

Effect of acoustic treatment on fan flutter characteristics

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ABSTRACT

This paper presents an investigation of the acoustic treatment effect on fan flutter characteristics. A three-dimensional model is proposed to predict the unsteady aerodynamic responses of an oscillating rotor in a finite-length annular duct with the acoustically treated duct walls. In this model, the aerodynamic couplings of oscillating rotor and acoustic liner are analysed in a strict sense by employing the transfer element method, with the unsteady aerodynamic loading distribution on rotor blades predicted by the three-dimensional lifting surface theory and the liner modelled as equivalent monopole sources distributed on hard wall. Moreover, combined with a boundary integral approach, the present model implicitly considers the reflections at duct openings, thus the aerodynamic interactions within such a finite-length duct system can be truly determined. Within the linear scope, the primary perturbations induced directly by blade oscillation and the scattering fields resulted from aerodynamic couplings are superposed to give the total blade loadings, such that the rotor flutter stability can be further evaluated by its unsteady aerodynamic work based on the energy method. A parametric study for the first-order torsion vibration has been conducted, whose results reveal the noticeable impact of the acoustic liner on aeroelastic stability under certain circumstances.

Keywords: Fan flutter stability, Acoustic treatment, Transfer element method

1. INTRODUCTION

Extensive research efforts have been paid to fan flutter problems since the middle of last century. However, a brake is on the development of blade flutter control due to the great complexities arising from the fact that any factors affecting the surrounding flow fields or the aeroelastic characteristics of cascade structure are likely to further alter the evolution of this kind of self-excited vibration. Passive suppression techniques of cascade flutter, which were comprehensively assessed by Bendiksen (1) and have revealed little appreciable breakthroughs in the recent decades, usually come with a limited range of utility under the extreme and variable operating conditions of aero engines. By contrast, the concept of suppressing fan flutter with the active control of acoustic treatment on duct wall (2–4) shows a promising prospective. Zhao and Sun (5) have experimentally achieved the active control of duct wall impedance with a kind of perforated liner with adjustable bias flow backed up by a hollow cavity of controllable depth, whose impedance characteristics have been investigated by considerable theoretical and experimental work (6–9). Then we can expect that, once the in-depth knowledge of the aerodynamic impact of acoustically lined wall on fan flutter stability is gained, the corresponding control strategy can be devised for flutter suppression purpose.

Watanabe and Kaji (3) have firstly proven that the change of duct wall impedance could considerably change the aerodynamic damping of oscillating blades. However, the three-dimensional semi-actuator disk model they used has been excessively simplified. By determining the Green's function for a duct comprised of two parallel infinite flat walls with one of locally reacting impedance, Sun and Kaji (4) further investigated the soft wall effect on the aeroelastic stability of a linear cascade. Their model cannot take into account the annular effect critical for real compressors or turbines as well as the acoustic liners of finite extent. Besides, a troublesome process involved for numerically computing the complex eigenvalues for soft walled duct considerably reduces its efficiency. Hence, more refined and efficient model is still in urgent demand.

In this paper, a three-dimensional semi-analytical model is proposed to predict the unsteady

aerodynamic responses of an oscillating rotor in a finite-length annular duct with the acoustically treated side walls. This model is established based on the transfer element method (10,11), in which each segment of a duct is modelled as a response function in a unified matrix form, called transfer element. Combined with a boundary integral approach (12), this method can derive an overall matrix equation for the entire disturbed field from the continuity conditions imposed on every interface of duct segments as well as on inlet and outlet duct terminations. By solving this matrix equation one can simultaneously determine the disturbed fields both inside and outside the duct, with the existing aerodynamic interactions and the reflections at duct openings thoroughly considered. Moreover, two analytical tools have been used to construct the specific transfer elements for rotor and liner, respectively. First of all, by using Namba's three-dimensional lifting surface theory (13), the unsteady aerodynamic loadings and the corresponding induced disturbances on blade surfaces in response to the oncoming perturbations can be determined from an upwash integral equation. Secondly, by treating an impedance wall as the corresponding hard wall distributed with the equivalent monopole of fluctuating strength, (14), the scattering effect of finite-length lined section can be effectively modelled. Finally, as indicated by the energy method, the possibility of cascade flutter is evaluated by the total unsteady aerodynamic work exerted on rotor blades, which is obtained by summing up the contributions from both the initial blade oscillation and the aerodynamic couplings of oscillating rotor, acoustic liner and duct openings.

