Substituting Traditional Piano Soundboard by Synthetic Layered Material Without Ribs

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Introduction

Traditional piano soundboards are complex structures where the main resonator (plate) is stiffened and slightly bowed with ribs on the bottom, while on the upper side bridges are installed. The main resonator is traditionally built from solid music wood, but nowadays, because of environmental and economical motives, manufactured materials are gaining importance. Not only wood resonators but also multi-layer composites have stochastic parameters through variation of fibre orientation and thickness.

The influence of the piano soundboard on the tone quality and the modal behaviour of the piano soundboard is widely examined. Measurements are taken (e.g. [5, 7]), and based on these results there are numerous articles discussing how to model a piano soundboard (e.g. [2, 3]. The typical material properties and modelling techniques of wood (e.g. [3, 4]) are well documented, but in all cases material properties are handled as deterministic. Frequency response and vibrational properties of wood pieces were also measured (e.g. [11]).

Since wood is also a hierarchically structured composite [17], and the technology has developed drastically in recent years, manufactured composite soundboards seem to be a good alternative. Numerous manufactured materials (e.g. polyester and bio-composites, modified woods) are recently examined for acoustical purposes concluding that their performance can be similar to wood (e.g. [1, 12, 13]. Stability conditions of hypothetical materials for simulation purpose are also documented [10].

In this paper we discuss the possibilities of using substitute materials in rib-less piano soundboards. In the next sections we briefly introduce our model and present some numerical simulation results.

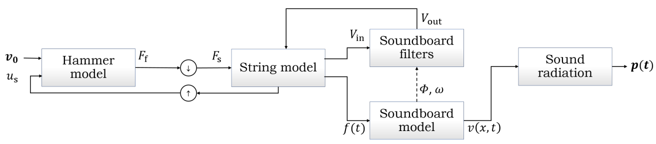

Figure 1: Piano model overview – block diagram.

Soundboard Model Overview

In our case the modelling goal of the soundboard is to determine the **stochastic modal description** (stochastic eigenfrequencies ($\omega_{k\theta}$) and mode shapes ($\Phi_{k\theta}$)) of the structure, caused by naturally variation of fibre orientation. Based on this the probable behaviour of the resonator can be estimated and the required filter set can be parametrised for further simulations (s.a. Fig. 1). For this purpose a stochastic standard eigenvalue problem of the form

$$\left(\mathcal{M}_\theta{}^{-1} \cdot \mathcal{K}_\theta - \omega^2 \cdot I\right) \cdot U_\theta(\omega, x) = 0\,, \qquad (1)$$

has to be solved, where $\mathcal{M}_\theta$ is the so-called stochastic mass-, $\mathcal{K}_\theta$ is the stochastic stiffness, I is the identity matrix and U_θ stands for the stochastic displacement.

The mass- and stiffness matrices are the outputs of the numerical model of the geometry based on the **finite element approach**. According to this the dynamics of an elastic volume (Ω) is determined through discretisation of the geometry and function space. In case of ribbed soundboard, we apply also the master-slave multi-freedom constrain theory, resulting in the same number of nodes as in rib-less case. The resulting stochastic equation of forces and moments is derived from the Kirchoff-Love thin plate theory. The solution for the stochastic displacement is given by a superposition of finite number of polynomial base functions defined over these standard elements.

The resulting mode shapes and modal weights ($\alpha_{k\theta}$) form a proper orthonormal base for each realisation θ of the uncertain parameter, that is modelled as **stochastic processes** using the Karhuen-Loève expansion:

$$\beta(x, \theta) = \sum_{k=1}^{\infty} \xi_k(\theta) \cdot g_k(x)\,. \qquad (2)$$

In Eq. (2) $\xi_k(\theta)$ are the coefficients and $g_k(x)$ are an orthonormal base over space x. For a given θ Eq. (1) can be solved in a deterministic way.

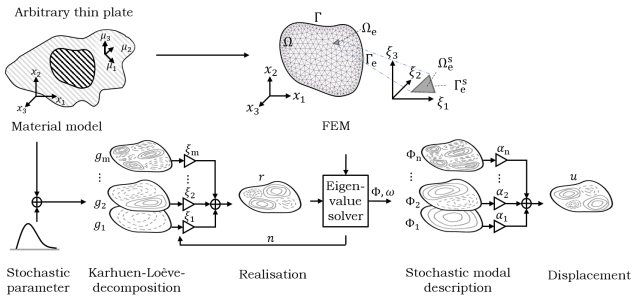

Figure 2: Soundboard model overview.

