A method for detecting convergence completion of adaptive filter cancelling feedback path appeared in active noise control systems

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ABSTRACT
We propose a method for detecting the convergence completion of the feedback control filter working as an adaptive filter. Generally, the feedback path is manually estimated before starting active noise control. This estimation is practically desired to be automatically completed. We first introduce a step size control method capable of completing the estimation with a designed error even where the power of disturbance fluctuates. We subsequently propose a method for detecting the convergence completion of the adaptive filter. In practical use, since the impulse response of the feedback path and its power gain are unknown, we cannot directly monitor the difference between the impulse response and the coefficients of the adaptive filter. For solving this problem, we apply two adaptive filters with two different step sizes to estimating the feedback path. Actually, we have already presented a recursive filter expression illustrating the structure of the normalized least mean square algorithm. According to the expression, the square error between the coefficients of the two adaptive filters converges on the designed error multiplied by the square of the difference between the two step sizes. We finally verify using computer simulations that the proposed method can successfully detect the convergence completion.

Keywords: Feedback, Path, Cancellation

1. INTRODUCTION
Feedforward type active noise control system (1) detecting a primary noise with a microphone, inevitably forms a feedback path, which is generally cancelled by the feedback control filter connected in parallel. The simultaneous equations method (2-4) is proposed as an algorithm not requiring the cancellation unless howling occurs. The feedback path, however, is practically desired to be sufficiently cancelled, because the cancellation error degrades the noise reduction effect (5). The feedback control filter used for the cancellation is generally operated as an adaptive filter before starting the noise control, whose coefficients are manually fixed after the feedback path is estimated with a sufficiently small error (6). In practical use, the operation is desired to be automatically stopped after the estimation error decreases to a designed level.

The automatic stop operation requires a method for detecting that the estimation error decreased to less than the designed level. Three methods for the detection are accordingly proposed. One is the method of tracking the transition of the gradient vector used for updating the coefficients of the adaptive filter (7). Another is the method of using the cross-correlation between the output of the adaptive filter and the remainder subtracted the previously mentioned output from the output of the feedback path (8). The other is the method of continuing to update the coefficients for a prescribed time (9). These methods, however, cannot be applied to practical systems in which the power of the primary noise arriving at the microphone is supposed to be unknown or to fluctuate. The methods also cannot guarantee the estimation error to decrease to the designed level.

The automatic stop operation moreover requires defining the upper limit of the estimation error not degrading the noise reduction effect. The upper limit depends on the shape of the impulse response of the acoustic paths formed in the system. As such examples, two upper limits are shown by using simulations. One is --10 dB derived by using the practical impulse responses whose direct sound

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component is considerably large in comparison to the reflection components (10). The other is --20 dB shown by using artificial impulse responses whose components consist of exponentially decayed random numbers (11). In practical use, the coefficients of the adaptive filter need to be estimated with the designed error even where the power of the primary noise is unknown or fluctuates.

In this study, we first present a method capable of steadily reducing the estimation error to the designed level under the fluctuation of the primary noise power. This method is characterized by applying the block implementation normalized least mean square (NLMS) algorithm (12, 13) to the evaluation of the short time average power of the primary noise disturbing the estimation of the feedback path. The power is then evaluated by substituting the short time square sum of the difference between the outputs of the adaptive filter and the microphone for the real one. The square sum is accordingly overestimated by the estimation error.

The overestimated power is next applied to the step size control method proposed in (14, 15). The step size is then estimated under the desired one. This underestimated step size is actually preferable, which operates so as to reduce the estimation error to smaller than the designed level. Applying the underestimated step size to the adaptive algorithm can decrease the estimation error to less than the designed level even under the power fluctuation of the primary noise.

The remained issue is the derivation of the method for detecting that the estimation error decreased to less than the designed level. A method for detecting the decrease is proposed (16). The method uses two adaptive filters, and applies two different step sizes to them. Then, the square sum of the difference between the coefficient vectors of the two adaptive filters converges on the designed estimation error multiplied by the square of the difference of the two step sizes, which is used as the detection threshold. The convergence of the adaptive filter can be judged by detecting that the square sum of the difference decreases to less than the threshold.

