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Modeling sound transmission through a periodic acoustic metamaterial grating of finite size

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ABSTRACT

Recent studies have discovered that acoustic metamaterial possesses great versatility in designing novel acoustic systems. This study investigates into the sound transmission through a periodic acoustic metamaterial grating of finite size. The single-layer grating is constructed by periodically arranging sub-wavelength unit cells in a slab, as a part of a large baffle between two acoustic domains. The metamaterial unit cell consists of an open duct decorated with coiled resonators, which intends to suppress sound transmission using its acoustic stop-band. The space-coiling structure allows the metamaterial to operate at low frequency with a compact size. Analytical approach to predict the sound transmission loss (STL) of the combined baffle is developed. The concept of metamaterial grating can be applied to the design of a novel ventilation window system with improved noise insulation.

Keywords: Acoustic metamaterial, Sound transmission loss, Ventilation window.

1. INTRODUCTION

Acoustic metamaterials are a type of artificial materials that are designed to provide exotic properties absent in nature. The study of using acoustic metamaterials to manipulate sound wave propagation has received much attention, particularly for the purpose of low-frequency absorption and isolation beyond the restriction of dimension and weight (1-3). Acoustic metasurfaces are a newer class of acoustic metamaterials which have surface profile and sub-wavelength thickness, allowing on-demand tailoring of sound propagation by imposing a specific boundary.

The class of metamaterial unit with an acoustic duct can be utilized for sound isolation without completely blocking air ventilation. The duct allows air passage while the resonators attached to the side-branch can provide attenuation effect on the sound transmission. For example, a two-dimensional array of low loss cylindrical cavities was studied, showing negative bulk modulus and dispersion relation of acoustic bands (4). The resonant tubular array demonstrated continuous tuning of compressibility in a wide range (5). Conceptually, the ensemble of duct and attached resonators can be considered as a metamaterial unit cell. If we periodically arrange such unit cells in a slab, an acoustic grating can be constructed to realize special acoustic functions by making use of the unusual property of the unit cells and also the interaction between the unit cells. Here, the acoustic grating is viewed as a subclass of acoustic metasurface where the constituting unit cells are identical rather than possessing a certain phase relationship. As the thickness of a single-layer acoustic grating equals to that of the constituting element, the slab structure has excellent geometric advantage benefiting from the sub-wavelength size of the unit cells.

In this study, we present a numerical method to predict the sound transmission through a metamaterial grating using semi-analytical approach. The single-layer grating is constructed by periodically arranging sub-wavelength unit cells in a slab, as a part of a large baffle between two acoustic domains. It is shown that a metamaterial grating can provide superior sound insulation while allowing air to ventilate through the openings distributed on the surface. By using such a principle, a full-scale metasurface ventilation window is designed and experimentally demonstrated. Substantial improvement in the acoustic performance shows the benefit of incorporating such metasurface concept into the design of sound insulation components.

2. Semi-analytical numerical approach

This section briefly outlines the necessary steps to model the sound transmission of a metamaterial grating system. Let us consider a two-dimensional slab comprised of a number of metamaterial unit cells arranged periodically, forming a single-layer grating as a part of a large baffle. The metamaterial unit cells are open acoustic ducts decorated with coiled resonators, as shown in Fig. 1. The boundaries of the unit cells and the separation between grating elements are assumed as rigid to simplify the theoretical modeling. Using the Cartesian coordinate system, let *x*-axis denote the slab normal and y-axis denote the slab tangential. The sound incidence angle α is defined relative to the slab normal, with $\alpha = 0^{\circ}$ being normal incidence case and the $\alpha = 90^{\circ}$ being the grazing incidence case.

Assuming the slab is excited by plane wave with incidence angle α, the incident sound pressure field on the left-hand side of the baffle can be expressed as:

$$p_{in}(x, y, \omega) = p_0 \mathrm{e}^{\mathrm{j}(\omega t - kx\cos\alpha - ky\sin\alpha)} \tag{1}$$

where p_{in} is the incident pressure field, p_0 is the amplitude of incident wave; $\mathrm{j} = \sqrt{-1}$; k is the wave number $k = \omega / c_0 = 2\pi f / c_0$, ω is the angular frequency and c_0 is the speed of sound in air. At the slab interface where *x*=0, the above expression reduces to $p_0 \mathrm{e}^{\mathrm{j}(\omega t - ky\sin\alpha)}$. The time dependence $\mathrm{e}^{\mathrm{j}\omega t}$ can be neglected if we carry out harmonic analysis.

