Generalization of Wave Field Synthesis theory with application for virtual moving sources

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Introduction

Sound field synthesis aims at the physical reproduction of arbitrary target or virtual sound fields over an extended listening area by employing a densely spaced loudspeaker ensemble, termed the *secondary source distribution (SSD)*. The loudspeakers are fed with properly chosen *driving functions*, so that the resultant field of the individual SSD elements ideally coincides with the target sound field in the intended receiving area [1]. *Wave Field Synthesis (WFS)* is one of the prominent SFS methods, extracting the required driving functions from a suitable boundary integral representation of the target sound field [2, 3].

The present paper gives a brief summary of the recent results of the author concerning the topic of WFS. First, it is presented how WFS theory can be generalized towards the synthesis of arbitrary target fields by employing an arbitrary shaped SSD and optimizing the amplitude of the synthesis on an arbitrary reference curve [4]. This overcomes the limitation of previous approaches, which allow only the synthesis of specific virtual field models, ensuring amplitude correct synthesis only on linear and point-like reference curves.

Then, a simple, asymptotical anti-aliasing strategy is presented, that can be employed in order to mitigate the effect of aliasing echoes present due to the application of a discrete SSD instead of the theoretically ideal continuous one [5].

Finally, as a complex application example, it is demonstrated that all the results can be adapted with minor modifications in order to synthesize the sound field generated by a moving virtual point source [6].

Generalized WFS theory

The starting point of the generalized Wave Field Synthesis theory is the Kirchhoff integral/approximation. The integral formulates an arbitrary 3D sound field P under high frequency assumptions in terms of a boundary integral, given by

$$P(\mathbf{x},\omega) = \oint_{\mathrm{d}\Omega} \underbrace{-2\frac{\partial}{\partial \mathbf{n}} P(\mathbf{x}_0,\omega)}_{\text{3D driv. func.}} G(\mathbf{x}-\mathbf{x}_0,\omega)\,\mathrm{d}s(\mathbf{x}_0), \quad (1)$$

where $G()$ is the Green's function, describing the sound field of a 3D point source. Obviously, the integral implicitly contains the required driving function if an enclosing surface of secondary point sources is applied.

In practical applications instead of a surface, a horizontal contour of secondary point sources is employed, enclosing the planar listening region. Hence, in order to arrive at the so-called 2.5D driving function, the Kirchhoff integral has to be reduced into a contour integral. This dimensionality reduction can be performed by applying the *stationary phase approximation (SPA)*, being a central asymptotical tool in WFS theory.

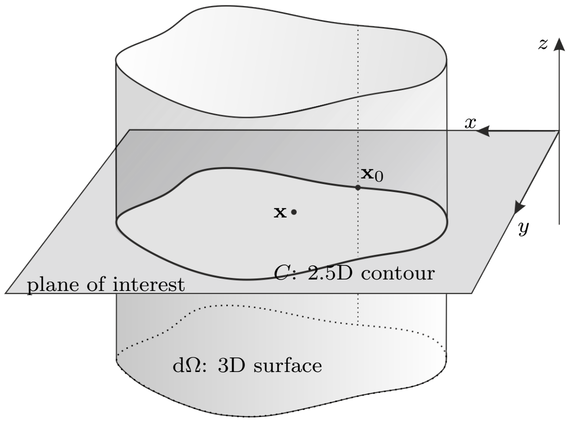

Figure 1: Geometry for the derivation of the 2.5D Kirchhoff integral

Loosely speaking, the SPA allows the evaluation of integrals of complex valued functions around critical points in the integral path, termed as the stationary points. The main idea behind the approximation is that at those regimes where the phase of the function changes, due to rapid oscillation the integral cancels out and the entire integral is dominated by those positions where the phase does not change, i.e. by the stationary positions.

In the geometry depicted in Figure 1 the Kirchhoff integral can be separated into a vertical line integral and a contour integral. By applying the SPA, the vertical integral can be evaluated as long as virtual fields propagating parallel to the horizontal plane of interest are assumed. This results in the 2.5D Kirchhoff integral, reading

$$P(\mathbf{x},\omega) = \oint_C \underbrace{w(\mathbf{x}_0)\sqrt{\frac{8\pi \mathrm{j}}{|\phi_{zz}^{P''}(\mathbf{x}_0)+\phi_{zz}^{G''}(\mathbf{x}-\mathbf{x}_0)|}}\, k_\mathrm{n}^P(\mathbf{x}_0) P(\mathbf{x}_0,\omega)}_{\text{2.5D driv. fun.}}\, G(\mathbf{x}-\mathbf{x}_0,\omega)\mathrm{d}\mathbf{x}_0. \quad (2)$$

