Uncertainty quantification analysis for torsional vibration of crankshaft based on generalized polynomial chaos expansion

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Introduction

Torsional vibration is the main vibration form for crankshaft with multi-support in power machine, which directly affects the operational reliability of power machine. The traditional torsional vibration analysis of crankshaft is conducted by considering the system parameters as deterministic values [1,2], however, there are some uncertain factors in the torsional vibration analysis, such as uncertainty in geometric parameters, materials and some error between real crankshaft and simplified analytical model. The deterministic analysis may cause the wrong estimation for torsional vibration and even cause serious security problem in engineering. Therefore, it is of great significance to carry out uncertainty quantification analysis for the torsional vibration of the crankshaft.

In recent years, generalized Polynomial Chaos (gPC) expansion has drawn a great attention due to its high-efficiency in addressing the uncertain problem [3–5]. Compared with the traditional sampling-based method such as Monte Carlo method which has a high computational cost, gPC expansion is more efficient [6]. It convert the uncertain problems to the deterministic problems by representing the uncertain parameters as the function of polynomials related to the random variables. Until now, some researchers have applied gPC expansion to many areas, including vibration problem. For example, Sepahvand studied the random harmonic analysis of composite plates using finite element method combined with gPC expansion [7]. Guerine et al. proposed a method that taking uncertainties into account based on the projection on polynomial chaos [8]. However, the torsional vibration of crankshaft considering the uncertainty still need further investigation.

In this paper, two kinds of gPC expansion methods, intrusive gPC and non-intrusive gPC, are introduced to torsional vibration analysis of crankshaft, respectively. For intrusive method, the constructed gPC expansion for uncertain input parameters are substituted into the analysis model to obtain the direct relationship between random variables and output, then the stochastic free torsional vibration response of crankshaft obtained. For non-intrusive method, the output is directly represented as function of polynomials with coefficients, in which these coefficients are calculated by the least square method at some special points, then the stochastic free torsional vibration response of crankshaft also obtained without a complex expression.

Torsional vibration model of crankshaft and random parameters

Lumped parameter model is commonly used for torsional vibration analysis of crankshaft, and it is usually viewed as two parts, one is moment of inertia while another is torsional stiffness, and damping is always ignored in free vibration response. The Lumped parameter model of crankshaft can be seen in Fig. 1.

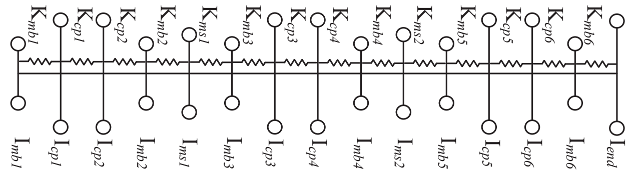


Figure 1: Lumped parameter model of crankshaft

To determine these parameters, some empirical formula are employed in the early stage [9], for example, torsional stiffness of crank part can be calculated by Eq. (1)

$$K = \frac{32}{\pi G}\Big[\frac{l_1 + 0.6h\frac{D_1}{l_1}}{D_1^4} + \frac{0.8l_2 + 0.2b\frac{D_1}{r}}{D_2^4} + \frac{r\sqrt{r}}{\sqrt{D_1}}\frac{1}{hb}\Big] \quad (1)$$

In which l, h, D, b, r are structural parameters, and G is shear modulus.

With the development of CAD and CAE software, these lumped parameters can be measured by CAD and CAE software, for example, the moment of inertia can be measured directly in CAD software, while torsional stiffness can be calculated by CAE technique [10,11].

However, whether it is the empirical formula method or CAD/CAE method, they are all not precise enough due to the model simplification. Therefore, parameters of analysis model are taken as uncertain parameters, and assuming these parameters are related to random variables, and the distribution of these parameters are predefined as shown in Table 1:

Basic theory for gPC expansion

The basic idea of gPC expansion is to project the variables of system onto a stochastic space spanned by a set of complete orthogonal polynomial which are the function of a random variable, for example, an uncertain parameter $\boldsymbol{\chi}$ can be represented as

