Optimized Sound Field Generation in the Time Domain – Validation for Source Arrays in 2D

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Introduction

Line Source Arrays (LSAs) are used for sound reinforcement aiming at the synthesis of homogeneous sound fields for the whole audio band width. The deployed loudspeaker cabinets are rigged with different tilt angles and/or are electronically controlled. The determination of the optimal geometric arrangement and electronic drive is an ill-posed inverse problem.
In two preceding contributions [5, 6], we introduced an adjoint-based approach for sound reinforcement applications in the time domain. By defining a target sound field within an objective function the method allows the optimization of acoustic sources also considering loudspeaker directivities, a base flow or thermal stratification. It is based on the Euler equations and the corresponding adjoint which are solved by means of computational aeroacoustic (CAA) techniques. Both, appropriate driving functions and appropriate positions of the sources for the generation of a desired sound field can be determined. In the following, we will present new validation examples for two-dimensional arrangements: a circular array consisting of monopole loudspeakers, and one speaker with a circular piston directivity. It will be shown that prespecified driving functions – amplitudes and phases – can be regained or that the corresponding target sound fields can be determined using the adjoint-based method.

Governing equations

The governing equations for the computations are the two-dimensional EULER-equations [3], abbreviated by E in terms of a space-time-solution. The full state, including density, velocities and pressure, as energy quantity, is accumulated in the state vector q. In order to model acoustic loudspeakers, the equation is extended by a right-hand-side $f(x_i, t)$, only affecting the pressure equation.

$$E(q) = f \tag{1}$$

The all-encompassing idea is to optimize this term by means of an adjoint-based approach and, thus, to determine optimal driving functions for real loudspeaker arrays.

Adjoint-based approach

The required high-dimensional gradient for optimization of the loudspeakers – represented by $f_i(t)$ – is determined by a continuous adjoint-based approach, cf. [3], which aims at minimizing the following objective function, (2) and (3). The objective is defined as integral difference between a current solution of the EULER-equations q and a desired target state q_{target}.

$$J = \frac{1}{2} \iint_{\Omega} \left(q - q_{\text{target}}\right)^2 \mathrm{d}\Omega \tag{2}$$

$$\delta J = \iint_{\Omega} \underbrace{\left(q - q_{\text{target}}\right)}_{g} \delta q \mathrm{d}\Omega = g^{\mathrm{T}} \delta q \tag{3}$$

By combining the (linearized) EULER-equations E_{lin} and the (linearized) objective in a LAGRANGIAN manner, and a simple rearrangement the adjoint EULER-equations are defined.

$$\delta J = g^{\mathrm{T}} \delta q - (q^*)^{\mathrm{T}} \underbrace{\left(E_{\text{lin}} \delta q - \delta f\right)}_{=0} \tag{4}$$

$$= \delta q^{\mathrm{T}} \underbrace{\left(g - E_{\text{lin}}^{\mathrm{T}} q^*\right)}_{=0,\ \text{adjoint eqn.}} + (q^*)^{\mathrm{T}} \delta f \tag{5}$$

Thus, the change of the objective becomes independent of the change of the state δq. The required high-dimensional gradient results in

$$\delta J = {q^*}^{\mathrm{T}} \delta f \rightarrow \frac{\delta J}{\delta f} = q^* \approx \nabla_f J. \tag{6}$$

It is given by the adjoint solution. More details on the derivation of the adjoint EULER-equations can be found in [3, 4, 5]
The adjoint-based gradient is used in an iterative manner, see Fig. 1. The desired optimized driving functions for the speakers are obtained by deconvolution of the optimal f_i with the input signal creating the target sound field.