In the following content, a general framework of transfer element will be introduced at first and then its tailored forms for rotor and liner will be discussed subsequently. With the other relevant theoretical tools briefly introduced, the complete process for establishing the present model will be presented. Numerical experiments will be discussed for an improved understanding about how the acoustic treatment on duct wall can influence the rotor flutter stability in a finite-length duct.

2. MODEL AND METHOD DESCRIPTION

2.1 Model Description

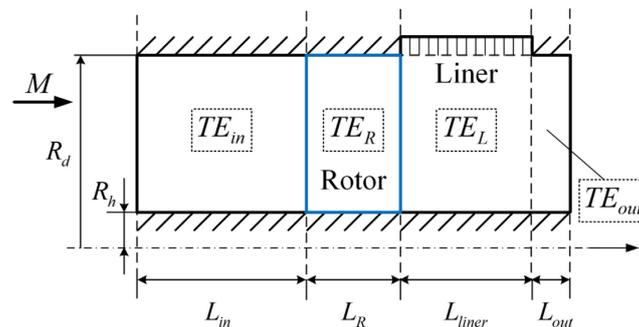


Figure 1 –Finite-length annular duct in question

In this paper, we consider a subsonic rotor cascade and a downstream lined section on the casing in an annular duct of finite axial extent, as shown in Figure 1. The outer and the inner duct radii are denoted by R_d^* and R_h^* , respectively. With the focus in this paper on the basic insights of the aerodynamic impact, on the onset of instabilities, of the partially lined wall upon the aeroelastic stability of an oscillating rotor in finite-length duct, the analyses have been developed within the linear scope with necessary assumptions made towards flow conditions, cascade geometry and blade vibration mode. Suppose the time mean flow is a uniform axial flow without separation, of static pressure p_0^* , density ρ_0^* and velocity $U^* = Mc_0^*$ (where $M < 1$ is its Mach number and c_0^* is the sound speed). Hereafter, with the asterisk denoting dimensional quantities omitted, all the quantities will be correspondingly scaled with respect to R_d^* , R_d^*/U^* and $\rho_0^*U^{*2}$. Further, consider the unstalled rotor rotating with angular velocity Ω has B identical, equally spaced, zero-thickness blades operating at zero mean incident. The B blades are vibrating in a given harmonic mode with the same angular frequency Ω_s but with a constant inter-blade phase lag σ , which gives birth to the excitation sources in our problem. All the induced perturbations are small compared to the steady counterparts. Also note that the unsteady aerodynamic loadings exerted on rotor blades have been investigated under fully three-dimensional consideration instead of subject to any simplifications required by strip theories, while the effect of steady blade loadings has been excluded in the entire discussion.

2.2 Method of Calculation

The difficulties in predicting unsteady rotor response in a partially lined duct of finite axial extent mainly arise from two respects. Firstly, the disturbances everywhere within a duct are tightly coupled and simultaneously changing, subject to all the scatterings of inner components as well as the reflections at duct open ends. So one cannot truly determine the disturbed fields unless the response function of the entire duct system is constructed as a whole, with all the coupling effects taken into account. Secondly, the capability of treating a wall boundary condition of non-uniform impedance is required, in the preference for a time-saving process. To these ends, we suggest a semi-analytical model developed by using the transfer element method combined with a boundary integral equation, in which the three-dimensional lifting surface theory is employed to derive the rotor response function while the acoustically lined section is equivalently modelled as a rigid wall with the monopole sources of fluctuating strength distributed on it.