Here, $w(\mathbf{x}_0)$ is a windowing function selecting the active secondary sources, $\phi_{zz}^{P''}$ and $\phi_{zz}^{G''}$ are the second derivatives of the phase function of P and G w.r.t the z-dimension and $k_\mathrm{n}^P(\mathbf{x}_0)$ is the normal component of the *local wavenumber vector* of wave field P, defined as

$$\mathbf{k}^P(\mathbf{x}) = [k_x^P(\mathbf{x}),\ k_y^P(\mathbf{x}),\ k_z^P(\mathbf{x})]^\mathrm{T} = -\nabla\phi^P(\mathbf{x},\omega). \quad (3)$$

The local wavenumber vector is a vector, being perpen-

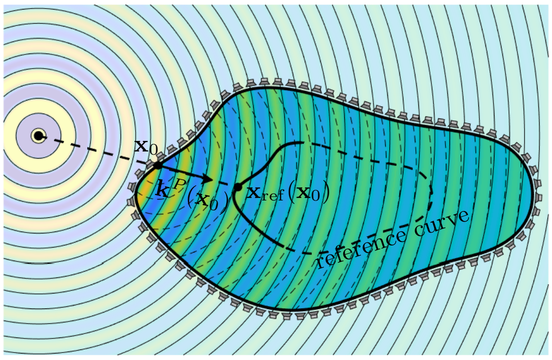

Figure 2: Position of the reference position of a single SSD element in case of a virtual point source

dicular to the wavefront, i.e. pointing into the local propagation direction in an arbitrary position. Under high frequency conditions for simple wavefields it satisfies the local dispersion relation $|\mathbf{k}^P(\mathbf{x})| = \frac{\omega}{c}$.

The 2.5D Kirchhoff integral contains the driving function for a secondary source contour implicitly, however, the driving function still contains the receiver position through the argument of $\phi_{zz}^{G''}(\mathbf{x}-\mathbf{x}_0)$. Hence, application of this driving function for WFS would result in amplitude correct synthesis only at a single receiver position, for which the driving function is evaluated. Within the validity of the SPA this *reference position* can be extended in order to achieve amplitude correct synthesis on an arbitrary *reference curve*, by assigning a unique reference position to each secondary source element. The stationary phase approximation states that a secondary source element at $\mathbf{x}_0$ dominates the synthesized field (described by the 2.5D Kirchhoff integral (2)) along those space regimes $\mathbf{x}_{\mathrm{ref}}(\mathbf{x}_0)$ for which $\mathbf{x}_0$ serves as a stationary position, satisfying

$$\mathbf{k}^P(\mathbf{x}_0) = \mathbf{k}^G(\mathbf{x}_{\mathrm{ref}}(\mathbf{x}_0) - \mathbf{x}_0). \tag{4}$$

Due to the spherical nature of the field G, this equation is satisfied along straight lines passing through $\mathbf{x}_0$ pointing into the direction of $\mathbf{k}^P(\mathbf{x}_0)$. Hence, along this line a unique reference position can be defined for the actual SSD element. As long as a continuous, convex SSD is assumed, the set of all the individual reference positions also form a continuous curve along which amplitude correct synthesis may be achieved, termed as the *reference curve*.

By substituting this asymptotical reference position into the 2.5D Kirchhoff integral, the final *2.5D WFS driving function* can be extracted, given by

$$D(\mathbf{x}_0,\omega) = \underbrace{w(\mathbf{x}_0)\sqrt{\frac{8\pi}{\mathrm{j}|\phi_{zz}^{P''}(\mathbf{x}_0) + \phi_{zz}^{G''}(\mathbf{x}_{\mathrm{ref}}(\mathbf{x}_0) - \mathbf{x}_0)|}}}_{\text{amplitude correction}} \underbrace{\mathrm{j}k_{\mathrm{n}}^P(\mathbf{x}_0)P(\mathbf{x}_0,\omega)}_{\approx \frac{\partial}{\partial \mathbf{n}}P(\mathbf{x}_0,\omega)}. \tag{5}$$

In practical scenarios, referencing to an arbitrary reference curve can be implemented by first, choosing the desired reference contour, C_{ref}. Afterwards, for each SSD element the corresponding reference position $\mathbf{x}_{\mathrm{ref}}(\mathbf{x}_0)$ has to be found, satisfying (4) and lying on the reference contour: $\mathbf{x}_{\mathrm{ref}}(\mathbf{x}_0) \in C_{\mathrm{ref}}$. Once all the reference positions are found, the driving function achieving amplitude correct synthesis on this curve can be evaluated. The referencing strategy is depicted in Figure 2

For a general SSD and reference curve geometry the reference positions can be only found numerically. The simulation of such a general scenario is depicted in Figure 3, verifying that with the presented approach phase correct synthesis can be achieved over the entire listening area (i.e. the wavefront shape is correct), while the amplitude error of the synthesis can be optimized only on a part of the reference curve for which a stationary SSD element can be found.