In practical systems, the power gain of the feedback path is supposed to be unknown. The method proposed in (16) accordingly normalizes the square sum of the difference with the power of either adaptive filter, which is approximate to that of the feedback path after the convergence. Unfortunately, some more simulation results show that the method cannot detect where the power gain of the feedback path is not unity. In this study, we show that non-normalized square sum is suitable for the detection, which is verified by computer simulation.

2. STEP SIZE CONTROL

In this study, the block implementation normalized least mean square (NLMS) algorithm (14, 15) formulated by

$$H_{n+1} = H_n + \mu_n \sum_{j=nf_0+1}^{(n+1)f_0} e_j X_j^T X_j,$$  \hspace{1cm} (1)

is applied to the estimation of the feedback path for evaluating the short time average power of the primary noise, where $H_n$ is the coefficient vector given to the adaptive filter at the $n$th block, $\mu_n$ is the step size, $j$ is the sample time index, $e_j$ is the difference between the outputs of the adaptive filter and the microphone, $f_0$ is the block length, $X_j$ is the vector of the reference signal provided to the adaptive filter and the feedback path. Accordingly, the short time average power of the primary noise can be approximately evaluated by

$$Q_n = \sum_{j=nf_0+1}^{(n+1)f_0} e_j^2.$$  \hspace{1cm} (2)

The short time average power is then overestimated by the estimation error. The estimation error, however, decreases along with the convergence of the adaptive filter, and finally to a negligible extent (14).

The step size is subsequently calculated (14, 15) by using

$$\mu_n = 2C_0 P_n f_0 / (Q_n + P_n C_0).$$  \hspace{1cm} (3)

The estimation error decreases to the designed level $C_0$ by applying the step size to Eq. (1), where $f_0$ is the number of taps of the adaptive filter and...
3. CONVERGENCE DETECTION METHOD

The impulse response of the feedback path is supposed to be unknown in practical systems. We cannot accordingly calculate the transition of the estimation error between the impulse response samples and the coefficients of the adaptive filter. We then require a method capable of indirectly detecting that the estimation error decreased to the designed level. In this paper, we next introduce the detection method proposed in (16). The method is characterized by applying two adaptive filters to the estimation of the feedback path.

The first order recursive filter expression of the NLMS algorithm (17) is used for the derivation of the threshold applied to the detection of the convergence of the adaptive filter. In the derivation process, the vector expression of the NLMS algorithm formulated by

$$ H_{j+1} = H_{j} + \mu e_{j} x_{j} / X_{j}^{T} X_{j} $$

is resolved into the element expression,

$$ H_{j+1}(m) = H_{j}(m) + \mu q_{j} x_{j}(m)/p_{j}, $$

where $H_{j}(m)$ is the $m$th element of the coefficient vector given to the adaptive filter at $j$th sample time, $\mu$ is the step size, $x_{j}(m)$ is the $m$th element of the reference signal vector and

$$ p_{j} = X_{j}^{T} X_{j}. $$

The element expression can be rearranged to the first order recursive filter expression (17).

$$ H_{j+1}(m) = H_{j}(m) a_{j}(m) + h(m) [1 - a_{j}(m)] + \mu \sum_{i=0}^{l-1} \{ \Delta_{j}(i) x_{j}(i) + n_{j} x_{j}(m) \} / p_{j}, $$

where $h(m)$ is the $m$th impulse response sample of the feedback path,

$$ a_{j}(m) = 1 - \mu x_{j}^{2}(m)/p_{j}, $$

$$ \Delta_{j}(i) = h(i) - H_{j}(i), $$

and $n_{j}$ is the primary noise disturbing the estimation. In this expression, $h(m)$ is supposed to be invariable until the estimation is completed, which can be regarded as a direct current. According to (18, 19), the transition of the estimation error can be described by substituting

$$ \alpha_{j}(m) = \sum_{m=0}^{l-1} a_{j}(m) / l = 1 - \mu / l < 1 $$

for $a_{j}(m)$. Then, the transition of the estimation error can be calculated as the step response of the first order low pass filter whose recursive coefficient is $\alpha_{j}(m)$.