For a more detailed soundboard model description s.a. [8] and [9].

The Material Model

The material model is based on the generalised Hooke's law:

$$\sigma = D \cdot \varepsilon, \tag{3}$$

where σ stand for stress, ε for strain and D for the material tensor. In our modelling case the material tensor can be reduced to 6×6 matrix.

The naturally grain patterns of **wood** is not only aesthetically pleasing but determines its mechanical properties. Wood is mostly described as orthotropic material: 'The longitudinal axis is parallel to the fiber (grain); the radial axis is normal to the growth rings (perpendicular to the grain in the radial direction); and the tangential axis is perpendicular to the grain but tangent to the growth rings.' ([14] p. 5-1) In case of thin plates orthotropic

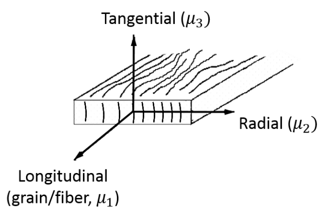

Figure 3: Wood section model.

materials can also be modelled as transversally isotropic, where the material properties are homogeneous in the 2-3 plane. Further simplification is possible in case of ribs since the longitudinal direction is dominant. They can be also modelled as isotropic materials.

High quality piano soundboards are made of spruce (especially Sitca or Norway spruce). The properties of a single wood species are constant within limits. [17] The typical mechanical parameters (density ρ, Young's moduli E_i, Poisson's ratio ν_ij and shear moduli G_ij) for simulated spruce are between the values shown in Tab. 1.

Table 1: Typical Mechanical Properties of Spruce

Property	Metric	Min.	Max.
ρ	kg m^{-3}	380	440
E_1	GPa	10	15.9
E_2	GPa	0.47	0.9
ν_{12}	1	0.26	0.44
G_{12}	GPa	0.72	1.2
G_{13}	GPa	3.9	4.2

The mechanical properties and frequency response of **manufactured materials** and developed constructions have to be as similar as possible to the traditional wood ones. To get a stable substitute of wood some criteria have to be reached. Schelleng determined two criteria for homogeneous materials with same flexural behaviour. One of these is about the stiffness

$$E_\mathrm{w} \cdot h_\mathrm{w}^3 = E_\mathrm{c} \cdot h_\mathrm{c}^3 \,, \tag{4}$$

and one for areal density

$$\rho_\mathrm{w} \cdot h_\mathrm{w}^3 = \rho_\mathrm{c} \cdot h_\mathrm{c}^3 \,, \tag{5}$$

where h_i-s are the thickness of wood and composite material. [15] Further it is shown that for inhomogeneous materials high degree of anisotropy and low internal friction also play an important role. [13]

While in most documented cases, the material properties of composites are predefined parameters Li and Barbic investigated to fine tuning of these parameters. [10] They defined a Poisson's ratio like parameter ν so that $-1 < \nu < 1/2$ and the Poisson's ratios are

$$\nu_{12} = \nu \cdot \sqrt{\frac{E_1}{E_2}}, \quad \nu_{23} = \nu \cdot \sqrt{\frac{E_2}{E_3}}, \quad \nu_{31} = \nu \cdot \sqrt{\frac{E_3}{E_1}}\,. \tag{6}$$

The shear moduli (G_ij) are predicted as

$$G_\mathrm{ij} = \frac{\sqrt{E_\mathrm{i} \cdot E_\mathrm{j}}}{2 \cdot \left(1 + \sqrt{\nu_\mathrm{ij} \cdot \nu_\mathrm{ji}}\right)}\,. \tag{7}$$

Fitting of mechanical behaviour is possible by laminating, carefully designing of thickness and fibre orientation profile and adopting sandwich structural concept. [6] In case of **laminated resonators** ply-wise determination of the material matrix is possible.

Numerical Example

In following paragraphs we present some numerical examples on a grand-piano-like plate. The plate is 1.50 m high, 8 mm thick and clamped along the boundary. We use four different set-ups, as shown in Fig. 4. In all cases deterministic and stochastic variant is also demonstrated. For stochastic examples we performed Monte Carlo simulations: because of the mode exchanges, application of sparse grid techniques would be limited. The realisations are created based on the first 100 KL-modes. Our examples are restricted to mode shapes and eigenfrequencies. The questions of modal damping are not discussed.

