Application of the Johnson-Champoux-Allard model for the calculation of the sound absorption coefficient of aerogel granules based on inverse characterization for the determination of the granules parameters

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Introduction

An important criterion for the choice of a sound insulation material is, among others, their absorption coefficient, which is an indicator how much sound energy is dissipated into thermal energy. To determine this, acoustic measurements are carried out and their results are compared with theoretical models. The impedance tube measurement results of a high porous material *aerogel* show adequate acoustic absorption properties comparing with the porous material glass wool. Especially the granular form of aerogels has higher sound absorption values in the medium frequency range than the aviation industry certified glass wool according to acoustic measurements. In order to reconcile these results with related physical phenomena, appropriate sound propagation models described in [1] are considered. One established sound absorption calculation approach are "equivalent fluid" models that assume a solid phase of the porous material during sound wave propagation as motionless [2]. Depending on the growing complexity of the propagation models, equations are derived that require increasing number of equivalent fluid material parameters. To analyze the absorption coefficient of porous materials with non-uniform microstructure, which is assumed to be the case for aerogel granules, the Johnson-Champoux-Allard (JCA) model [1] seems to be appropriate for theoretical considerations. This semi-phenomenological model provides sound absorption coefficient values for porous materials with straight and slanted cylindrical pores, as well as for porous materials with non-uniform microstructure [2] that shows a high degree of agreement with the impedance tube measured values, since it takes the visco-inertial dissipative and thermal dissipative effects inside the porous media also into account. The model requires five material parameters to be inserted into its mathematical formulations: porosity, flow resistivity, tortuosity and (for the consideration of the porous material non-uniform microstructure) viscous as well as thermal characteristic lengths. Neglecting thermal effects at low frequencies [2] and assuming that aerogel granules have a non-uniform microstructure, the application of the JCA model for the calculation of the sound absorption coefficient for different granules sizes is investigated and presented in this work. The model parameter tortuosity, viscos and thermal charac-teristic lengths for the JCA model are rather difficult to measure. In addition, their calculation models suggested by Johnson et al. [1], which will be discussed in the following sections of this paper, only deliver an order of magnitude of their values because of the missing information of the so called pore shape parameters [3] for the aerogel granules. Due to the high thermal and acoustic insulation performance of the aerogels and aerogel granules, which is an effect of their high porosity of about 90% and the size of the open pores in nanometer order of magnitude, several experimental and theoretical investigations on aerogel granules specially for their application in the building industry have been done, nevertheless aerogel granules microstructure parameters can not be found in literature. In order to determine the required unknown input parameters for the JCA model, the method of inverse characterization is used in this paper. According to Richter [4] the inverse problem method can be applied when a given mapping describes a causal relationship between a cause and a corresponding effect and the task to be solved is to deduce a cause from the known effect. Analogously, the given mapping is the applied JCA model and the known effects are the sound absorption coefficient curves obtained from impedance tube measurements, whereas the cause to be deduced are the unknown parameters of the aerogel granules. Further explanation is presented in the section "Using the Inverse Characterization Method“ of this paper, followed by a discussion on the results and the conclusion of the research.

Basic equations of JCA model

In this section the basic equations for mapping the sound absorption coefficient of porous media used in the JCA model and coupled with the inverse characterization method are summarized. Since the inverse characterization method is based on impedance tube measured data, for which a normal incidence of sound waves (related to the porous medium surface) is valid, the equation for the sound absorption coefficient α_{sac} refers also to the normal incidence case, which according to [1] is given by

$$\alpha_{\mathrm{sac}} = 1 - \left|\frac{Z - Z_0}{Z + Z_0}\right|, \tag{1}$$

where Z is the surface acoustic impedance of the porous layer and Z_0 the impedance of the air. For the calculation of Z the following equation is used:

$$Z = -\mathrm{j}\, Z_{\mathrm{c}}(\omega) \cdot \operatorname{cotg}\, (k_{\mathrm{c}}(\omega) \cdot d). \tag{2}$$

In equation (2) d is the thickness of the porous medium whereas the characteristic impedance Z_{c} and the wave number k_{c} can be obtained by

$$Z_{\mathrm{c}}(\omega) = \frac{1}{\Phi}\sqrt{\rho(\omega) \cdot K(\omega)} \tag{3}$$

$$k_{\mathrm{c}}(\omega) = \omega \sqrt{\frac{\rho(\omega)}{K(\omega)}}, \tag{4}$$

where ϕ is the open porosity of the material. Furthermore $\rho(\omega)$ is an equivalent effective density for a limp material, which is approximately

$$\rho(\omega) \approx \frac{\rho_\mathrm{t} \cdot \rho_\mathrm{eq} - \rho_0^2}{\rho_\mathrm{t} + \rho_\mathrm{eq} - 2\rho_0}, \tag{5}$$

