dir}^{(2)} \right]^{-1} f_2 \tag{16}$$

With zero external forcer $f_2 = 0$ the total stiffness is:

$$\mathbf{D}_{tot}^{SP} = \left[\mathbf{D}_s + \mathbf{D}_{ss}^{SP} + \mathbf{D}_{dir}^{(1)} \right] - \mathbf{D}_{s2}^{SP} \left[\mathbf{D}_{22}^{SP} + \mathbf{D}_{dir}^{(2)} \right]^{-1} \mathbf{D}_{2s}^{SP} \tag{17}$$

So equation (17) provides the new version of the total stiffness used in equation (1). The next step is to replace $\mathbf{D}_{dir}^{(2)}$ by a version that considers the radiation via the trim. An appropriate way to start this discussion is to determine the power radiated be the degrees of freedom from side two.

$$\Pi = \frac{\omega}{2} \operatorname{Im} u_2^H f_2 = \frac{\omega}{2} \operatorname{Im} u_2^H \, \mathbf{D}_{dir}^{(2)} \, u_2 \tag{18}$$

With equation (15) the radiated power can be expressed by structural coordinates u_s

$$\Pi = \frac{\omega}{2}\,\mathrm{Im}\{u_s^H\,\mathbf{D}_{2s}^{SP,H}\left[\mathbf{D}_{22}^{SP}+\mathbf{D}_{dir}^{(2)}\right]^{-H}\mathbf{D}_{dir}^{(2)} \left[\mathbf{D}_{22}^{SP}+\mathbf{D}_{dir}^{(2)}\right]^{-1}\mathbf{D}_{2s}^{SP}\,u_s\} \quad (19)$$

The term between both u_s is called the reduced radiation stiffness $\mathbf{D}_{red}^{SP}$ and replaces $\mathbf{D}_{dir}^{(2)}$ in equation (1).

$$\mathbf{D}_{red}^{SP} = \mathbf{D}_{2s}^{SP,H}\left[\mathbf{D}_{22}^{SP}+\mathbf{D}_{dir}^{(2)}\right]^{-H}\mathbf{D}_{dir}^{(2)}\left[\mathbf{D}_{22}^{SP}+\mathbf{D}_{dir}^{(2)}\right]^{-1}\mathbf{D}_{2s}^{SP} \quad (20)$$

So, the CLF with trim on side two given by:

$$\eta_{21}^{SP} = \frac{2}{\pi n_2 \omega}\sum_{i,j}\mathrm{Im}\,\mathbf{D}_{dir,ij}^{(1)}\left(\mathbf{D}_{tot}^{SP,-1}\,\mathrm{Im}\,\mathbf{D}_{red}^{SP}\,\mathbf{D}_{tot}^{SP,-H}\right)_{ij} \quad (21)$$

Modal space

The structural stiffness matrix can be simplified by using a modal base given by a truncated set of mass normalised modes

$$\mathbf{\Phi} = [\Phi_1\Phi_2\cdots\Phi_N] \text{ with } \Phi_m^H\,\mathbf{M}\,\Phi_n = \delta_{mn} \quad (22)$$

The structural stiffness matrix gets diagonal with

$$\mathbf{D}' = diag(\omega^2(1+j\eta)-\omega_n) \quad (23)$$

η and ω_n are the damping loss factor and the model frequency, respectively. The prime $'$ denotes the modal coordinates. The modal radiation stiffness is fully populated and symmetric.

$$\mathbf{D}_{dir}^{(m)'} = \mathbf{\Phi}^H\,\mathbf{D}_{dir}^{(m)}\,\mathbf{\Phi} \quad (24)$$

The same is true for the block matrices of the trim.

$$\mathbf{D}_{ab}^{SP'} = \mathbf{\Phi}^H\,\mathbf{D}_{ab}^{SP}\,\mathbf{\Phi} \quad (25)$$

From the conversion to modal space follows a useful property for the interpretation of this stiffness, the modal radiation efficiency, given by:

$$\sigma_n = \frac{1}{\omega\Delta A\rho_2 c_2}\frac{\mathrm{Im}\,\mathbf{D}_{dir}^{(2)}}{\Phi_n^H\Phi_n} \quad (26)$$

For configurations with trim the reduced radiation stiffness from equation (20) shall be considered.