For more simple geometries—e.g. linear/circular SSD with linear/circular reference curve—the reference position can be expressed analytically and closed form driving functions can be found. This allows the comparison of the presented WFS driving function with different SFS methods, namely with the Spectral Division Method (SDM) and Near-Field Compensated Higher Order Ambisonics. With SDM the presented WFS driving function was proved to exactly coincide in the high frequency region [7]. Also, in the particular geometries used in previous WFS approaches the presented driving function results in the driving functions given by the literature, therefore, the presented methodology is a generalization of the existing methods.

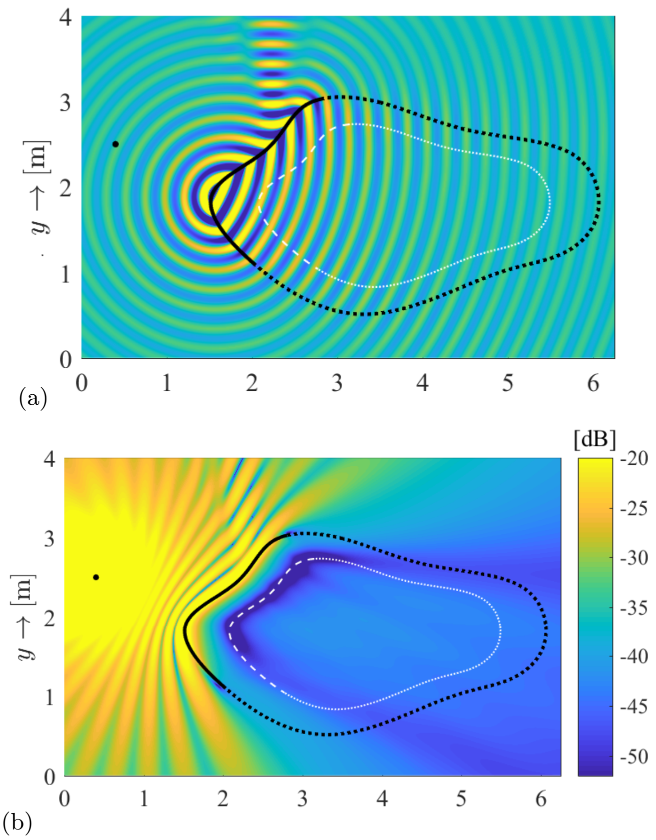

Figure 3: 2.5D WFS of a virtual point source with (a) depicting the real part of the synthesized field, (b) depicting the amplitude error in a logarithmic scale

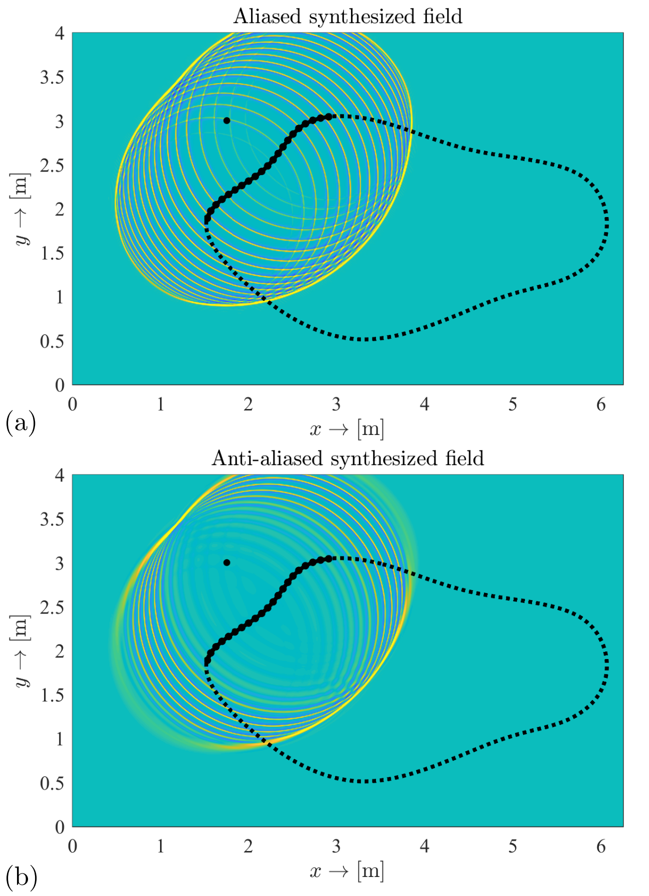

Figure 4: Aliasing artifacts due to the SSD discretization (a) and the result of anti-aliasing filtering (b).