Table 1: Distribution of lumped parameters

Parameters	value
Moment of inertia $/kg \cdot m^2$	$Imb \sim U(0.255, 0.275)$ $Icp \sim U(0.476, 0.496)$ $Ims \sim U(0.33, 0.35)$ $Iend \sim U(0.204, 0.224)$
Torsional stiffness $/MN \cdot m/rad$	$Kmb \sim N(44, 0.3)$ $Kcp \sim N(38.1, 0.3)$ $Kms \sim N(21.9, 0.3)$ $Kend \sim N(63, 0.3)$

$$\begin{aligned}\chi =& x_0\Psi_0 + \sum_{i_1=1}^{\infty} x_{i_1}\Psi_1(\xi_{i_1}) + \sum_{i_1=1}^{\infty}\sum_{i_2=1}^{i_1} x_{i_1 i_2}\Psi_2(\xi_{i_1},\xi_{i_2}) \\ &+ \sum_{i_1=1}^{\infty}\sum_{i_2=1}^{i_1}\sum_{i_3=1}^{i_2} x_{i_1 i_2 i_3}\Psi_3(\xi_{i_1},\xi_{i_2},\xi_{i_3}) + \cdots\end{aligned} \tag{2}$$

Or, it can be written as,

$$\chi = \sum_{i=0}^{\infty} x_i \Psi_i(\boldsymbol{\xi}) \tag{3}$$

In which x_i are unknown coefficients of polynomial Ψ_i, while Ψ_i are a set of multidimensional polynomials with respect to the multidimensional random variables ξ with the orthogonality relationship, and in practical simulation, the series in Eq. (3) is commonly truncated to a finite number of terms and denoted by N. In addition, the selection of polynomial Ψ_i depend on the distribution type of random variable , for example, the Legendre orthogonal polynomial are the optimal basis for the random variable with uniform distribution, while the Hermite orthogonal polynomial are the optimal basis for the random variable with normal distribution, and so on [3].

Usually, there are two kind of methods for gPC expansion: intrusive and non-intrusive. For intrusive method, the related equations must be accessed, which means the relationship between the input parameters and output targets must be known. For non-intrusive method, the specific function between input and output can be considered as a black-box, which are not necessary to known. The details and their application on torsional vibration of crankshaft will be discussed in subsequent sections.

Intrusive gPC expansion for torsional vibration of crankshaft

Intrusive method is substituting the polynomial of random variables into the detailed relationship between input parameters and output parameters, where this relationship can be solved in deterministic analysis.

The torsional vibration of crankshaft are usually described as

$$[I]\{\ddot{\theta}(t)\} + [C]\{\dot{\theta}(t)\} + [K]\{\theta(t)\} = \{T(t)\} \tag{4}$$

where $[I]$, $[C]$ and $[K]$ represent the matrix of moment of inertia, damping and torsional stiffness, $\{\ddot{\theta}\}$, $\{\dot{\theta}\}$ and $\{\theta\}$ represent the vector of angular acceleration, angular velocity and angular displacement.

Then, the undamped torsional natural frequency (TNF) of crankshaft can be solved by Eq. (5)

$$[\omega_n^2] = [I]^{-1}[K] \tag{5}$$

Taking the undamped TNF as the output target, then the deterministic result can be solved by substituting the deterministic value into Eq. (5). But, when considering the input parameters as random variables, the Eq. (5) can be rewritten as

$$[\omega_n^2(\boldsymbol{\xi})] = [I(\boldsymbol{\xi})]^{-1}[K(\boldsymbol{\xi})] \tag{6}$$

here, $\boldsymbol{\xi} = \{\xi_1, \xi_2, \cdots, \xi_n\}$ is a vector of random variables. And the element of matrix of moment of inertia and torsional stiffness related to $\boldsymbol{\xi}$ can be represented as

$$I_i = \sum_{j=0}^{N_{I_i}} I_{i_j}\Psi_j(\boldsymbol{\xi}) \tag{7}$$

$$K_i = \sum_{j=0}^{N_{K_i}} K_{i_j}\Psi_j(\boldsymbol{\xi}) \tag{8}$$

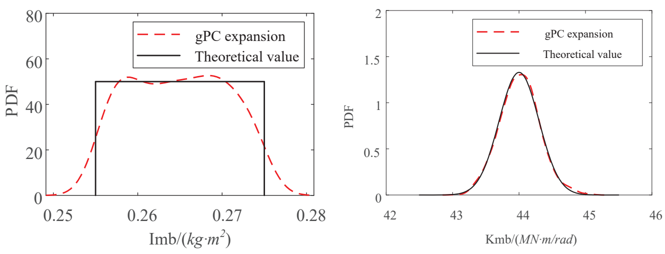
where I_{i_j} and K_{i_j} are the coefficients of elements which can be solved by Galerkin projection, for example, the solving process of coefficients of moment of inertia by Galerkin projection like Eq. (9)

$$I_{i_j} = \frac{\langle \boldsymbol{\chi}, \Psi_j(\boldsymbol{\xi})\rangle}{\Psi_j^2(\boldsymbol{\xi})} \tag{9}$$