In the first case we simulated a solid spruce resonator without ribs to be able to show the dominant effect of the ribs on mode shapes and eigenfrequencies later. The material properties are defined as documented in [16]. The longitudinal axis is set parallel to the grain ($-60°$) and perpendicular to ribs. For the deterministic simulation the first three dynamic modes are presented in Fig. 5. Stochastic simulation carried out applying 20° deviation on the previous defined fibre orientation resulting slightly change in the expected values of the modes by 5% deviation as shown in Fig. 6.

Thirteen beech ribs are installed. The material properties are based on [4]. Each rib is 1 cm wide and 2 cm

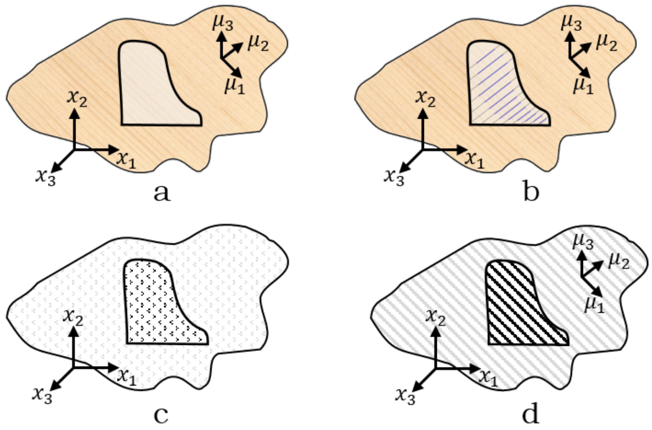

Figure 4: Example geometry and materials. a: solid spruce plate, b: solid spruce plate with ribs, c: composite lamina plate without ribs, solid plate using hypothetical material

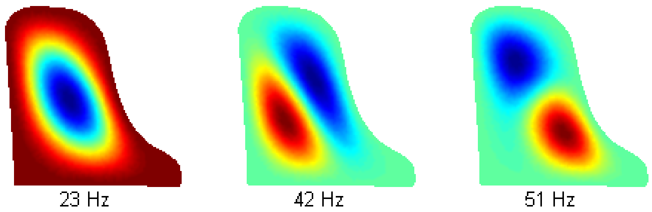

Figure 5: Solid plate modes.

high in the middle section. In the boundary section the height is set to 1 cm to imitate a more realistic constellation. The positioning of the ribs and the sections are shown in Fig. 4b. As expected eigenfrequencies are shifted upwards and the mode shapes deform because of the stiffening effect of the ribs (s.a. Fig. 7). Although the fibre orientation varies in an unrealistic wide range (same 20° is set) the standard deviation of the eigenfrequencies is decreased (s.a. Fig. 8). The ribs are dominant in the radial direction. Our goal is to substitute this structure by manufactured layered material without the necessity to install ribs.

As a first step we choose a parameter set for unidirectionally carbon fibre reinforced composite classified as spruce like by Ono [12] (p.139, Table 2, CF/UF T1-1). The missing Poisson's ratio is estimated by Eq. (6) with an arbitrary chosen $\nu = 0.1$ parameter. The geometry is split in 8 equally thick layers. The fibre orientation is set symmetric from the most outer to inner one: $[30°, -40°, 70°, 0°]$. Because of the rib-less construction, the fibre orientation is selected parallel to original ribs in the most outer layer.

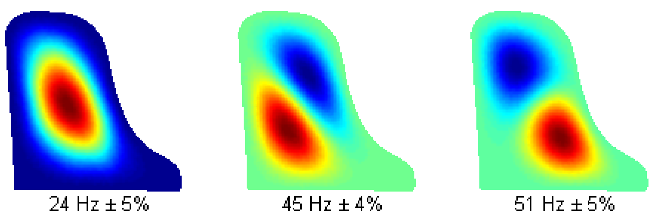

Figure 6: Stochastic solid plate modes.

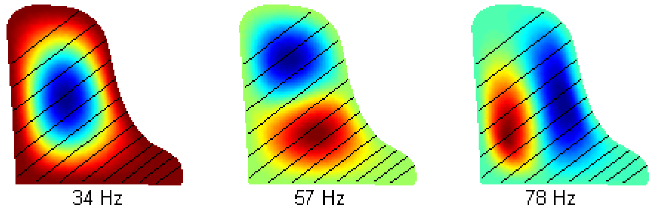

Figure 7: Solid plate modes with ribs.