Applications

In principle the above described method works well with any FE-structure model that has flat or slightly curved surfaces as shown in [3]. In addition the mode shape will be mapped on a regular mesh that makes the calculation much more efficient, especially the derivation of the radiation stiffness using Langleys method and the trim stiffness matrix. Both matrices depend only on distances between nodes and the regular mesh leads to several equal distances tremendously reducing the numerical problem.

Table 1: Simulated trim setups

type	thickness	material	η
	50 mm	air	1%
Trim1	-	$m' = 150\mathrm{g/m^2}$	-
	50 mm	microlite AA	
Trim2	-	$m' = 150\mathrm{g/m^2}$	-

As test application a curved aluminium panel of radius $R = 2$m, thickness $t = 4$mm and an area of $0.8x1.25\mathrm{m^2}$ was selected. The aim of trim1 is to show the deficiencies of the local approach, for this trim the effect is stronger because of less damping in the fluid making the cross-coupling more important. In figure 5 the TL of the bare structure is shown and compared to results from the commercial software VAOne [7]. The agreement is excellent, the slight difference may come from a different surface area that results from different treatment of edge nodes.

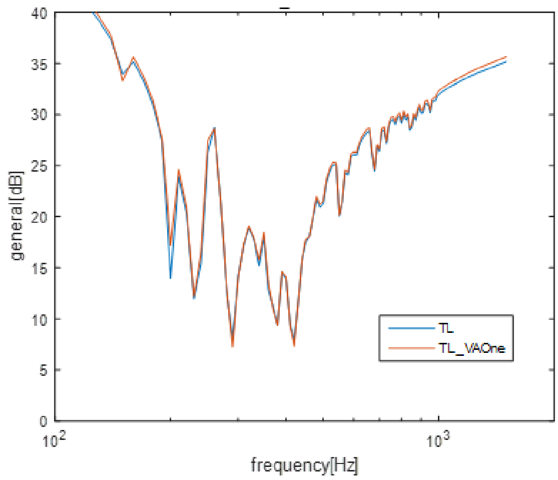

Figure 5: Transmission loss of bare panel

The radiation efficiency of the first five modes agrees also very well and shows that there is efficient radiation in the high frequency regime (figure 6).

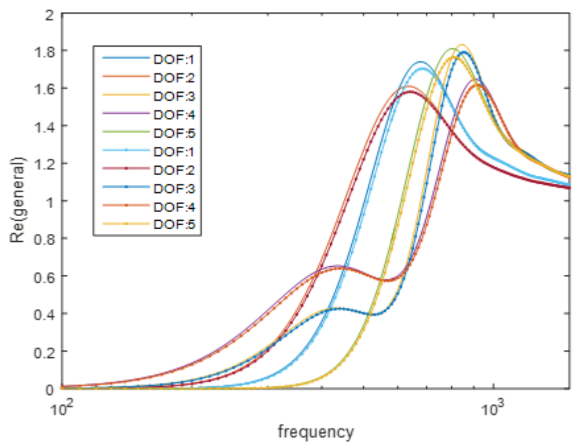

Figure 6: Modal radiation efficiency of the first five modes

The TL results of trim1 express the limitations of the local approach. It underestimates the TL for low and overestimates for high frequencies (figure 7). In figure 8 the differences in the modal radiation efficiency are even more obvious.

