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Pertinence of a simplified plane wave model for reverberation energy decays in rooms with a pair of parallel surfaces

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ABSTRACT

An analytical model has been developed recently to take into account specular and diffuse reflections in rooms with a pair of parallel surfaces. Analytical models can help to better understand the influence of scattering coefficients' values in those rooms in which unusually long reverberation times can be created by locked-in rays. Also, analytical models can be used complementary to ray-tracing methods to quickly estimate the late part of reverberation decays. The developed model is based on the formalism of sound particles emitted in the room by a point isotropic source. Some approximations lead to an intuitive analytical model including the influence of absorption and scattering coefficients on reverberation. The purpose of this paper is to develop and analyse the pertinence of an additional simplification to the model, based on the assumption of plane waves propagating between the two parallel walls. In this case, the model predicts a reverberation in the room composed of two exponential decays. The pertinence will be analysed by comparing this simplified plane wave model with reverberation decays computed by a sound ray program.

Keywords: Reverberation decays, scattering

1. INTRODUCTION

The influence of surface scattering on the reverberation in rooms has been deeply investigated in the last decades. In particular, the author of this paper has developed a geometrical acoustics approach linking surface scattering and reverberation decays (1). More recently, a simple analytical model of this approach was proposed for rooms with one pair of parallel surfaces (2). In these rooms, long and periodic propagation paths or 'locked-in rays' can create non-exponential reverberation decays which strongly depend on the surfaces' scattering properties.

This analytical model will be summarized in the next section. It is shown that the reverberation energy decay can be expressed as the sum of two exponential decays, one of them being weighted by a function $\left(\frac{1}{t^2}\right)$, where 't' is the time after the source's cut-off. The purpose of this study is to analyze the pertinence of a further simplification to this model consisting in assuming a plane wave propagation of the remaining energy between the two parallel surfaces.

2. THE MPW MODEL (MIXED REFLECTIONS BY PARALLEL WALLS)

2.1 Sound particles

The model first considers a sound source emitting an impulse of energy E_s at time t=0. This energy is represented by sound particles. Some particles are emitted in the direction of the two parallel surfaces A_1 or A_2 (see Figure 1) and a fraction of them may enter a back-and-forth mode of propagation between both surfaces, depending on their scattering coefficients. The energy transported by all those particles is called the 'specular flux' $\Phi_{spec}(t)$. All the other particles (reflected by the other surfaces, either specularly or diffusely) are supposed to follow a 'diffuse' mode of propagation characterized by an exponential energy decay $e^{-\gamma t}$: this is an assumption of the model partly justified by the diffuse reflections in the room and partly by the quasi-isotropic distribution of the image sources not created by the pair of parallel surfaces (1,2). The value of the decay rate γ only depends on the absorption properties of all surfaces in the room (1,2) and is given by:

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$$\gamma = m\,c + \frac{c}{4V}\int_S \frac{\alpha(dS)}{1-0.5\,\alpha(dS)}\,dS \tag{1}$$

In this formula, m is the air attenuation constant, c is the sound propagation speed, V and S are the volume and total surface of the room respectively and $\alpha(dS)$ is the absorption coefficient of the surface element dS.

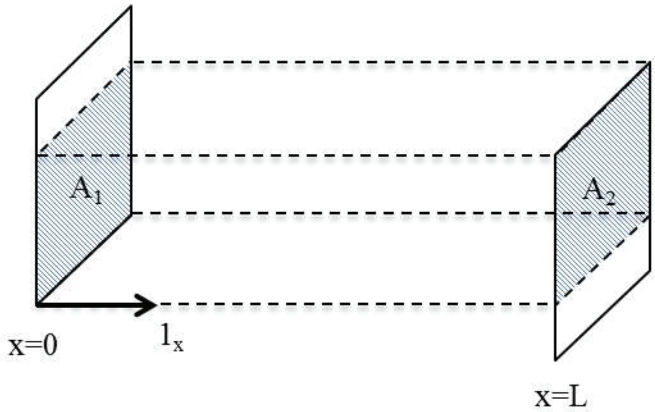

Figure 1 – The two parallel surfaces in the room are L meters apart. The sections A_1 and A_2 are included in these surfaces and they define the largest prism connecting them perpendicularly. The sound source must be situated inside this prism.