where

$$\rho_\mathrm{t} = \rho_1 + \phi\rho_0 \tag{6}$$

is the apparent total density of the equivalent fluid limp medium, ρ_1 is the granules bulk density, ρ_0 the air density and

$$\rho_\mathrm{eq} = \alpha(\omega) \cdot \rho_0 \tag{7}$$

is the equivalent density and $\alpha(\omega)$ the dynamic tortuosity, calculated by

$$\alpha(\omega) = \frac{\nu\,\phi\,\sigma}{\mathrm{j}\,\omega\,\eta}\sqrt{1 + \left(\frac{2\,\alpha_\infty\,\eta}{\phi\,\Lambda\,\sigma}\right)^2 \frac{\mathrm{j}\omega}{\nu}} + \alpha_\infty\,. \tag{8}$$

In the above expression ν is the kinematic viscosity of air, α_∞ is the material tortuosity, whereas η is the dynamic viscosity of air and σ is the matertial flow resistivity. Moreover, in equation (8) Λ is the viscous characteristic length. A further quantity appearing in equations (3) and (4) is the dynamic bulk modulus

$$K(\omega) = \frac{P_0}{\left(1 - \dfrac{\gamma - 1}{\gamma\,\alpha'(\omega)}\right)} \tag{9}$$

where P_0 is the standard air pressure, γ is the heat capacity ratio and $\alpha'(\omega)$ as the homologue of the dynamic tortuosity $\alpha(\omega)$ obtained by

$$\alpha'(\omega) = \frac{8\nu'}{\mathrm{j}\omega\Lambda'^2}\sqrt{1 + \left(\frac{\Lambda'}{4}\right)^2 \frac{j\omega}{\nu'}} + 1 \tag{10}$$

In the above expression ν' is the kinematic viscosity of air ν devided by the Prandtl number N_P

$$\nu' = \frac{\nu}{N_\mathrm{P}} \tag{11}$$

and lastly Λ' is the thermal characteristic length, which as well as the viscous characteristic length Λ and the tortuosity α_∞ are unknown parameters of the aerogel granules and therefore need to be determined by the method of inverse characterization.

Using the Inverse Characterization Method

The set of JCA model equations are used to map the sound absorption coefficient α_sac of the aerogel granules as a function of frequency. For determination of the required, but unknown parameters reasonable magnitudes, so called start values need to be chosen. Furthermore, for obtaining parameter values that are not only mathematically appropriate but also physically realistic, lower and upper boundaries for each parameter to be determined have been defined as following:

- Porosity ϕ: Start value of 95%, as specified by the used aerogel granules manufacturer. The order of magnitude is in accordance with considered values given by Parale in [5]. Lower and upper limits: 50% and 100% respectively as adequate values.
- Tortuosity α_∞: Start value as an average (for example 2,5) of the constraint $1 \leq \alpha_\infty \leq 4$, pointed by Atalla and Panneton [3].
- Viscous and thermal characteristic lengths Λ and Λ': As start values again average magnitudes of the limit values (see equations (12) and (13)) can be selected. Lower and upper limits: According to Atalla's and Panneton's approaches, using the boundaries of the so called pore shape parameters c and c':

$$\Lambda = \frac{1}{c}\sqrt{\frac{8\,\alpha_\infty\,\eta}{\sigma\phi}} \quad ; \qquad 0{,}3 \leq c \leq 3{,}3 \tag{12}$$

$$\Lambda' = \frac{1}{c'}\sqrt{\frac{8\,\alpha_\infty\,\eta}{\sigma\phi}} \quad ; \qquad 0{,}3 \leq c' \leq c. \tag{13}$$

For the thermal and viscous characteristic lengths the relation $\Lambda \leq \Lambda'$ should be imposed [6].

In order to minimize the measuring errors, some further parameters, namely the material thickness d, the bulk density ρ_1 and the flow resistivity σ, which can basically be measured however containing measurement deviations, also are put in the set of parameters to be determined with inverse characteristic method. The start values are the measured data and the lower and upper limits $\pm 15\%$ of the measured values.

The start and the boundary values for all the parameters to be determined applying the inverse characterization method are listed in the Table 1.