Computational setup

The governing and the corresponding adjoint equations are discretized by finite differences. A sixth-order accurate compact symmetric derivation stencil is used. The sound reinforcement area under consideration ranges over 1.2 m × 1.2 m and is discretized by a uniform grid which consists of 181 × 181 grid points. The computational time span is separated into equidistant time steps using a CFL-condition equal to 0.96, which corresponds to a sampling rate of ∼53.33 kHz. An explicit fourth-order Runge-Kutta scheme is used for the time-wise integration. All boundaries are treated as non-reflecting using characteristic boundary conditions in combination with a quadratic sponge layer. Initial and boundary conditions are chosen to realize a speed of sound equal to 343 m/s.

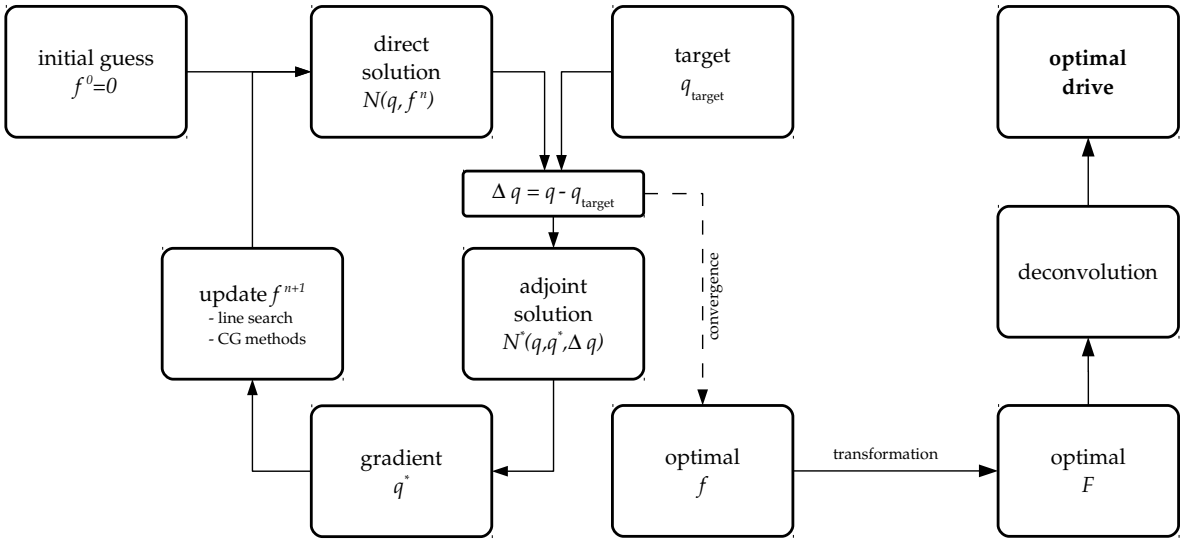

Figure 1: Iterative process for determining optimal driving functions: Starting from an initial guess for the loudspeakers, the governing equations are solved forward in time. The result is compared to the desired target state, while the actual difference drives the adjoint equations, which are solved backward in time. Based on the adjoint solution, the gradient $\nabla_f J$ is used to update the actual forcing. Once convergence is reached, deconvolution of optimal f's with the target input signal results in optimal drives for the speakers.

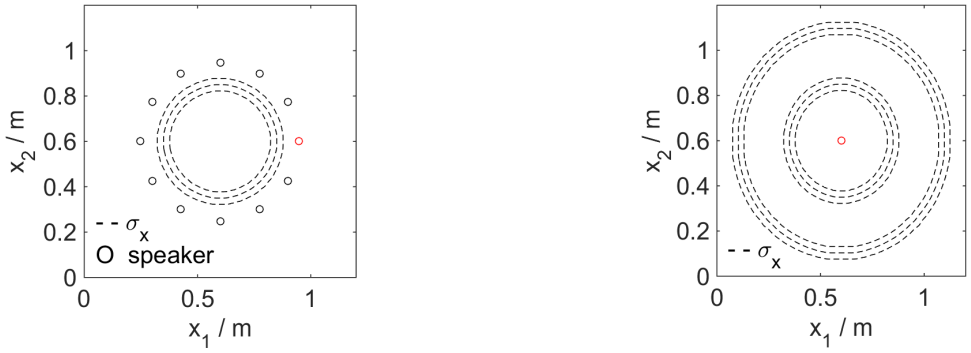

Figure 2: Setups under consideration: (Left): 12 monopole speakers in circular arrangement. (Right): Single speaker with circular piston directivity. In both graphics, the dashed contour lines mark the area, where the objective function is evaluated.