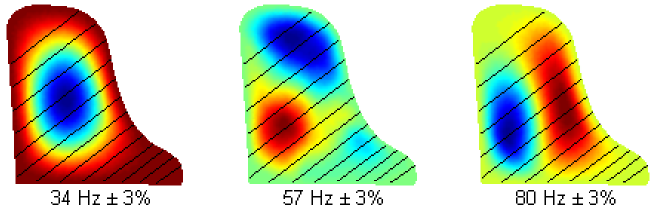

Figure 8: Stochastic solid plate modes with ribs.

As shown in Fig. 9, except the first one, the modes are

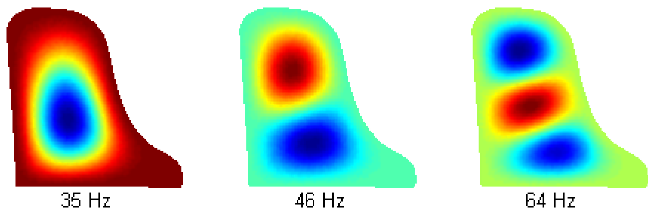

Figure 9: Laminated plate modes.

not met through this naive strategy. But the deviation of the modes in stochastic case is decreased to about 2%. The layered structure compensates the differences (s.a. Fig. 10). The modal behaviour could be fitted better by careful selection of layer thickness and fibre orientation.

As second step we tried to estimate a parameter set based on spruce and beech applying Eq. (6) and (7). Because of the insufficient knowledge of the mechanical parameters of the ribbed construction, desired parameters for estimation are determined as following: Young's moduli are estimated for each direction by algebraic mean. Material density is the average of the density parameter of the two given materials. The Poisson's ratio like parameter is set as $\nu = (\nu_{12_{spruce}} + \nu_{12_{beech}})/2$. Despite the simple approach the modes are quite well represented (s.a. Fig. 11). The variation of the eigenfrequencies in the stochastic case surprisingly is approaching for 0%. This can be caused by the resulting value of Young's moduli: the difference between the longitudinal and radial values is much less than in the original material selection. The homogeneous selection of the material has greater effect on stochastic behaviour than the naive selected fibre orientation in layered variant or the stiffening effect of the installed ribs. The stability and practicability of such an estimated material are not examined.

Conclusion and Future Work

We presented a stochastic soundboard model applying different materials. The ribbed wood structure is sub-

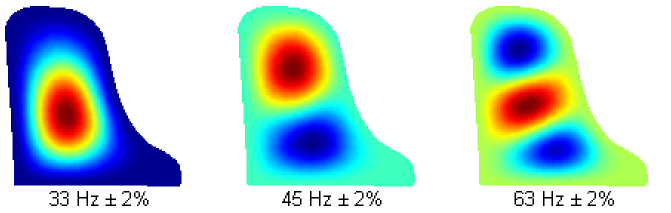

Figure 10: Stochastic laminated plate modes.

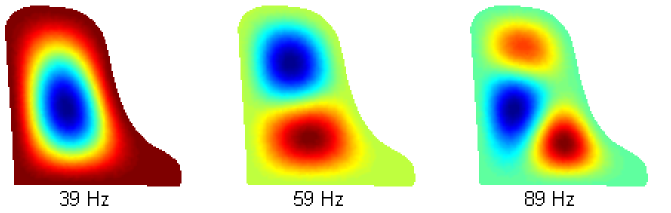

Figure 11: Solid plate modes in case of hypothetical material valid in deterministic and stochastic case as well (Standard deviation is approaching for 0%).

stituted by multi-layer carbon fibre reinforced composite and by a hypothetical parameter set. The similarity is demonstrated by the first three mode shapes and eigenfrequencies. Higher order modes and modal damping is not further examined. Manufacturing questions are not discussed.

In our simple example was shown that not only the ribs becomes dominant in the radial direction, so the effect of stochastic fibre orientation is decreased, but these effects can also be compensated through material selection and layering.

In the future there are many open questions to solve. Monte Carlo simulation is of great computational effort. The simulations are affected by very intensive mode exchanges especially in case of higher order modes, so sparse grid are not directly applicable. So we plan to develop a suitable and efficient computation method to solve this problem. Furthermore we would like to define tolerances for stochastic parameters under given quality measures.

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