2.2.1 Transfer Element Method

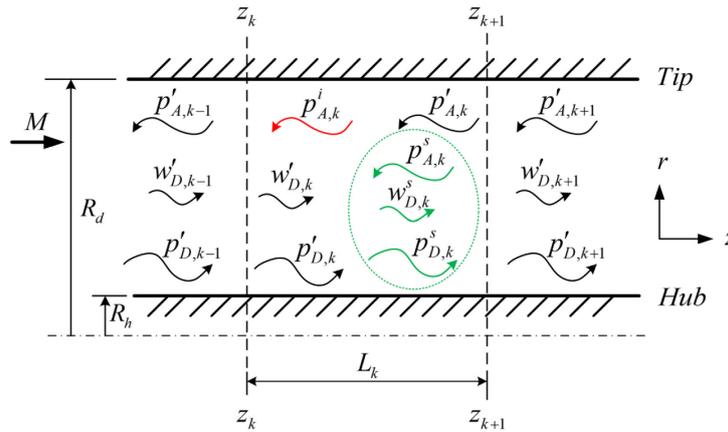


Figure 2 – An arbitrary segment of an annular duct

As depicted in Figure 2, consider a finite-length segment of an annular duct, denoted by the index k . In the cylindrical coordinates (r, φ, z) fixed to the duct, the axial coordinates of its front and the back interfaces are given by z_k and z_{k+1} , respectively. As is mentioned earlier, by treating the acoustic liners on the casing wall as the equivalent monopole source distributions, the impermeable boundary condition is imposed on the entire duct wall, and thus in the present approach the acoustic fields are expressed consistently in terms of rigid duct modes.

Suppose a forward-propagating acoustic wave $p_{A,k}^i$ emanates from certain sources within the k^{th} duct segment. The primary disturbances transmitting within this segment, i.e., the acoustic wave $p_{D,k}^s$ propagating downstream, the acoustic wave $p_{A,k}^s$ propagating upstream and the vortical wave $w_{D,k}^s$, whose complex modal coefficients are respectively denoted by D_{mn}^k , A_{mn}^k and V_{mn}^k , further arouse the scattering disturbances $p_{D,k}^s$, $p_{A,k}^s$ and $w_{D,k}^s$ by interacting with its inner structures. Here the subscripts D and A are used to indicate the fluctuations propagating downstream and upstream, respectively. m denotes the circumferential mode number and n denotes the radial mode number (noting that the lowest radial mode is defined as $n = 1$ in this paper). The adjacent segments, denoted by $k - 1$ and $k + 1$, are assumed to be free from any inner sources or scattering effects. The continuity conditions of acoustic pressure, axial sound particle velocity as well as circumferential vortex velocity on the front and the back interface of the k^{th} segment imply

$$\left. \begin{aligned} p_{A,k-1}^s + p_{D,k-1}^s &= p_{A,k}^s + p_{D,k}^s + p_{A,k}^s + p_{A,k}^i \\ u_{A,k-1}^s + u_{D,k-1}^s &= u_{A,k}^s + u_{D,k}^s + u_{A,k}^s + u_{A,k}^i \\ w_{D,k-1}^s &= w_{D,k}^s \end{aligned} \right\} \text{for } z = z_k; \quad \left. \begin{aligned} p_{A,k}^s + p_{D,k}^s + p_{D,k}^s &= p_{A,k+1}^s + p_{D,k+1}^s \\ u_{A,k}^s + u_{D,k}^s + u_{D,k}^s &= u_{A,k+1}^s + u_{D,k+1}^s \\ w_{D,k}^s + w_{D,k}^s &= w_{D,k+1}^s \end{aligned} \right\} \text{for } z = z_{k+1} \quad (1)$$

It can be proven that the complex modal coefficient $X_{m'\mu'}^{s,k}$ of an arbitrary scattering mode (m', μ') is the function of the modal coefficients C_{mn}^k of the primary waves in the form of

$$X_{m'\mu'}^{s,k}(z) = \sum_m \sum_n \sum_{C=D,A,w} \mathfrak{F}_{C_{mn}}^{X_{m'\mu'}}(z) C_{mn}^k \quad (2)$$

where $\mathfrak{S}_{C_{mn}}^{X_{m'\mu'}}$ is referred to as the ‘‘scattering multiplier’’. It is well-known that the orthogonality of the eigenfunctions of a hard-walled duct can transform Eqs. (1) into a series of mode-matching equations. Then, we can further define the multiplier of the unknown primary mode coefficient C_{mn}^k in the resulting mode-matching equations as the ‘‘transfer multiplier $\wp_{C_{mn}^k}$ ’’. All the interface matching conditions lead to an overall matrix equation

$$[\wp] \cdot [C_{mn}] = \mathbf{I} \quad (3)$$

The submatrix $[\wp^k]$ of its coefficient matrix, composed of all the transfer multipliers of the unknown modal coefficients C_{mn}^k ($C = A, D$ and V), is defined as a transfer element. Then we know that once the scattering multiplier \mathfrak{S} embodied the inner scatterings of a segment is determined, the corresponding transfer element can be obtained subsequently.