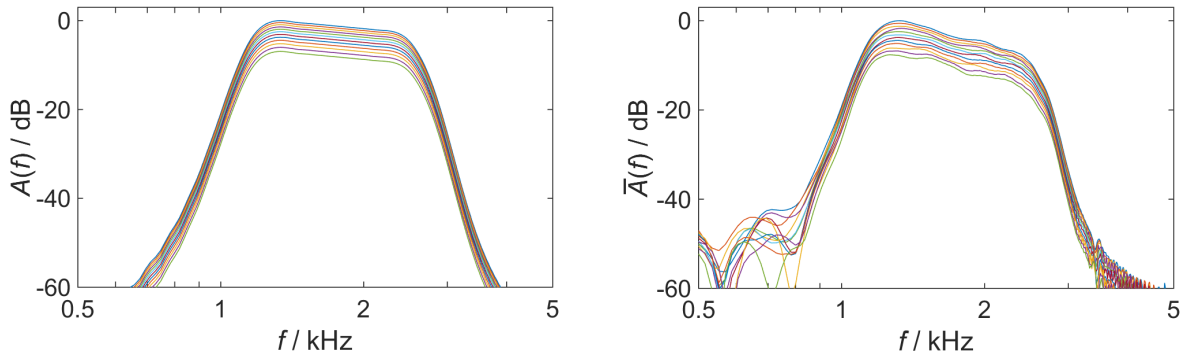

Figure 3: (Left): Amplitude frequency responses of the reference input signals. (Right): Corresponding curves resulting from adjoint-based optimization. Please note the different definitions: resulting pressure fluctuation (Left) and forcing amplitude (Right).

Results

Two different configurations are examined. The first configuration includes 12 monopole speakers arranged in a circular manner, see Fig. 2. The objective is restricted to the area enclosed by the speakers. As reference input signal, a logarithmic sweep from 1 kHz to 3 kHz is used. After 30 iterations the reference sound field, encoded

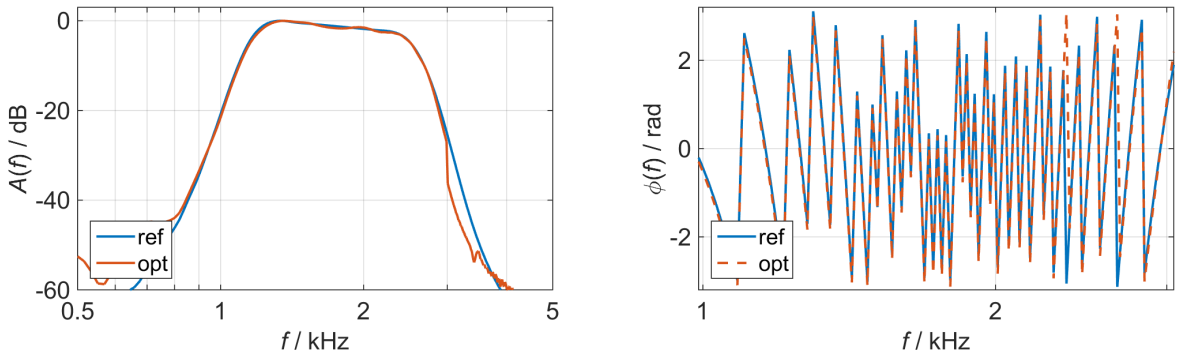

Figure 4: (Left): Amplitude frequency response of speaker (1) using quadratic conversion for the optimization results. Within the considered frequency range from 1 kHz to 3 kHz, the deviation is smaller than 1 dB. (Right): Phase frequency response of speaker (1) using a constant phase shift of $\pi/2$. Within the considered frequency range from 1 kHz to 3 kHz, the deviation is smaller than 0.15 rad.