The filter expression shown in Eq. (8) can be easily developed to the block implementation type,

$$ H_{n+1}(m) = H_{n}(m) a_{n}(m) + h(m) [1 - a_{n}(m)] + g_{n}(m), $$

where

$$ a_{n}(m) = 1 - \mu \sum_{j=n+l+1}^{(n+1)l_{0}} x_{j}^{2}(m)/p_{n}, $$

$$ g_{n}(m) = \mu \sum_{j=n+l_{0}+1}^{(n+1)l_{0}} \sum_{i=0}^{l-1} \{ \Delta_{n}(i) x_{j}(i) + n_{j} x_{j}(m) \} / p_{j}, $$

$$ \Delta_{n}(i) = h(i) - H_{n}(i). $$

It should be noted here that the white noise is used as the reference signal for the estimation of the feedback path. Then, the transition of the estimation error calculated by Eq. (12) is equal to that
provided by Eq. (5) (12, 13). Equation (12) indicates that the adaptive filter coefficient $H_{n+1}(m)$ is estimated as the step response generated by applying $m$th element of the impulse response of the feedback path to the first order recursive filter. As seen from the expression, the coefficient $H_{n+1}(m)$ fluctuates around $h(m)$ after the convergence, as
\[
\lim_{n \to \infty} H_{n+1}(m) = h(m) + \mu_n \sum_{j=nJ_0+1}^{(n+1)J_0} \sum_{i=0(i \neq m)}^{l-1} \frac{\delta_n(i)x_j(i) + n_jx_j(m)}{P_j}, \quad (16)
\]
where $\delta_n(i)$ denotes the estimation error added to the coefficient $h_n(i)$. The second term of Eq. (16) shows that the coefficient $H_{n+1}(m)$ fluctuates around to $h(m)$, which expresses the designed estimation error. The fluctuation equal to the estimation error can be moreover expressed as
\[
\delta_{n+1}(m) = \mu_n \sum_{j=nJ_0+1}^{(n+1)J_0} \sum_{i=0(i \neq m)}^{l-1} \frac{\delta_n(i)x_j(i) + n_jx_j(m)}{P_j}. \quad (17)
\]
In practical use, the power of the reference signal is supposed to be less than that of the primary noise so as not to be perceived. $\delta_n(i)$ is then expected to be negligible in comparison with $n_j$ after the convergence. Equation (17) can be accordingly approximated to
\[
\delta_{n+1}(m) \approx \mu_n \sum_{j=nJ_0+1}^{(n+1)J_0} \sum_{i=0(i \neq m)}^{l-1} n_jx_j(m) / P_j. \quad (18)
\]
We next apply another step size $a \mu_n$ to either adaptive filter, where $a < 1$
\[
a \mu_n \quad (19)
\]
Then, the fluctuation of the coefficient of the either adaptive filter can be expressed by
\[
\delta'_{n+1}(m) \approx a \mu_n \sum_{j=nJ_0+1}^{(n+1)J_0} \sum_{i=0(i \neq m)}^{l-1} n_jx_j(m) / P_j = a \delta_{n+1}(m). \quad (20)
\]
It should be noted here that
\[
d_{n+1}(m) = \delta_{n+1}(m) - \delta'_{n+1}(m) = (1-a)\mu_n \sum_{j=nJ_0+1}^{(n+1)J_0} \sum_{i=0(i \neq m)}^{l-1} n_jx_j(m) / P_j = (1-a)\delta_{n+1}(m) \quad (21)
\]
is equal to the difference of the coefficients of the two adaptive filters. The square sum of the differences can be accordingly evaluated by,
\[
D_n = (H^a_n - H^b_n)^T(H^a_n - H^b_n), \quad (22)
\]
where $H^a_n$ and $H^b_n$ denote the coefficient vectors of the adaptive filters, respectively. Equations (21) and (22) show the square sum $D_n$ converges on the $(1-a)^2$ times of the designed estimation error $C_0$. The convergence of the adaptive filter can be judged by monitoring the transition of $D_n$.