Intuitively, since disturbances propagate in a hollow rigid duct without any scatterings, the scattering multipliers will vanish when it comes to an empty duct segment with hard side walls, thus leading to the simplest form of the corresponding transfer element. The application of the transfer element method to a finite-length segment lined with locally reacting liner has been derived in detail in (12), where Eqs. (22-25) can yield the corresponding scattering multipliers.

Table 1 – Mode and frequency characteristics

At the oscillating rotor	Circumferential mode numbers	Frequencies
Primary waves	$m = sB + V_\sigma$	$\omega = \Omega_s + m\Omega$
Scattering waves	$m' = s'B + m = (s + s')B + V_\sigma$	$\omega' = s'B\Omega + \omega = \Omega_s + m'\Omega$

Note: the inter-blade phase parameter $V_\sigma = \sigma B / 2\pi$.

Based on the three-dimensional lifting surface theory, the scattering multipliers in the transfer element for a rotor of B identical, equally spaced blades in an annular hard-walled duct can be written as follow:

$$\mathfrak{S}_{C_{mn}}^{X_{m'\mu'}}(z) = \int_{R_h}^{R_d} \int_0^{C_a} \Delta p^{C_{mn}}(r', z', \omega) \cdot K_X(r', r, z - z' | B, C_a, \omega', m', \mu) \cdot dz' dr' \quad (4)$$

where $C/X = D, A$ and V and C_a denotes the axial length of blade chord, which is assumed to be constant along the blade span. The coordinates of source point and observation point in the cylindrical frame of reference fixed to the duct are respectively given by (r', φ', z') and (r, φ, z) . $\Delta p^{C_{mn}}$ is the unsteady blade loading on rotor blade surface resulted from the primary mode (m, n) of the upwash disturbance ($p'_{D,k}$, $p'_{A,k}$ or $w'_{D,k}$) with the incident frequency ω and the modal coefficient equal to unit. Moreover, the kernel function K_X indeed denotes the mode coefficient $X_{m'\mu'}$ of the scattering disturbances induced by a row of pressure dipoles of unit fluctuating amplitude.

It is known that the circumferential mode numbers and the angular frequencies of the scattering waves generated at such a rotor are related to those of the primary disturbances by

$$\begin{aligned} m' &= s'B + m \\ \omega' &= \omega + s'B\Omega \end{aligned} \quad (5)$$

Therefore, it can be proven that both the primary disturbances and the scattering disturbances generated at the oscillating rotor in question are comprised of an infinity number of the modal components in the uniform mode and frequency characteristics shown in Table 1.

So far, all the scattering multipliers for the duct segments involved in the present model have been briefly discussed, and the relevant transfer elements can be built up correspondingly.

2.2.2 Boundary Integral Equation

In order to match the disturbed fields inside of the duct with the radiation fields outside of it, the matching conditions on the duct openings based on the classic Helmholtz integral (14)

$$C(P)p(P) = \int_S \left[\Psi(Q) \frac{\partial G^0(P, Q)}{\partial n} - \frac{\partial \Psi(Q)}{\partial n} G^0(P, Q) \right] \cdot dS(Q) \quad (6)$$

where the velocity potential $\Psi = p' e^{-ik_0 Mz / (1-M^2)}$ ($k_0 = \omega / c_0$) and G^0 is the free-space Green's function, is applied in combination with the transfer element matrix in Eq. (3) to construct an overall response function of the duct system in a matrix form (10).