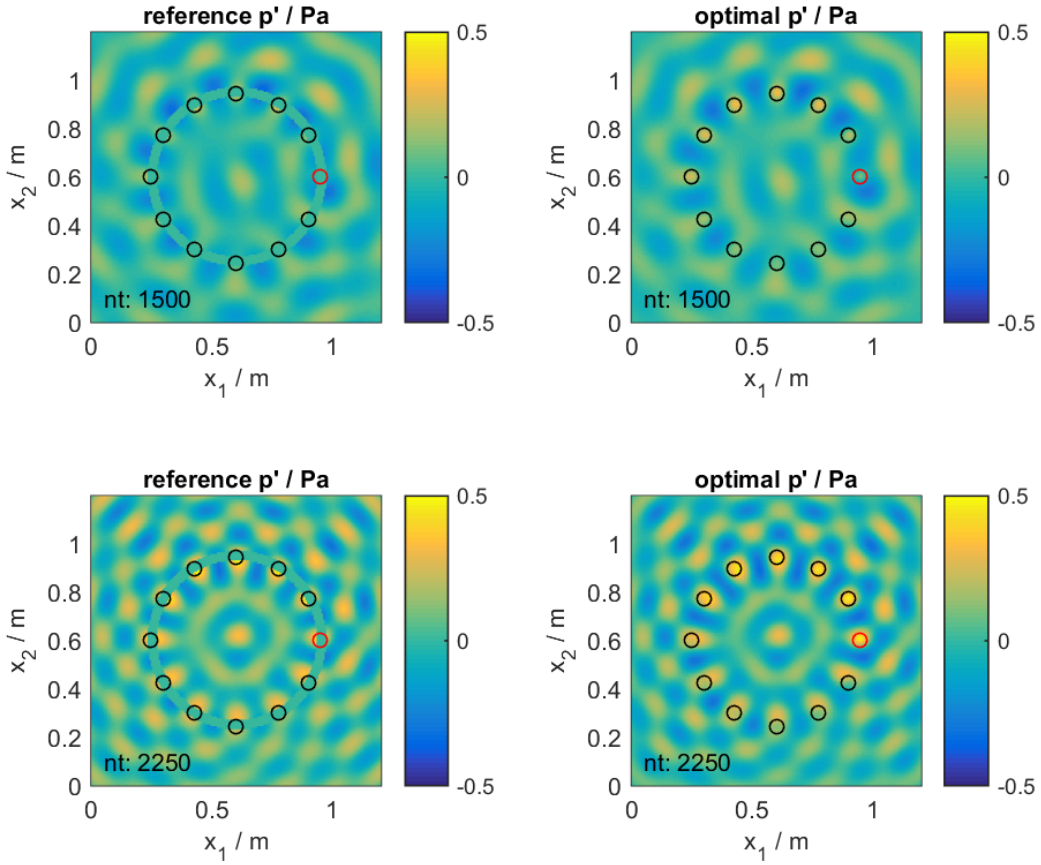

Figure 5: (Left): The reference sound field and (Right): the optimized sound field after 30 iterations of the adjoint-based approach for the time steps 1500 (Top) and 2250 (Bottom) for the circular array configuration. Please note, the sponge region is also shown.

in q_{target}, is reproduced, see Fig. 5. The frequency-dependent amplitude responses for all speakers are identified, see Fig. 3. Within the sweep frequency range, the optimized amplitude ratios are in accordance with the reference ratios. The required conversion from disturbance (in the reference curve) to source definition (as result of the optimization) is identified as quadratic function corresponding to the analytic solution [1].
A detailed analysis of a single speaker, marked in red in Fig. 2, proves that the optimized amplitude and phase responses vastly match the reference ones, see Fig. 4. The resulting amplitude deviations are smaller than 1 dB within the sweep frequency range. The results are representative for all speakers.
The second selected configuration consists of a single speaker centered in the sound reinforcement area. This speaker features a circular piston directivity. In order to reproduce the resulting target sound field, the