Avoiding spatial aliasing

So far it has been assumed that the SSD is a continuous set of secondary point sources. In practical applications loudspeakers are placed in discrete locations, leading to artifacts in the synthesized field termed as *spatial aliasing*. Speaking in the time domain, spatial aliasing manifests in high-pass filtered ,,echoes" following the intended virtual wavefront, as illustrated in Figure 4 (a). Due to the short time interval between the arrival of the echo wavefronts, aliasing is perceived rather as the coloration of the virtual sound field, than actual reverberation. The suppression of aliasing phenomena is the central goal of *Local SFS* methods, well studied in the related literature. Here a simple SPA based anti-aliasing strategy is presented.

Traditionally, a discrete SSD can be modeled by the discretization of the WFS driving function. This process can be simply modeled mathematically in case of a linear SSD in the wavenumber domain: By performing a spatial Fourier transform the wavenumber spectrum of the driving function is obtained as the sum of the continuous spectrum, repeating on the multiples of the sampling wavenumber $k_{x,s} = \frac{2\pi}{\Delta x}$

$$\tilde{D}^{\mathrm{S}}(k_x, \omega) = \frac{1}{\Delta x} \sum_{\eta=-\infty}^{\infty} \tilde{D}\left(k_x - \eta \frac{2\pi}{\Delta x}, \omega\right), \qquad (6)$$

where Δx is the loudspeaker spacing. Aliasing can be explained by the overlapping of the repeating spectra. Obviously, by requiring that the driving function spectrum does not contain components above the half of the sampling wavenumber, i.e

$$\tilde{D}(k_x, \omega) = 0, \quad \text{if } |k_x| \geq \frac{\pi}{\Delta x}, \qquad (7)$$

the spectral overlapping may be avoided.

Application of the SPA to the spatial Fourier transform of the driving function allows one to establish the connection between spectral components and spatial positions: The spectrum of the driving function at a given spectral wavenumber k_x is dominated by that part of the space, where $k_x^P(x_0) = k_x$ is satisfied. Hence, the above anti-aliasing criterion can by reformulated asymptotically in the spatial domain as

$$D(x_0, \omega) = 0, \quad \text{if } |k_x^P(x_0)| \geq \frac{\pi}{\Delta x}. \qquad (8)$$

Furthermore, the local dispersion relation relates the local wavenumber vector with the angular frequency as $k_x^P(x_0) = \frac{\omega}{c}\hat{k}^P(x_0)$, with $\hat{k}^P(x_0)$ being a unit vector, pointing into the local propagation direction of P. Reformulation the above expression in terms of the angular frequency leads to the final anti-aliasing criterion

$$D(x_0, \omega) = 0, \quad \text{if } \omega \geq \frac{\pi}{\Delta x} \frac{c}{|\hat{k}_x^P(x_0)|}. \qquad (9)$$

Note that this requirement can be satisfied by simple temporal low-pass filtering of the driving function with the cut-off frequency of the filter defined by the local propagation direction of the virtual field, measured on the SSD.

Figure 4 (b) illustrates the effect of the above anti-aliasing strategy, with the criterion extended for non-linear SSDs by substituting $k_x^P \rightarrow k_t^P$, being the tangential component of the wavenumber vector. As it can be observed, aliasing echoes can be suppressed at least into one particular direction over the listening area. Into this direction full-band synthesis is achieved, while into other directions the virtual wavefront is bandlimited due to the anti-aliasing filtering. The remaining lateral echoes can not be mitigated with simple pre-processing, but would require the application of directive secondary sources.

Synthesis of moving point sources

Finally, as a complex application example for the foregoing, the synthesis of moving sound sources is discussed. In this case the main challenge is the proper reconstruction of the Doppler effect, occurring due to the constant speed of sound. However, the problem is inherently solved once an appropriate analytical source model is applied. For a moving 3D point source this analytical description is available.

The derivation of the 2.5D WFS driving function starts from time domain version of the Kirchhoff integral, formulating the field of a harmonic point source, moving on an arbitrary trajectory restricted to the horizontal plane of synthesis. By applying the SPA to this formulation along with the same referencing strategy as given in the previous sections, it can be proven that the 2.5D WFS driving function (5) holds without modification for the synthesis of moving sources as well.