2.2.3 Upwash Velocity

Assuming all the blades are vibrating harmonically with the same angular frequency Ω_s , the blade displacement function $\xi(r', z')$ in this paper is chosen to be exactly the same with Eq. (24) in Ref.(15):

$$\xi(r, z) = H \cdot C_a \cdot h_\alpha(r) / \sqrt{1 + (\Omega r / U)^2} + \Theta \cdot \theta_\beta(r) \cdot (z - z_e) \cdot \sqrt{1 + (\Omega r / U)^2} \quad (7)$$

The first term on the right-hand side of Eq. (7) represents the blade displacement due to the bending vibration of order α while the second term is contributed by the torsion vibration of order β . The normalized radial distribution functions of the bending vibration and the torsion vibration are $h_\alpha(r)$ and $\theta_\beta(r)$, and the complex vibration amplitudes are H and Θ , respectively. In this paper, the torsional axis $z = z_e$ is assumed to be located on the mid-chord position. Taking the blade oscillation in the given manner $\xi(r, z)e^{i\Omega_s t + (j-1)\sigma}$ ($j = 1, \dots, B$) as the excitation source, the upwash velocity of the disturbances acting on rotor blade surfaces can then be determined by

$$\tilde{\mathbf{u}}_\phi = D \left[\xi(r', z') \cdot e^{i\Omega_s \tau + (j-1)\sigma} \right] / D\tau \quad (8)$$

2.2.4 Unsteady aerodynamic work

With Namba's definition (15) of the complex work coefficient

$$C_w = \pi \int_{R_h}^{R_d} \sqrt{1 + \Omega^2 r^2 / U^2} \int_0^{C_a} \Delta p(r, z) \cdot \bar{\xi}(r, z) \cdot dz dr \quad (9)$$

introduced to the present model, the likelihood of cascade flutter can be predicted by its imaginary part $I_m[C_w]$ (here the overline represent complex conjugate while $\text{Re}[\cdot]$ and $I_m[\cdot]$ in this paper denote the real and the imaginary parts of a complex value, respectively). Indeed, $I_m[C_w]$ is the unsteady aerodynamic work exerted on rotor blade during one period of blade oscillation, thus referred to as the unsteady aerodynamic work coefficient. According to the energy method, the cascade can maintain stable if $I_m[C_w] < 0$; otherwise, a flutter is expected.

3. RESULTS AND DISCUSSION

3.1 Case Configuration

For illustrative purpose, a subsonic rotor subject to the first-order torsion vibration (i.e., $\beta = 1$ and $H = 0$ in Eq. (7)) is considered in this paper. The case configurations are given as follow: $R_h = 0.2$ and $R_d = 1$; $M = 0.35$; $\Omega = 2.4744$, $B = 30$, $C_a = 0.2$ and $\Omega_s C_a = 0.5$; the complex amplitude of the torsion vibration $\Theta = e^{i\Phi}$ where $\Phi = -10^\circ$. Four transfer elements should be constructed respectively for the four duct segments depicted in Figure 1, i.e., the inlet segment empty with hard walls, the second segment containing the oscillating rotor, the third segment containing a locally reacting liner and the outlet segment empty with the hard walls. Their default axial lengths are $L_{in} = 3C_a$, $L_R = C_a$, $L_{liner} = 2C_a$ and $L_{out} = C_a$, respectively. Two inter-blade phase parameters are investigated in this work: first, $V_\sigma = 1$ where only the lowest radial, first circumferential mode (i.e., $m = 1, n = 1$) is cut-on in the hard-walled duct of infinite axial extent, and second, $V_\sigma = 2$ where all the hard-walled duct modes are cut-off.