Table 1: Start and boundary values for the inverse characterization parameters for granule size a) 1 – 3 mm, b) 0,5 – 1 mm, c) 0,25 – 0,5 mm d) < 0,25 mm

Parameter	Start value				Lower limit				Upper limit			
	a	b	c	d	a	b	c	d	a	b	c	d
ϕ	0,95				0,5				1			
α_∞	2,5				1				4			
Λ [µm]	89	39	25	20	28	12	8	6	10130	447	285	223
Λ' [µm]	536	236	151	118	28	12	8	6	11143	4918	3139	2451
d [mm]	50				43				58			
ρ_1 [kg/m³]	70	80	72	78	60	68	61	66	81	92	83	90
σ [kNs/m⁴]	15	77	189	310	13	66	161	264	17	89	217	357

After mapping the sound absorption coefficient α_{sac} of the aerogel granules applying the JCA model and using the start values described in Table 1, the mapping curve is compared with the measured impedance tube sound absorption data. Now the sum of the squared differences of both curves at every single position, which is more specifically the objective function of an optimization task, needs to be minimized by varying the JCA model parameters within the above described lower and upper boundaries. For the optimization of the parameters the MATLAB function "lsqcurvefit" for non-linear curve fitting has been used. Furthermore, for obtaining more representative parameters three series of impedance tube sound absorption measurements for each different size of aerogel granules have been considered.

Since the impedance tube measured data within the frequency range 200 to 1600 Hz are reasonable, also the JCA model absorption coefficient curve to be fitted need to be regarded within the same frequency range.

Results discussion

The presented approach was used to determine the aerogel granules parameters (from the manufacturer CABOT Corporation, see Figure 1a) in order to map its sound absorption coefficient analytically for 4 different granules sizes, 1 to 3 mm, 0,5 to 1 mm, 0,25 to 0,5 mm and lastly lower than 0,25 mm. The introduced optimization procedure results are shown in Figure 2 to 4 and Table 2. As it can be seen the optimized curve after the implementation of the inverse characteristic method for the granules size 1 to 3 mm is nearly perfectly fitted the measurement curves (see Figure 2). However, the evaluation of the curve shape and how precise it is fitted to the measurement curves is not the main criterion to validate the optimization method and the outcomes. Also, the results must be physically reasonable and realistic. Since for the optimization algorithm lower and upper boundaries are defined within which the parameters are varied, the outputs of the algorithm are within a reasonable range. Likewise, the optimized parameter values (shown in Table 2) can be evaluated as realistic. A decrease of the porosity (from 95% start value to 65% optimized value) means a decrease of the peak value of the absorption coefficient, which can be observed in Figure 2.

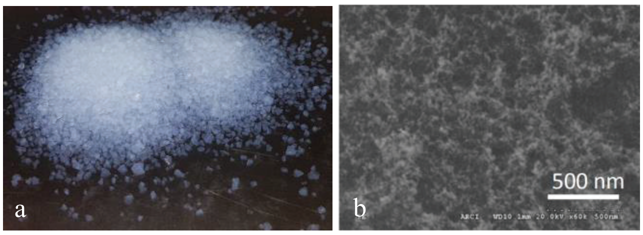

Figure 1: a) CABOT aerogel granules 1 to 3 mm
b) Scanning electron microscope image showing a section of highly porous aerogel granule of 1 mm size [8]

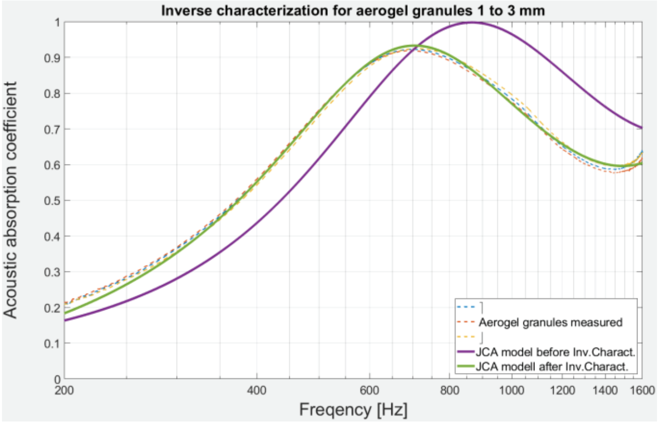

Figure 2: Sound absorption coefficient for the aerogel granules 1 to 3 mm

Knapen [7] explains that this effect occurs since at high values of the porosity, the friction surface increases and more energy is dissipated. Knapen also points out that at higher values of the tortuosity, the peak value shifts towards lower frequencies and lower values of the absorption, which is also obvious in Figure 2. While the characteristic thermal length has a minor influence on the width of the absorption peak, an increase of the flow resistivity corresponds to a decrease of the peak value of the absorption coefficient [7], that can be noticed too in Figure 2. The same behaviour can be observed for the aerogel granules of sizes 0,5 to 1 mm and 0,25 to 0,5 mm shown in Figure 3.