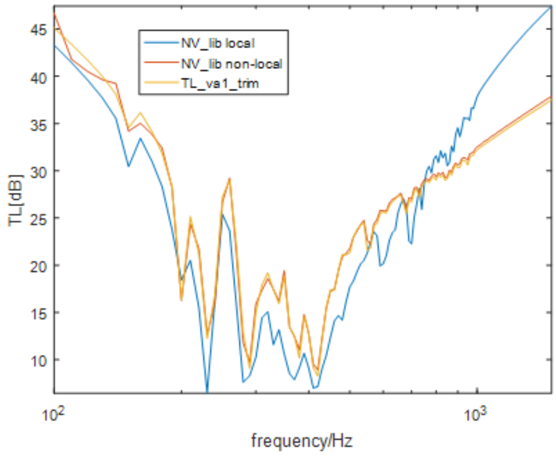

Figure 7: Transmission loss of panel with trim 1

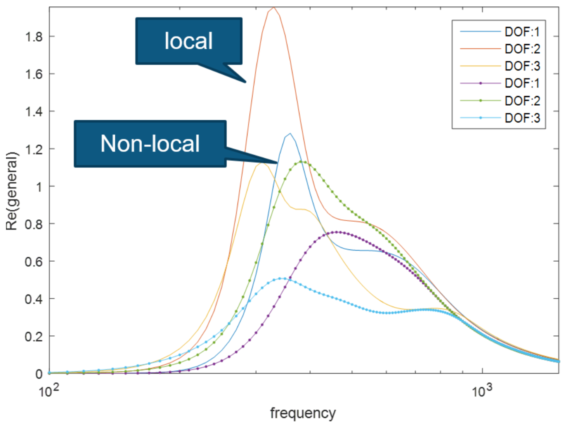

Figure 8: Modal radiation efficiency of the first three modes

The second trim case with the more realistic aerospace fibre material shows a lower deviation between both approaches due to the high damping in the wave propagation of the fluid layer. Thus, for fast estimations the local approximation might by an option.

In order to explain the effect of acoustic treatment the bare and trim2 modal radiation efficiency is shown. In figure 10 one can see that the global performance is improved, but the low frequency is made worse. For example a propeller noise excitation from 100-300 Hz would be better transmitted with that trim 2 than without.

Conclusion

The TMM implementation of trim in the hybrid context is an appropriate tool for the evaluation of acoustic measures. It is much faster than full FEM implementations and may be used for first design. In the near future the implementation of more complicated layers as foam material, plates or perforate layers is intended.

References

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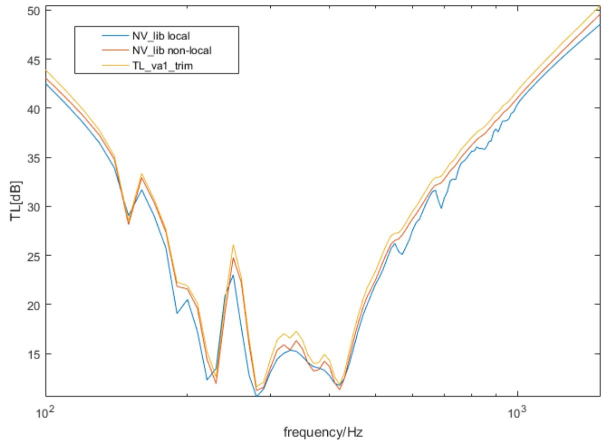

Figure 9: Transmission loss of panel with trim 2

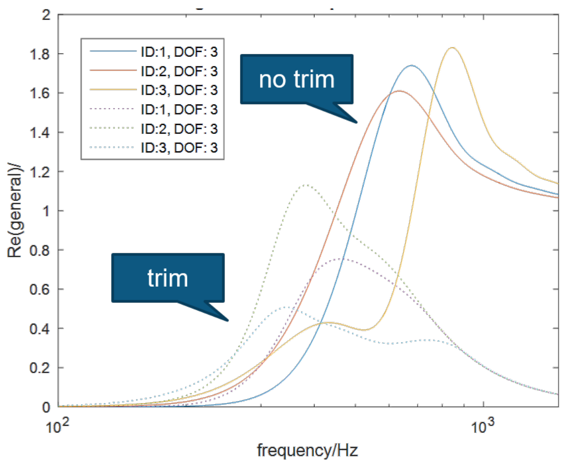

Figure 10: Modal radiation efficiency of bare and trim 2

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