3.2 Numerical Experiments

Figure 3 shows the unsteady aerodynamic work coefficient $I_m[C_w]$ plotted against the variation of the liner reactance $I_m[Z_s]$ with its resistance $\text{Re}[Z_s]$ held constant as 0.01, 0.10, 0.50, 1.00 and 3.00, respectively. The impedance ranges in these two plots are given to be very broad, out of aeroelastic stability concern rather than any practical acoustic consideration. For the oscillating rotor of concern in the finite-length hard-walled duct, the unsteady aerodynamic work coefficient $I_m[C_w] = 0.02071$ with $V_\sigma = 1$. With the liner added on the casing, a critical value of the specific reactance around -2.4 is found in Figure 3(a). Within a proper range, where the reactance is smaller than the critical value, the rotor which would flutter in the rigid duct can remain stable with the stabilizing effect of the liner; otherwise, the rotor will flutter and the existence of the liner may even considerably exaggerate the unbalance of the unsteady aerodynamic blade loading distribution due to blade oscillation. For $V_\sigma = 2$, the rotor is deeply unstable in the finite-length rigid wall, where $I_m[C_w] = 0.2579$. As shown in Figure 3(b), with the liner placed on the prescribed section of a small resistance

and a favourable specific reactance around -3, the rotor is predicted to be effectively stabilized for $V_\sigma = 2$.

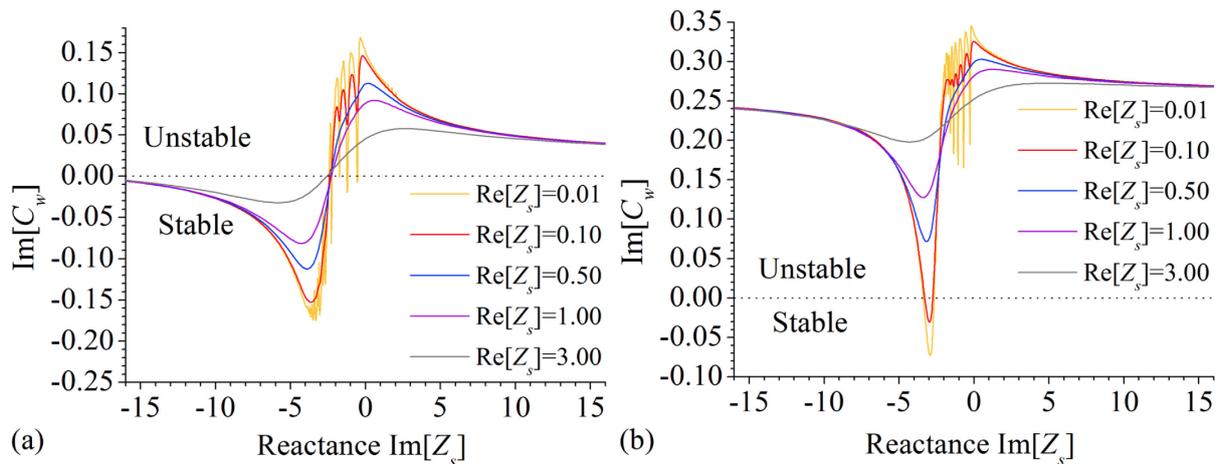


Figure 3 – Variation of $\text{Im}[C_w]$ against the change of the specific impedance Z_s : (a) $V_\sigma = 1$; (b) $V_\sigma = 2$.

Moreover, the unsteady aerodynamic responses of the oscillating rotor is found to be very sensitive to the reactance change when the specific resistance is small. When $Z_s = 0.01$, as the reactance varies from -5 to 5, an oscillating curve of $\text{Im}[C_w]$ is observed both in Figure 3(a) for $V_\sigma = 1$ and in Figure 3(b) for $V_\sigma = 2$. With the resistance increasing to 0.10, the drastic oscillation of unsteady aerodynamic work against the increase of the reactance is damped a little, while the reactance values corresponding to those peaks appearing in the curves almost remain unchanged. With the resistance further increased up to 0.50, 1.00 and 3.00, the curves of $\text{Im}[C_w]$ become smooth. Moreover, larger the resistance is, weaker the liner impact upon the rotor responses becomes. Therefore, we guess the phase change of disturbances, which is usually more sensitive to reactance than resistance and turns out to be evident only if less disturbances are absorbed on the liner surface, may take a dominant role in the actual effect of the aerodynamic interactions between oscillating rotor and acoustic liner on flutter stability.