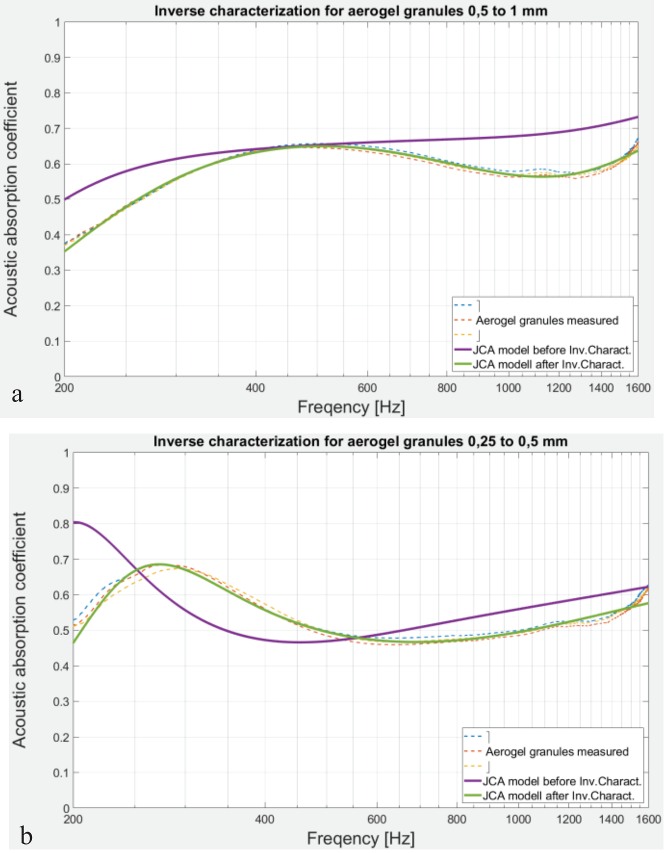

Figure 3: Sound absorption coefficient for aerogel granules a) 0,5 to 1 mm and b) 0,25 to 0,5 mm

Table 2: Start and optimized values obtained by the inverse characterization for granule size a) 1 – 3 mm, b) 0,5 – 1 mm, c) 0,25 – 0,5 mm d) < 0,25 mm

Parameter	Start value				Optimized value			
	a	b	c	d	a	b	c	d
ϕ	0,95				0,65	0,75	0,84	0,81
α_∞	2,5				2,2	3,9	3,8	4
Λ [µm]	89	39	25	20	77	44	50	56
Λ' [µm]	536	236	151	118	280	227	295	136
d [mm]	50				57	47	43	43
ρ_1 [kg/m³]	70	80	72	78	80	68	61	66
σ [kNs/m⁴]	15	77	189	310	17	89	200	264

By comparing the optimized curves for different granules sizes it can be noticed that the smaller the granules size, the less precise is the fitted curve referred to the measured sound absorption curves. The optimization process based on the JCA model, using the presented inverse characteristic method by implementing the above defined start and limit values, fails up to now for the granules sizes smaller then 0,25 mm (see Figure 4). The optimized curve is not fitted to the measured data, getting stuck in a local minimum. A modification of the start and limit values as well as a detailed investigation on the applied methods is required to solve this problem.

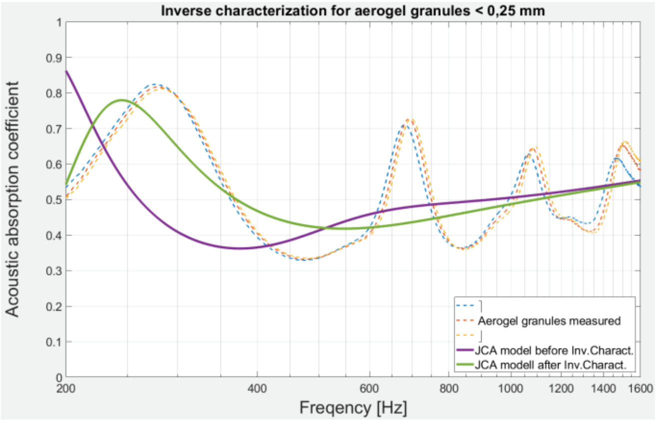

Figure 4: Failed determination of sound absorption coefficient parameters for the aerogel granules smaller than 0,25 mm

Conclusion

Based on the JCA model an inverse characterization method was applied to determine characteristic material parameters of aerogel granules complying with the model and with values listed in literature. The considered material parameters comprise porosity, flow resistivity, tortuosity, viscous and thermal characteristic lengths of different granular sizes. The most important result of this investigation is that a specific combination of allowed parameters leads to an optimal fit between theoretical and experimental sound absorption curves. For the granules 1 to 3 mm, 0,5 to 1 mm and 0,25 to 0,5 mm the JCA model and the applied inverse characteristic method delivers reasonable and realistic values, that needs to be validated by further measurements of the parameters or new model approaches. The presented approach is not suitable for aerogel granules sizes smaller than 0,25 mm. Parameter variations for this granule size showed that a revision of the start and boundary values for the viscous and thermal characteristic lengths Λ and Λ‘, or more precisely for the pore shape parameters c and c‘, introduced in equations (12) and (13), eventuates at most partly in a qualitative convergence of the impedance tube measured sound absorption curves.

Literature

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