2.2 Specular and diffuse fluxes

After n reflections (n even) on A_1 and A_2, it is shown in (2) that the specular flux is expressed by:

$$\Phi_{spec}(t_n) = E_s\,\frac{\Omega_n}{2\pi}\,(R_1R_2)^{n/2} \tag{2}$$

where t_n is the time between specular reflections of orders (n) and (n+1) on A_1 and A_2, which can be approximated by $\left(\frac{n\,L}{c}\right)$ if the number of reflections n is 'great enough', Ω_n is the solid angle subtended by A_1 or A_2 from the corresponding image sources of order n and R_1 and R_2 are the specular reflection coefficients of these two surfaces. In (2), the decrease of the solid angle Ω_n is taken into account as $\Omega_n \cong \frac{A}{(nL)^2}$ if n is 'great enough' (A being the common area of A_1 and A_2). Introducing the decay constant $\beta = -\frac{c}{2L}\ln(R_1R_2)$ finally leads to:

$$\Phi_{spec}(t_n) = E_s\left(\frac{\mathrm{A}}{2\pi\,c^2}\right)\frac{e^{-\beta t_n}}{t_n^2} \tag{3}$$

Equation 3 gives the remaining energy of the specular flux at time t_n after the impulse emission by the source. This function of time decreases more or less by discrete steps, but an approximated continuous decrease is simply obtained by replacing t_n by t in eq.3.

The diffuse flow of sound particles contains all particles not travelling between A_1 and A_2 through specular reflections. By assumption, the corresponding energy decreases proportionally to $e^{-\gamma t}$. Moreover, at each new reflection on A_1 and A_2, additional sound particles (and thus energy) join this flow, mostly those diffusely reflected by A_1 and A_2. It is shown in (2) that, if $\left(\Delta t = \frac{2\,L}{c}\right)$ is the time interval including two successive specular reflections on A_1 and A_2, the energy remaining in the diffuse flux at time $(t_n+\Delta t)$ is:

$$\Phi_d(t_n+\Delta t) = \Phi_d(t_n)\,e^{-\gamma\Delta t} + \Phi_{spec}(t_n)\,D_{12} \qquad D_{12} = \nu^{-3/2}\,(\overline{D} + \nu\,\overline{RD}) \tag{4}$$

D_1 and D_2 are the diffuse reflection coefficients of A_1 and A_2 respectively, $\overline{D} = 0.5 * (D_1 + D_2)$,

$\overline{RD} = 0.5 * (R_2 D_1 + R_1 D_2)$ and $\nu = e^{\gamma L/c}$.

2.3 Solution for the total remaining energy and the reverberation decay

In (2), a continuous expression is found for $\Phi_d(t)$ by solving a differential equation based on eq.4. This expression is combined with eq.3 to give the total remaining energy in the sound particle flow at time t after the impulse emission by the source, provided t is 'great enough'. Indeed, some approximations and assumptions have been used, such that the early part of the decay cannot be predicted by the resulting analytical expression.

The model, called in (2) the MPW model, is rather expressed not as an impulse response, but as the usual energy decay curve or reverberation decay. Applying the Schroeder backward integration to the previous 'impulse response' leads to:

$$D_s(t) = C_d \frac{e^{-\gamma t}}{\gamma} + W_s Q_{12} \frac{e^{-\beta t}}{\beta t^2} \tag{5}$$

In this expression, W_s is the acoustic power of the source and Q_{12} is a parameter depending on the acoustic properties (absorption and scattering) of the parallel surfaces (A_1 and A_2) and on the decay constants γ and β. The constant C_d in eq.5 could be fixed if the decaying energy could be evaluated at some particular time of the reverberation decay, for example at t=0. However, as the MPW model cannot predict the early part of the decay, another approximation is used in (2).