To further investigate the influence of phase shift as well as mode propagation characteristics on this issue, the predictions have also been made by numerically changing the rotor-liner gap as well as the axial length of the liner. For the calculations based on the varying gap, an additional hard-walled hollow segment of the axial length L_{gap} is introduced between the rotor and the liner, with $L_{in} = 3C_a$, $L_R = C_a$, $L_{liner} = 2C_a$ and $L_{out} = C_a$ kept constant. And when the axial length of the liner L_{liner} is changed, we still take $L_{in} = 3C_a$, $L_R = C_a$ and $L_{out} = C_a$. Firstly, for $V_\sigma = 1$ with the liner of the specific impedance $Z_s = (0.01, -0.36)$, the dependencies of $\text{Im}[C_w]$ upon the rotor-liner gap and the liner length are shown in Figure 4, where and the results obtained for the infinitely long duct (denoted by ILD) and those for the finite-length duct (denoted by FLD) are presented in comparison. In a similar fashion, Figure 5 shows the corresponding results for $V_\sigma = 2$ with $Z_s = (0.1, -3.0)$.

For $V_\sigma = 1$, the mode of $m = 1$ and $n = 1$ is cut-on in the duct with entirely rigid walls. By gradually increasing the gap, the phase of the disturbances incident onto the lined section will change sinusoidally, which justifies the periodicity observed in Figure 4(a). Besides, the change of the rotor's unsteady aerodynamic work against the increase of the liner length also reveals a quasi-sinusoidal pattern, whose periodicity is likely due to the relatively evident effect of the reactance embodied by the phase shift while the small but finite resistance may contribute to its slowly decreasing amplitude.

For $V_\sigma = 2$, where all the modes generated in the hard-walled duct are cut-off, when the gap between the rotor and the liner is not too large, the decaying primary disturbances emitted at the rotor cascade can still arrive at the liner segment and further arouse the scattering waves by interacting with the lined wall. If the scattering fields can significantly alter the primary blade loading distribution, a stabilizing or destabilizing effect will be observed. On the other hand, since all the primary disturbances of cut-off modes will attenuate exponentially on their way to the acoustically lined wall, if the rotor-liner gap is large enough, the liner cannot affect the rotor responses anymore, which is exactly consistent with the results given in Figure 5(a). Besides, remarkable quasi-periodic change of the unsteady aerodynamic work against the increase of L_{liner} is observed in Figure 5(b) with $V_\sigma = 2$ and $Z_s = (0.1, -3.0)$. One possible reason behind it can be the modification of mode propagation characteristics in a duct with the side walls of considerable lined area, compared with the duct of the same geometry but with entirely hard walls.

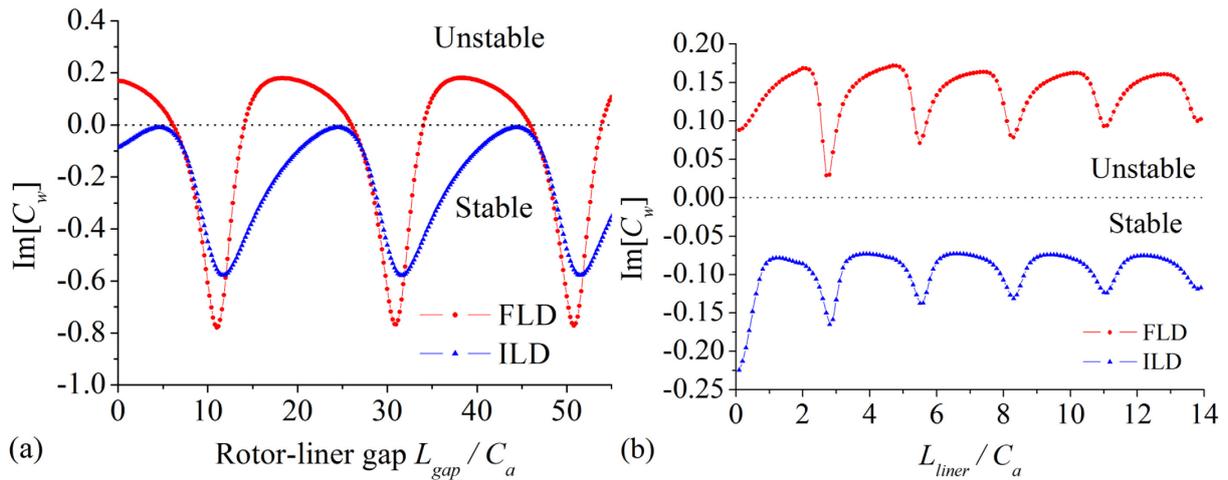


Figure 4 – Dependency of $\text{Im}[C_w]$ on (a) the rotor-liner gap and (b) the axial length of the liner, with $Z_s = (0.01, -0.36)$, $V_\sigma = 1$.