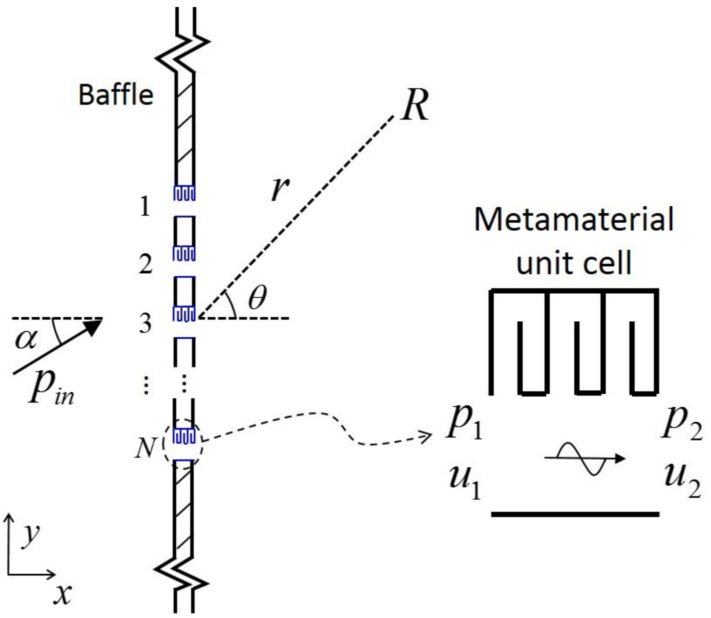


Figure 1. A slab comprised of metamaterial unit cells arranged periodically to form a single-layer acoustic grating. The metamaterial unit cell is an open duct decorated with coiled resonators.

On the right-hand side of the slab, the transmitted sound through the unit cell causes an air velocity disturbance at the outlet aperture and radiates into the receiving field. Under the baffled condition, sound radiation from the periodic outlet apertures can be modeled as cylindrical waves radiating into a semi-infinite free space. To determine the radiation impedance between the radiated pressure field and the unit cell velocity, the governing Helmholtz equation is:

$$[\nabla^2 + k^2(\vec{r})]p_{rad}(\vec{r}, \omega) = 0 \tag{2}$$

where p_{rad} is the radiated sound pressure, ∇ is the Laplacian operator, $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ for Cartesian coordinate system and $\nabla^2 = \frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial}{\partial r}$ for cylindrical coordinate system, $\vec{r}$ is the position vector of receiving point R. By omitting the time-dependent term $e^{j\omega t}$, the solution to the above equation is:

$$p_{rad}(\vec{r}) = aH_0^{(2)}(kr) \tag{3}$$

where a is the amplitude determined by the strength of sound radiation source, $H_0^{(2)}$ is the zero order Hankel of the second kind.

Assuming the outlet aperture of a unit cell is small compared to the acoustic wavelength, and the receiving point R is located in the far field, the outlet aperture can be approximated as a point source with uniform velocity. The radiation strength is $a = k\rho_0 c_0 q / 2$ and Eq. (3) can be further written as:

$$p_{rad}(\vec{r}) = \frac{k\rho_0 c_0 q}{2} H_0^{(2)}(kr) \tag{4}$$

where q is to describe the strength of air velocity disturbance at the outlet aperture: $\mathrm{d}q = \bar{u}\mathrm{d}y$, with d here being the differential operator, $\bar{u}$ being the normal air velocity averaged over the aperture area h_o.

Assuming the radiated sound pressure field behind the baffle is a linear superposition of the sound fields generated by all the periodic unit cells. The sound pressure p_n at a specific unit cell n due to excitation from all the unit cells, including cell n itself, inversely affects the pressure continuity relation. This coupling effect has to be apprehended to solve the coupled system response. The mutual radiation impedance between a receiving unit cell n and an exciting unit cell m is $Z_{nm} = p_n / \bar{u}_m$, and the self-radiation impedance is $Z_{mm} = p_m / \bar{u}_m$. By translating the origin of the coordinate system to the center of the outlet aperture at the exciting cell m, i.e., let y=0, the acoustic pressure on the receiving cell n due to an exciting cell m, provided that $m \neq n$ is:

$$p_n(y_n) = \frac{k\rho_0 c_0}{2} \int_{-h_o/2}^{h_o/2} \bar{u}_m H_0^{(2)}(ky)\mathrm{d}y \tag{5}$$

where the integration is taken over the radiating outlet aperture at cell m from $-h_o / 2$ to $h_o / 2$. The mutual radiation impedance between cell m and n is thus:

$$Z_{nm} = \frac{p_n}{\bar{u}_m} = \frac{k\rho_0 c_0 h_o}{2} H_0^{(2)}(kd_{nm}) \tag{6}$$

where d_{nm} is the separation distance between the cell n and m.