We must here recall the main purpose of this model. We are not searching for a detailed model of the early part of the decay, but rather for an analytical model of the total energy decay in the room, from which we could extract an 'average' reverberation time or a 'global' ('non-local') description of the reverberation decay including the influence of absorption and scattering coefficients.

With this aim in mind, the early part of the reverberation decay is simply approximated by an exponential decay with γ as the decay rate. This approximation is extensively motivated in (2). Also, the initial time t_i from which the eq.5 can be applied has been chosen as the time corresponding to the first 10dB decrease in the energy decay curve ($t_i = \ln(10)/\gamma$). This has been considered as a good compromise between the 'asymptotic' assumption of the model (t great enough) and the fact that t_i should also belong to the early part of the decay (see (2) for more details). With this choice, the constant C_d in eq.5 could be fixed.

2.4 Test of the MPW model

As several approximations have led to the MPW analytical model, a great number of comparisons have been made with reverberation decays computed with a ray tracing program, in order to validate the model. These comparisons included tests in shoebox rooms with different dimensions and combinations of absorption and scattering coefficients. Results are discussed in (2), some comparisons will be illustrated in the next section.

The adequacy of the model is measured by the correspondence between the reverberation time T_{30} computed by both models (MPW and ray tracing), the second one being considered as the reference. The correlation between the corresponding values of T_{30} has been shown to be excellent, except for one particular room with a very flat geometry. In any case, the predictions of the MPW model were much better than the classic diffuse field theory. Except for the very flat room, it is shown in (2) that the average deviation between the values of T_{30} computed by both models was less than 15%.

3. SIMPLIFIED PLANE WAVE MODEL FOR THE ENERGY DECAY CURVE

3.1 Development of the model

A further simplification of the MPW model can be developed, which leads to a reverberation energy decay expressed by the sum of two exponential decays. This kind of 'pure' exponential model has already been suggested by some authors to simplify the reverberation decay analytical expression (3,4). The relevance of a 'pure' exponential model will be investigated in the following, but first of all we develop the model.

Starting from the sound particles model described in the previous section, suppose that, after N (even integer) reflections, the flow of 'specular' particles travels perpendicularly to the surfaces A_1 and A_2, just like plane waves would propagate between both surfaces. Another way of expressing this is to assume that the corresponding image sources are 'very far' from A_1 and A_2, close to infinity. In this case, eq.2 expressing the energy of the specular flux becomes (for n even):

$$\Phi_{spec}(t_{n>N}) = E_s \frac{\Omega_N}{2\pi} (R_1 R_2)^{n/2} \tag{6}$$

Regarding the diffuse flux of particles, eq.4 can be converted with the 'plane wave' assumption at time (t_N+$2\Delta t$) as:

$$\Phi_d(t_N + 2\Delta t) = \left(\Phi_d(t_N)\, e^{-\gamma\Delta t} + \Phi_{spec}(t_N)\, D_{12}\right) e^{-\gamma\Delta t} + \Phi_{spec}(t_N)\ (R_1 R_2)\, D_{12} \tag{7}$$

which can be generalized as (for any integer value m >0):

$$\Phi_d(t_N + m\Delta t) = \Phi_d(t_N)\, e^{-\gamma m\Delta t} + \Phi_{spec}(t_N)\, D_{12} \sum_{k=0}^{m-1} (R_1 R_2)^k\ e^{-(m-1-k)\gamma\Delta t} \tag{8}$$

or after evaluation of the sum by:

$$\Phi_d(t_N + m\Delta t) = \Phi_d(t_N)\, e^{-\gamma m\Delta t} + \Phi_{spec}(t_N)\, D_{12}\, e^{-(m-1)\gamma\Delta t} \left(\frac{B_\nu^{2m} - 1}{B_\nu^2 - 1}\right) \tag{9}$$

In the previous formula:

$$B_\nu^2 = (R_1 R_2)\, e^{\gamma\Delta t} = e^{(\gamma-\beta)\Delta t} \tag{10}$$