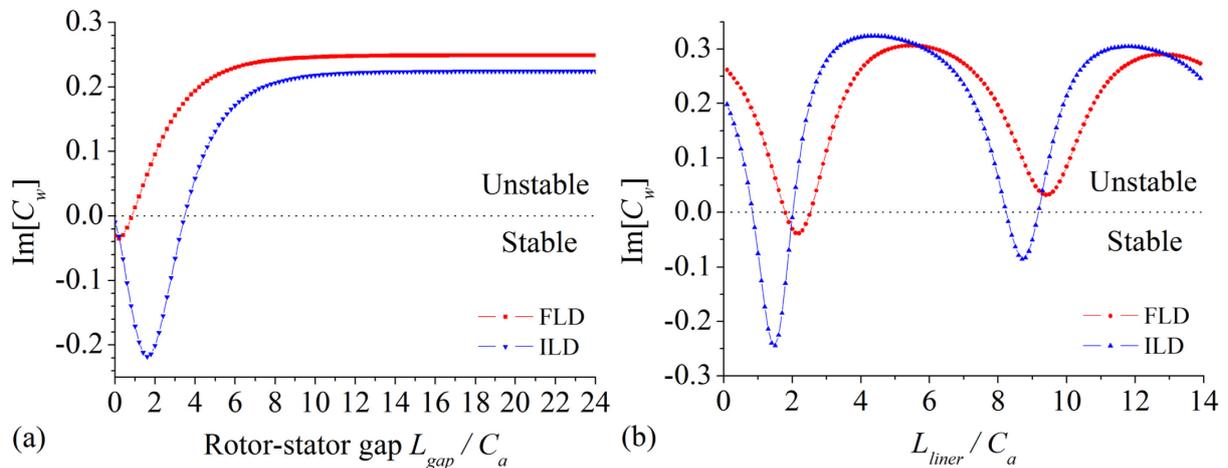


Figure 5 –Dependency of $\text{Im}[C_w]$ on (a) the rotor-liner gap and (b) the axial length of the liner, with $Z_s = (0.1, -3.0)$, $V_\sigma = 2$.

Furthermore, the remarkable derivations of the results obtained for the finite-length duct from those obtained for the duct of infinite axial extent have been observed for all the predictions of $\text{Im}[C_w]$ in Figures 4 and 5. This implies that the effect of acoustic treatment on fan flutter stability is strongly affected by the reflections at duct openings. Besides, it is found that the difference in $\text{Im}[C_w]$ are not a constant as the liner distribution changes, which indicates there is a coupling effect of the acoustically treated wall and the duct terminations. Since the unsteady aerodynamic responses of fan blades in a real engine nacelle are subject to the mutual acoustic couplings with both the acoustically treated side walls and the duct terminations, implementing the boundary conditions reflecting the physical essence is then of vital importance for the accurate fan flutter predictions.

4. CONCLUSIONS

Primarily on the basis of the transfer element method, a semi-analytical model is developed in this paper to predict the unsteady aerodynamic responses of an oscillating rotor in a partially lined annular duct of finite axial extent. A parametric study exploring the space of acoustic impedance and liner distribution has been performed. Under certain circumstance, both the stabilizing and the destabilizing effects of the acoustically lined section on the rotor flutter stability have been observed, where the duct end reflections are found to play a crucial role. In most cases, the impact of a liner turns out to be evident only when its resistance is small, and under such condition the unsteady aerodynamic responses of the oscillating rotor is found to be very susceptible to the reactance variation. It is the combined condition of duct geometry, the structural and aerodynamic state of rotor as well as the

configurations of acoustic treatment that mutually determines the actual impact of acoustic treatment on the flutter stability of a ducted fan.

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