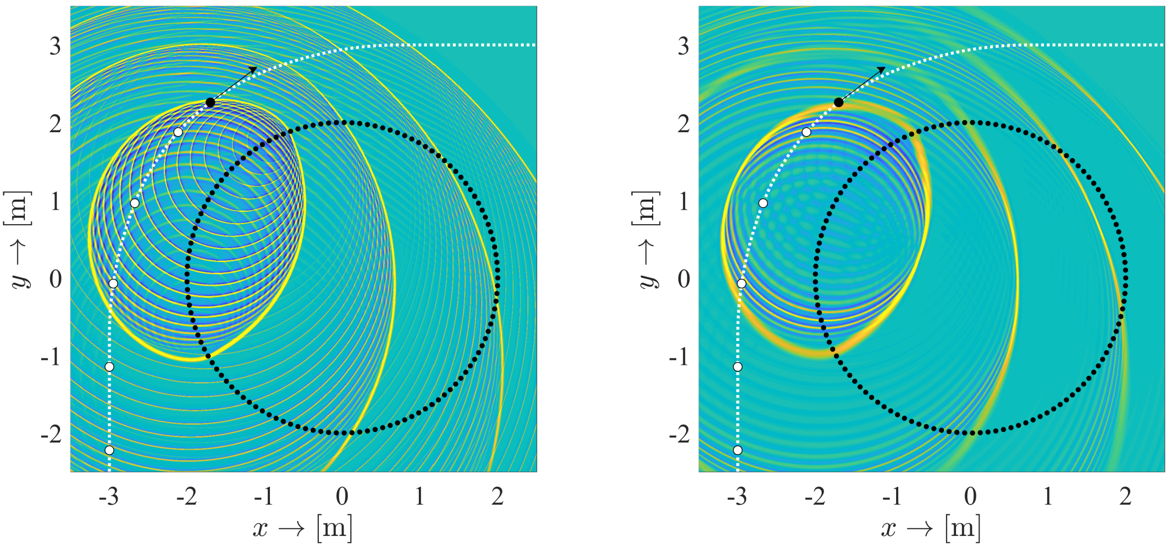

Figure 5: Synthesis of a point source moving on a curved trajectory, emitting a train of pulses at positions, denoted by white dots (a) and the result of anti-alasing filtering (b)

In the particular case of synthesizing moving sources the suppression of aliasing artifacts is of critical importance. While in the stationary case aliasing results in the colouration of the virtual sound field and in a slight increase of the apparent source width, in this dynamic case even frequency distortion is present, being clearly audible at high source velocities. This distortion occurs because the aliasing echoes suffer a different Doppler shift than the intended virtual wavefront.

In order to avoid aliasing phenomena, the anti-aliasing strategy introduced in the previous section may be applied. Similarly to the stationary case, anti-aliasing can be performed by simple temporal low-pass filtering of the driving function, by accounting for the time dependency of the local wavenumber vector and the perceived temporal frequency. Hence, in this case the cut-off frequency is time dependent:

$$D(x_0, \omega) = 0, \qquad \text{if } \omega(\mathbf{x}_0, t) \geq \frac{\pi}{\Delta x} \frac{c}{|\hat{k}_t^P(\mathbf{x}_0, t)|}, \qquad (10)$$

where $\hat{k}_t^P(\mathbf{x}_0, t)$ is the tangential component of the normalized local wavenumber vector, being unit vector pointing into the local propagation direction of the wavefront. For a moving point source the normalized local wavenubmer vector can be expressed analytically. Hence, in order to achieve anti-aliased synthesis, time-varying filtering of the driving function is needed.

The result of synthesis with a discrete SSD can be seen in Figure 5 (a), while (b) presents the result of the anti-aliasing approach. It is verified that similarly to the stationary case along one particular direction full-band, anti-aliased synthesis can be achieved. In the present case of a circular SSD, this direction always points towards the center of the array, making this particular choice of SSD shape feasible in the aspect of practical applications. Furthermore, by referencing the synthesis to the center of the array, here, amplitude correct synthesis can be achieved during the entire virtual source pass-by.

Conclusion

The paper summarized the theoretical basics of a generalized WFS framework. The methodology allows the synthesis of an arbitrary virtual sound field by an arbitrarily shaped SSD and optimizes amplitude correct synthesis on an arbitrary reference curve. It was shown that the arbitrary virtual sound field can be the field generated by a moving sound source as well. Furthermore, a simple, asymptotic anti-aliasing strategy was presented, valid for both stationary and dynamic sound fields as well.

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