Equations 6 and 9 can then be added to obtain the remaining energy in the flow of sound particles at time (t_N+$m\Delta t$) after the impulse emission by the source. As explained in section 2, the remaining energy decreases more or less by successive steps as a function of time, but we can approximate this stepwise decrease by a continuous function, if ($m\Delta t$) is simply replaced by any $\tau > 0$ in the model.

$$\Phi_{tot}(t_N + \tau) = \Phi_d(t_N)\, e^{-\gamma\tau} + \Phi_{spec}(t_N) \left(e^{-\beta\tau} + K_\nu\, e^{-\gamma\tau}\left(e^{(\gamma-\beta)\tau} - 1\right)\right) \tag{11}$$

with the parameter $K_\nu = \frac{D_{12}\, e^{\gamma\Delta t}}{B_\nu^2 - 1}$. If we group the terms proportional to each exponential decay and approximate $\Omega_N \cong \frac{A}{(NL)^2} = \frac{A}{c^2 t_N^2}$, the following expression is obtained (for $t > t_N$):

$$\Phi_{tot}(t) = \left(\Phi_d(t_N) - \Phi_{spec}(t_N) K_\nu\right) e^{\gamma t_N}\, e^{-\gamma t} + E_s \frac{A}{2\pi\, c^2 t_N^2} (1 + K_\nu) e^{-\beta t} \tag{12}$$

Finally, the reverberation energy decay is obtained by backward integration of eq.12 between $t > t_N$ and infinity. If the symbol C_d is used for the product of all constant factors multiplying $e^{-\gamma t}$, then the final expression for the energy decay is very similar to the MPW model (see eq. 5):

$$D_s(t) = C_d \frac{e^{-\gamma t}}{\gamma} + W_s \frac{A\,(1 + K_\nu)}{2\pi\, c^2} \frac{e^{-\beta t}}{\beta\, t_N^2} \tag{13}$$

The significant difference with the MPW model is that we have now the sum of two 'pure' exponential reverberation decays (t_N instead of t in the denominator of the second term).

3.2 Model for the early decay

The constant C_d in eq.13 will be fixed as explained in section 2.3 for the MPW model. The same assumption of an exponential early energy decay is used between times 0 and t_N, with γ as the decay rate. In other words, the early reverberation decay is supposed to be fixed by the average absorption in the room only. We recall that this assumption is not intended to accurately model the early reverberation decay, but the goal is only to give an initial condition to eq.13. The justification is the same as in (2).

$$C_d = W_s \left(1 - \frac{A\,(1 + K_\nu)}{2\pi\, c^2} \frac{\gamma\, e^{(\gamma-\beta) t_N}}{\beta\, t_N^2}\right) \tag{14}$$

Regarding the initial time t_N, a first proposal is to adopt the same value as in (2), i.e. ($t_N = \ln(10)/\gamma$) corresponding to the first 10dB of the decay. Double this value will also be tested (corresponding to the first 20 dB of the reverberation decay).

An additional procedure is needed if $C_d<0$ in eq.14. This situation must be avoided as it can lead to unrealistic energy decays, as shown in (2). In that case, the same initial value of the decay t_N is used, but the constant C_d is set to zero and another decay rate is selected for the exponential early decay ($e^{-\mu t}$ with $\mu < \gamma$).

3.3 Test of the simplified model

The simplified model expressed by eqs. 13 and 14 is now tested by comparison with a ray tracing program and the MPW model.

A shoebox room similar to the one used in (2) is selected: the length L in Figure 1 is set to 30m, while the two other dimensions (width and height) are 10m. All the lateral surfaces are diffusely reflecting, with a variable absorption coefficients. Only the surfaces A_1 and A_2 can reflect specularly, with a scattering coefficient value being equal to either 0, 0.3 or 1. The sound source is situated in the middle of the room in this study and several receptors are distributed in the volume for the ray tracing program.

Regarding the distribution of absorption in the room, eight tests cases are defined in Table 1: A_1 and A_2 share the same value α_{12} while all other surfaces have the same absorption coefficient α_{others}.