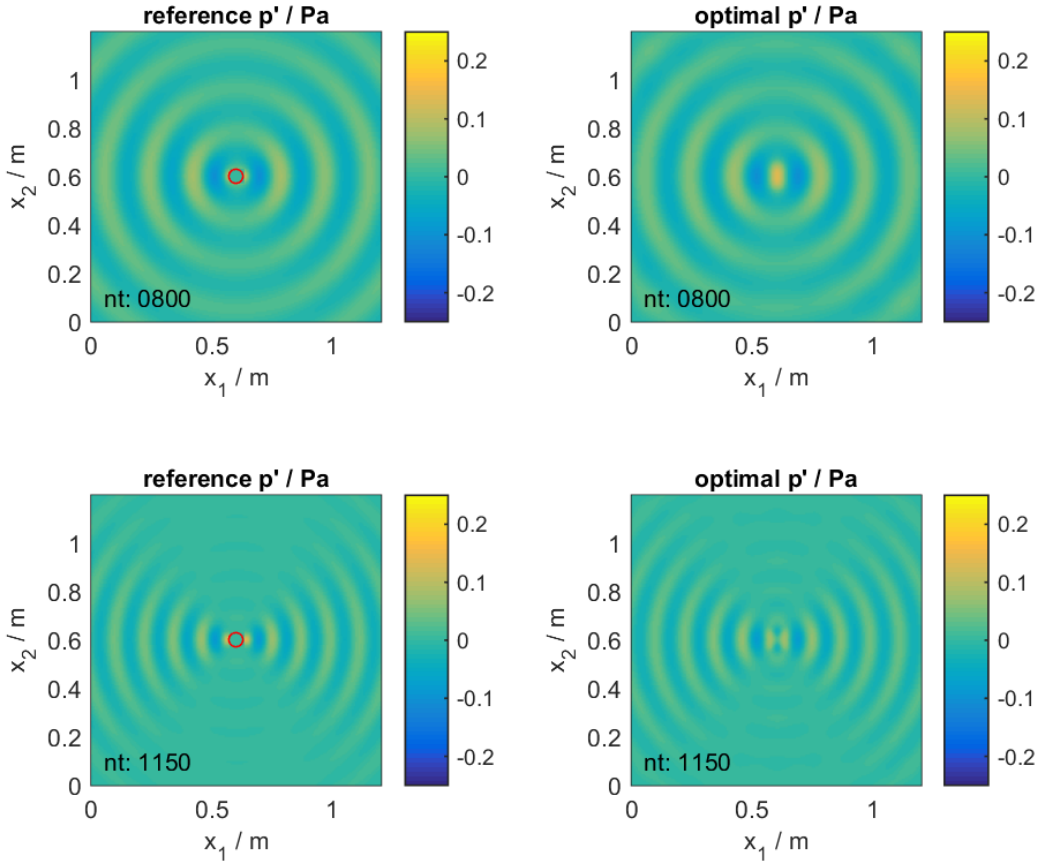

Figure 6: (Left): The reference sound field and (Right): the optimized sound field after 150 iterations of the adjoint-based approach for the time steps 800 (Top) and 1150 (Bottom) for the single speaker configuration with circular piston directivity. Please note, the sponge region is also shown.

adjoint-based framework is used to optimize more than 60 monopole sources, located at the grid nodes in the center region of the computational domain, following the monopole synthesis approach [2, 7].
In Fig. 6, it can be seen, that the reference sound field is reproduced in good quality. Using more monopole sources for the synthesis improves the results, while requiring the same computational time. Thus, it is shown, that the adjoint-based approach is capable of handling loudspeaker directivities.

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The authors gratefully acknowledge the support of the Deutsche Forschungsgemeinschaft (DFG) within the projects LE 3888/2-1 and WE 4057/16-1.