After the coefficients obtained, the uncertain parameters can be described by gPC expansion. Among that, the coefficients of uncertain parameter I_{mb} and K_{mb} described by 2 order IgPC can be seen in table 2 and their Probability Density Function (PDF) can be seen in Fig. 2.

Table 2: The coefficients of gPC of Imb and Kmb

Parameters	Coefficients					
	C_0	C_1	C_2	C_3	C_4	C_5
I_{mbi}	0.265	0.01	0	0	0	0
K_{mbi}	44	0.3	0	0	0	0



(a) Imb (b) Kmb

Figure 2: gPC expansion and theoretical value of Imb and Kmb

From the picture, it is showed that the 2 order PC expansion can well meet the MC simulation. In addition, according to the accuracy calculation method defined in reference [3], the error in evaluation of statistical moments are all 0, which is reasonable for the optimal polynomial. Otherwise, the representation maybe need higher order expansion with lower accuracy for the non-optimal polynomial.

Then, the TNF under uncertainty can be solved by substituting Eq. (7) and (8) into Eq. (6). The comparison of PDF of of 1^{st} order TNF between different order IgPC expansion and MC simulation with 1000 points can be seen in Fig.3. However, the PDF representations for different order representations shows no remarkable difference, and are all close to that for MC simulation. This is due to the reason that coefficients after C_1 are all zero, which means 1 order is accurate enough for the representation of parameters. The problem which can not be ignored is that the complexity of response function described by IgPC will be rapidly grows with the increasing dimension of randoms variables and complexity of deterministic response function.

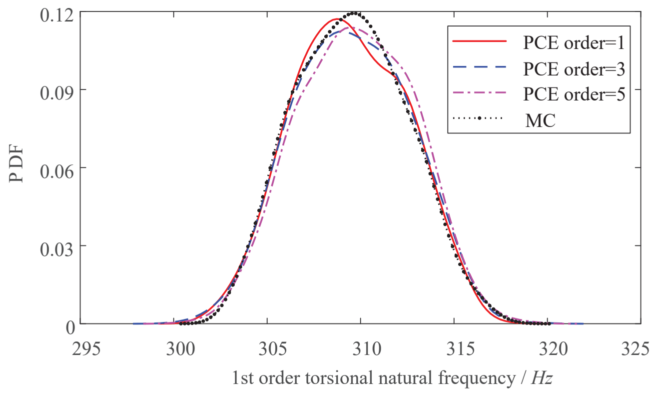


Figure 3: IgPC expansion and MC of 1^{st} order TNF

Non-intrusive gPC expansion for torsional vibration of crankshaft

Non-intrusive method is suitable for the system without the specific function between input and output, or the function can be seen as a black-box. In this method, the output is directly represented as gPC expansion like Eq. (10)

$$\omega_{n_i} = \sum_{j=0}^{N_{\omega_{n_i}}} \omega_{n_{i_j}} \Psi_j(\boldsymbol{\xi}) \tag{10}$$

Here, $\Psi_j(\boldsymbol{\xi})$ is polynomial, which can be determined by the types of multidimensional random variables $\boldsymbol{\xi}$, for example, if $\boldsymbol{\xi} = \{\xi_1, \xi_2\}$, and ξ_1 is random variable with normal distribution which optimal polynomial is Hermite polynomial $H_j(\xi_1)$ while ξ_2 is random variable with uniform distribution which best polynomial is Legendre polynomial $L_j(\xi_2)$, then polynomial of output can be assumed as

$$\Psi_j(\boldsymbol{\xi}) = H_j(\xi_1) \otimes L_j(\xi_2) \tag{11}$$

Then, Eq. (10) can be viewed as a function with $N_{\omega_{n_i}}$ unknown coefficients, in which $N_{\omega_{n_i}}$ is the number of term of polynomial. In order to solve these coefficients, at least $N_{\omega_{n_i}}$ equations is demanded, in other words, at least $N_{\omega_{n_i}}$ response samples should be known. As mentioned above, these samples can be solved through the black-box.