Table 1 – The different combinations of absorption coefficients

Test case	1	2	3	4	5	6	7	8
α_{others}	0.2	0.2	0.2	0.5	0.5	0.8	0.8	0.8
α_{12}	0.1	0.2	0.4	0.1	0.4	0.1	0.2	0.4

If the scattering coefficients of A_1 and A_2 are both equal to 1, then all reflections in the shoebox room are of the diffuse type. In that case, the energy decays computed by the sound ray program are very similar from one receptor to another and also quite exponential (linear decay in dB), whatever the combination of absorption coefficients. Also, both the MPW and the simplified model lead to the same results: since R_1=R_2=0, the decay rate $\beta \rightarrow \infty$ and the decay is simply exponential in both cases, with the decay rate γ. The resulting reverberation times T_{30} are close to those predicted by the ray tracing program (averaged over all receiving positions).

More interesting is the test case in which specular reflections are defined on A_1 and A_2 (scattering coefficients equal to 0). Now, the shape of the reverberation energy decays predicted by the ray tracing program strongly depends on the combination of absorption coefficients, as shown in Figure 2 (green curves). It also significantly depends on the receiving position in this shoebox room, so only the average decay is compared with the MPW and the simplified models. Indeed, those models do not predict local variations of the reverberation, but only the decay of the global energy.

It is first shown in Figure 2 that the MPW model (purple curve) developed in (2) correctly predicts the reverberation decays (compared with ray tracing), for all combinations of absorption coefficients. This is just an illustration of the discussion made in section 2.4 about the validity of this model. The reverberation times T_{30} have also been computed from these energy decays. The average relative difference between the T_{30} values issued from ray tracing and the MPW model is 8% for the cases illustrated in Figure 2, the maximum being 20% for test case number 4.

However, the predictions of the simplified model developed in sections 3.1 and 3.2 are far from accurate. If the deviations with the ray tracing results are quite limited if the absorption of the lateral walls is weak (cases 1 to 3), they are highly significant for all other cases. Not only the energy decay, but also the reverberation times are badly predicted. Another point is the clear discontinuity of the slope between the early decay and eq.13 in most cases, when the decay rate β is much less γ. This discontinuity can also exist in the energy decays computed by the MPW model, but it is much less visible, due to the smoothing effect of the factor $\left(\frac{1}{t^2}\right)$.

In Figure 2, the initial time has been defined as $(t_N = 2\ln(10)/\gamma)$, i.e. corresponding to the first 20dB of the early decay. Changing this value to $(t_N = \ln(10)/\gamma)$ does not modify the conclusions of the comparison.