For the self-radiation impedance where $m = n$:

$$Z_{mm} = \frac{p_m}{\bar{u}_m} = \frac{k\rho_0 c_0}{2} \int_{-h_0/2}^{h_0/2} H_0^{(2)}(ky)\mathrm{d}y \tag{7}$$

The metamaterial unit cell is modeled as a waveguide to connect the incident and receiving sound fields analytically described as above. The sub-wavelength property of the unit cell makes it reasonable to consider its interaction with the adjacent acoustic fields as a one-dimensional waveguide. For the metamaterial unit cell as shown in Fig. 1, its acoustic wave propagation in the duct is one-dimensional if the frequency range is below the cut-off frequency determined by the aperture size: $f_c = c_0 / 2h_o$. The one-dimensional waveguide can be described by the four-pole transfer matrix, which relates the pressure and velocity conditions on both ends of a unit cell as:

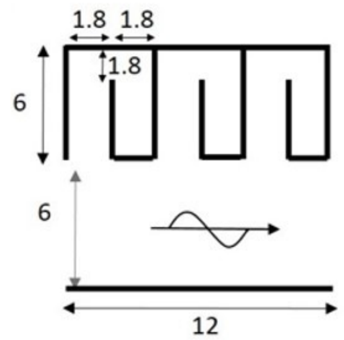
$$\begin{bmatrix} p_1 \\ u_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} p_2 \\ u_2 \end{bmatrix} \tag{8}$$

The four-pole parameters can be obtained either analytically or numerically, depending on the complexity of the unit cell geometry. After the unit cell parameters in Eq. 8 and the radiation impedances in Eqs. 6 & 7 are obtained, the system response can be determined by coupling the incident and receiving field together using a sub-structuring approach (6). Then the sound transmission loss of the single-layer metamaterial can be calculated.

3. TL of a 2D metamaterial grating

As a numerical example, the sound transmission characteristics of a metamaterial grating composed of unit cells as shown in Fig. 2 are studied using the proposed approach. The STL of a single baffled unit cell is first calculated. In figure 2, the STL shows a sharp peak between 700 and 900 Hz, owing to the stop-band of the unit cell. In this frequency range, the coiled resonators along the duct impose strong impedance mismatch along the wave propagation direction, thus the incoming wave is strongly reflected back to the incident field. The unit cell is blocking noise transmission, while at the same time offering a transparent path for air movement.

Then, the STL of the acoustic grating system comprised of five identical unit cells is calculated. First, we consider normal sound incidence and the unit cells are closely adjacent to each other. With d = 0 m, the STL in Fig. 2 shows a similar sharp peak, whereas TL at other frequencies are more flattened compared to the single unit cell STL. This indicates that with more periodic arrangement of identical unit cells, the metamaterial grating is able to provide the same noise isolation ability as the unit cells. The metamaterial block can then be scaled up into a linear or planar structure to provide ideal acoustic function.



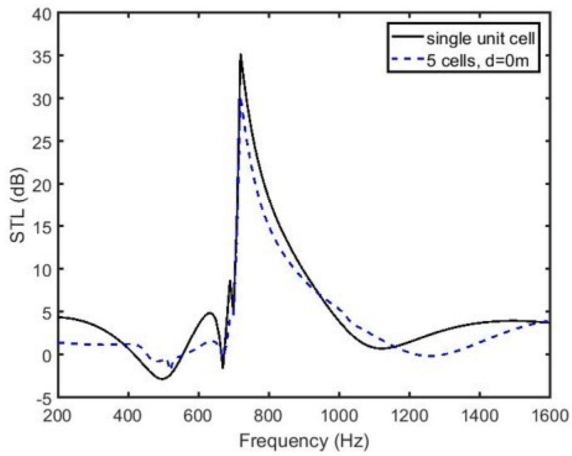


Figure. 2. Unit cell geometry with three coiled resonators (in cm) and STL of a single unit cell and acoustic gratings comprised of five unit cells.

4. Design of metamaterial ventilation window

Using such design principle, a metamaterial ventilation window is designed and experimentally demonstrated at the room scale. Using the center opening of a conventional window combination, a 3x3 metamaterial array is designed. Each unit cell has a two-layer resonant chamber as sketched in Fig. 3 (a), where an opening occupying a quarter of the unit cell surface area is designed to allow air ventilation. The dimension of the chamber is tuned using numerical model to optimize the attenuation performance for traffic noise frequency near 1000 Hz (7). The fabricated prototype is shown in Fig. 3 (b), the structure made of 2 mm acrylic panels provides good transparency.