### 4. SIMULATION RESULTS

Figure 1 shows the impulse response of the feedback path applied to simulations, whose samples are given the series of exponentially decayed regular numbers, and whose power gain is adjusted to 0 dB. The other conditions are shown below:
- Reference signal and Primary noise: white noise
- Power ratio of reference signal to primary noise: -6 dB and 0 dB
- Block length $J_0$: 1000
- Designed estimation error $C_0$: 0.001 (-30 dB)
- Step size ratio $a$: 0.9
Equation (23) states that the square sum of the differences, $D_n$, decreases to 20 dB less than the designed estimation error $C_0$. In the conditions shown above, the designed estimation error $C_0$ is -30 dB, which states that the convergence of the adaptive filter can be judged by detecting that $D_n$ decreased to less than -50 dB.

Figure 2 shows the transition of the estimation errors, $D^a_n = (h - H^a_n)^T(h - H^a_n)$, $D^b_n = (h - H^b_n)^T(h - H^b_n)$, and the square sum of the differences, $D^c_n = (H^a_n - H^b_n)^T(H^a_n - H^b_n)$, respectively. In this result, the square sum of the differences, $D^c_n$, increases from less than -60 dB, which is caused by initializing the coefficients of the adaptive filters to zero. The square sum of the differences, $D^c_n$, subsequently decreases to less than -50 dB again. This result shows that the convergence with the designed estimation error, -30 dB, can be judged by detecting that the square sum of the differences, $D^c_n$, decreases to less than -50 dB.

Figure 3 shows the transitions of the estimation errors and the square sum of the differences calculated by reducing the power gain of the feedback path to -10 dB. In this result, the estimation errors decrease to less than the designed level, -30 dB, and consequently the convergence of the adaptive filter can be judged by detecting that the square sum of the differences decreases to the threshold, -50 dB. On the other hand, normalizing the estimation errors and the square sum of the differences with the power gain of the feedback path increases the transitions by 10 dB. Consequently, the estimation errors and the square sum of the differences do not decrease to less than the designed level, -30 dB, and to less than the detection threshold, -50 dB, respectively.
Figure 3 – Transition of the estimation errors and the square sum of the differences (the power gain: -10 dB)

Figure 4 also shows the transition of the estimation errors and the square sum of the differences calculated by increasing the power gain of the feedback path to 10 dB. Similarly, normalizing the estimation errors and the square sum of the differences with the power gain decreases the transitions by 10 dB. Consequently, the estimation errors decrease to less than -40 dB, which is 10 dB less than the designed level, -30 dB. The square sum of the differences also decreases to less than -60 dB, which is 10 dB less than the detection threshold -50 dB. The two results shown in Figs. 3 and 4 demonstrate that the normalization presented in (16) is unnecessary for the detection where the power gain of the feedback path is not unity.

Figure 4 – Transition of the estimation errors and the square sum of the differences (the power gain: 10 dB)

Figure 5 shows the transitions of the estimation errors and the square sum of the differences calculated by changing the power ratio of the reference signal to the primary noise from 0 dB to -6 dB at the indicated point, where the power gain of the feedback path is 10 dB. This result shows that the estimation error decreases steadily to the designed level, -30 dB, though the power ratio changes at the point and that the convergence of the adaptive filter can be judged by detecting that the square sum of the differences decreases to less than -50 dB.
Figure 5 – Transition of the estimation errors and the square sum of the differences calculated changing the power ratio of the reference signal to the primary noise at the indicated point (the power gain: 10 dB)

5. SUMMARY

In this paper, we have proposed the method capable of detecting the convergence of the coefficients of the adaptive filter with the designed estimation error, and then have shown that the method successfully works. In the near future, we will apply the proposed method to re-estimating the feedback path under active noise control.

REFERENCES