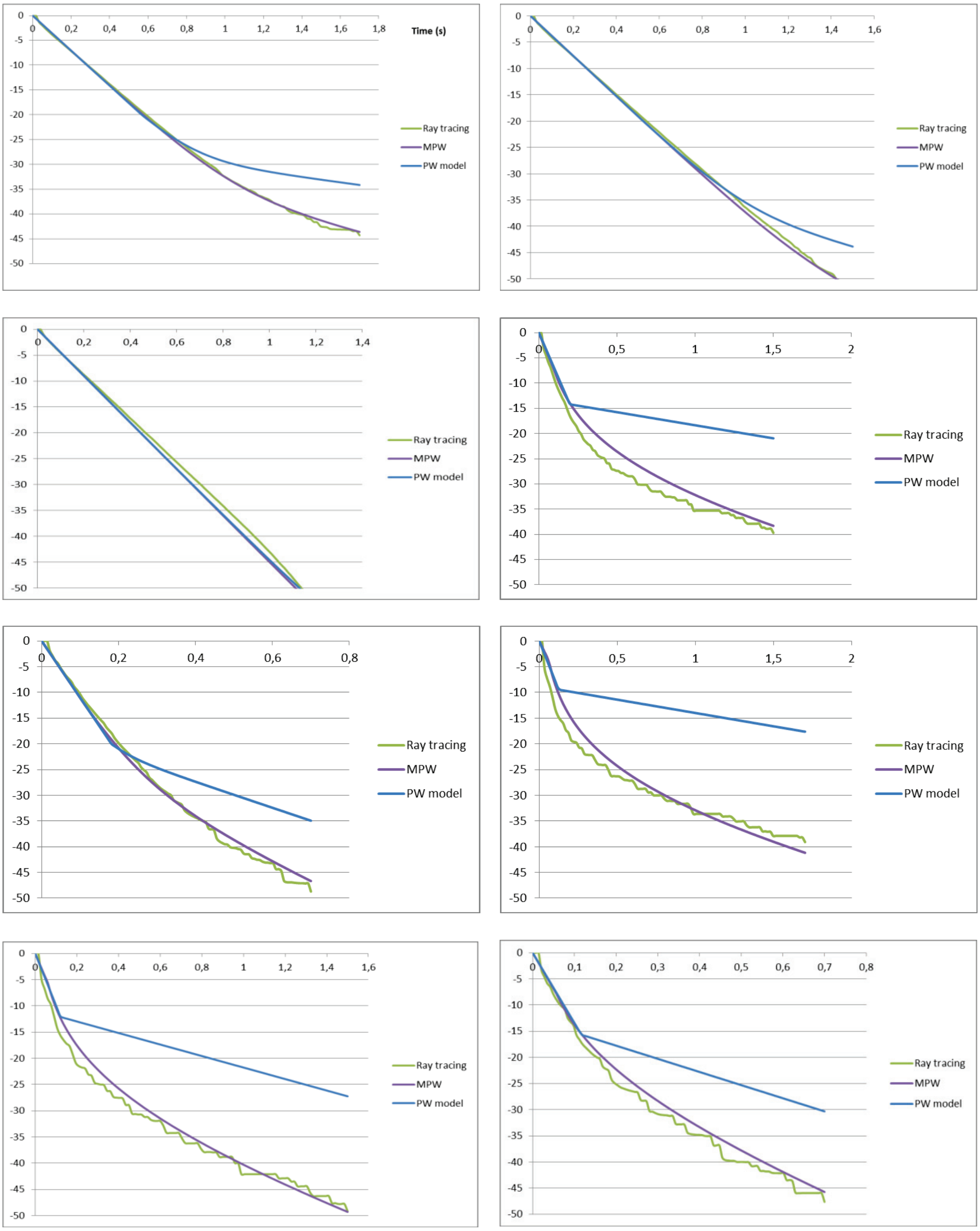

Figure 2 – Comparisons of the reverberation energy decays computed in the shoebox room by the ray tracing program (at a given receptor, close to average results), the MPW model and the simplified "plane wave" (PW) model with $(t_N = 2\ln(10)/\gamma)$. The eight tests cases (depending on absorption coefficients) are presented in order (see table 1), from left to right and up to down. The surfaces A_1 and A_2 are specularly reflecting.

4. CONCLUSIONS

The question asked at the beginning of this study was if it was relevant to still simplify the MPW analytical model of the reverberation energy decay in a room with a pair of parallel surfaces. This model was developed by the author of this paper in (2) and led to an expression of the decay as the sum of two exponential decays, the second one being multiplied by $\left(\frac{1}{t^2}\right)$. The simplification consists in assuming plane wave propagation between the two parallel surfaces, from a certain time of the decay. This results in a modified expression of the decay as the sum of two 'pure' exponential decays, a model that has already been proposed by other authors.

Several comparisons have been done with the decays predicted by a sound ray program which is considered as the reference method. In particular, it has been shown in section 3 that, for a particular shoebox room, the simplified model gives significantly worse predictions than the MPW model. This example is the only one illustrated in this paper, but bad predictions have also been observed in many other cases.

It appears that no clear adaptation of the model's parameters could improve the accuracy of the simplified model. The main problem is well illustrated in Figure 2, i.e. the (reference) energy decays predicted by the ray tracing program have a continuously decreasing slope (in dB), something that cannot be accounted for by the 'pure' exponential decay, but well by the factor $\left(\frac{1}{t^2}\right)$.

This confirms the necessity of taking this into account in the MPW model.

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