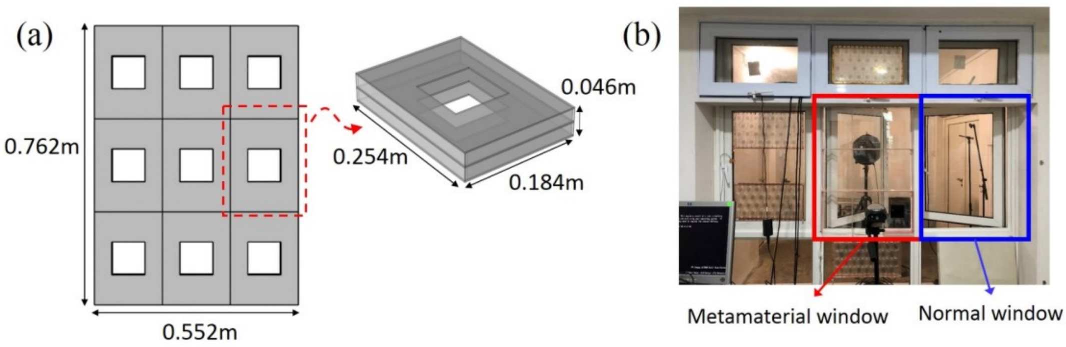


Figure 3. (a) Design and dimensions of a meta-material ventilation window; (b) The fabricated prototype.

The performance of the meta-material window is first measured in linear frequency range by using tonal noise excitation to verify the design and prediction. Fig. 4(a) shows the predicted transmission loss and the measured data, where good agreement is found in both frequency range and attenuation level. Thus the numerical model can provide an accurate tool to design such kind of ventilation window for different applications with different noise features. In Fig. 4(b), the Sound Reduction Index (SRI) of the metamaterial window is compared with that of a normal window, as described in Fig. 4(b). The SRI is measured with either metamaterial or normal window opened alone, and both windows are opened with a similar ventilation area. Measurement is conducted according to ISO 10140. The improvement by metamaterial window is shown to be very significant, and it provides an enhanced attenuation region between 800 Hz and 1600 Hz, exactly as we designed. The Single Number Quantity (SNQ) rated by ISO 717 is 21.7 dB for the metamaterial window and 14.6 dB for the normal window, i.e., 7.1 dB higher. Using broadband white noise and real traffic noise recording, it was verified that meta-material window can perform 6-8 dB better than normal window. Therefore, the metamaterial window is concluded to be capable of improving the noise attenuation by 7 dB.

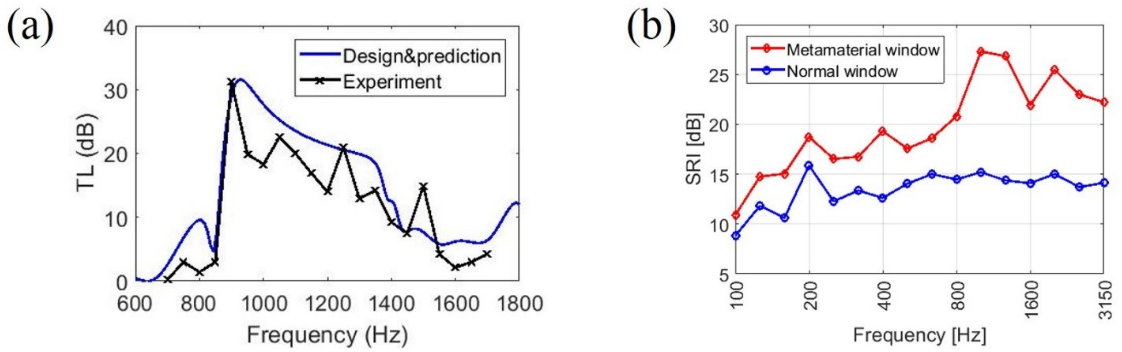


Figure 4. (a) Comparison between predicted attenuation performance (design) and actual measured data; (b) Comparison between the measured SRI of the metamaterial window and the normal window with the same opening area.

5. CONCLUSIONS

This paper has presented a numerical approach to model the sound transmission through an acoustic grating system comprised of sub-wavelength unit cells. The unit cell was constructed based on an acoustic duct decorated with coiled resonators. A number of unit cells were arranged periodically in a slab to form an acoustic grating. Numerical results showed that the proposed metamaterial grating can provide desired sound insulation in the design frequency range without entirely blocking the airflow. The design concept has been applied to the design and tuning of a metamaterial ventilation window prototype. Experimental measurement demonstrated the validity of the proposed numerical approach. Improved sound reduction performance has been achieved by using metamaterial window design.

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