After that, these coefficients can be solved by mathematical method like the least squares method. This kind of method used to solve coefficients is called collocation method. The problem what is worth to pay attention in this method is that the selection of these sample points since the inappropriate points may affect the accuracy of coefficients. In such a case, zero and the root of one higher order polynomial are selected as the collocation points to minimize the approximation error [12].

When NIgPC is employed to uncertain torsional vibration analysis of crankshaft, the process of calculating the TNF is viewed as black-box, and the $\Psi_j(\boldsymbol{\xi})$ can be determined according to the distribution of parameters defined in previous section,

$$\begin{aligned} \Psi_0(\boldsymbol{\xi}) &= H_0(\xi_1) * L_0(\xi_2) = 1 \\ \Psi_1(\boldsymbol{\xi}) &= H_0(\xi_1) * L_1(\xi_2) = \xi_1 \\ \Psi_2(\boldsymbol{\xi}) &= H_1(\xi_1) * L_0(\xi_2) = \xi_2 \\ \Psi_3(\boldsymbol{\xi}) &= H_1(\xi_1) * L_1(\xi_2) = \xi_1 * \xi_2 \\ &\vdots \end{aligned} \tag{12}$$

The coefficients of 1^{st} order TNF described by 3 order NIgPC can be seen in table 3. The PDF of 1^{st} order TNF for different order NIgPC expansion and MC simulation with 1000 points can be seen in Fig. 4. Mean value and variance of these simulation can be seen in table 4.

Table 3: The coefficients of NIgPC for TNF of crankshaft

Parameters	Coefficients				
w_{n_1}	C_0	C_1	C_2	C_3	C_4
	309.61	1.69	-4.35	-0.01	-0.02
	C_5	C_6	C_7	C_8	C_9
	0.06	0.00	0.00	0.00	0.00

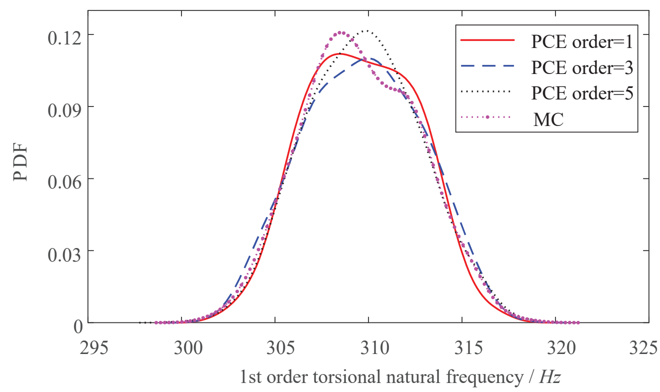


Figure 4: NIgPC expansion and MC of 1^{st} order TNF

It can be seen from table 3,table 4 and Fig. 4 that the coefficients after C_2 are close to zero, therefore, the PDF of different order NigPC expansion reveal no remarkable

Table 4: Mean value and variance of different order NIgPC and MC

Parameters	Value			
	1 order	3 order	5 order	MC
mean value	309.55	309.64	309.6	309.54
variance	9.08	9.20	9.04	9.1

difference. And the mean value and variance of different order NIgPC are close to that result of MC.

Conclusion

The torsional vibration of crankshaft under uncertainty is studied in this paper. Among that, the intrusive and non-intrusive gPC expansion method are employed to investigate the uncertain free vibration response caused by random system parameters, respectively. It is concluded that both of these two kind of gPC expansion can well describe the stochastic process of torsional vibration of crankshaft and have a better efficiency compared with MC simulation.

In addition, IgPC expansion reveals a better effect on torsional vibration model of crankshaft with a detailed response function and less random variables, because it can directly obtain the description of response with random variables, but this obtained equation will become more complex with the increasing dimension of randoms variables and complexity of deterministic response function.

As for NIgPC, it shows a better effect on the model with unknown or complex response function, because it only need to calculate a few set of response samples used to calculate the coefficients while the process of building the samples can be viewed as black-box, which can avoid the process of deriving the complex expression, although the process of building samples still need some computational cost, in particular for the case that the expansion need a higher accuracy.

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