Simulation of cochlear response by bone conducted tone

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Abstract
To investigate cochlear responses to bone-conducted (BC) tones, a two-dimensional nonlinear cochlear model is proposed in this paper. The proposed model comprises macromechanical and micromechanical behaviors that can be described by the Laplace equation and Neely and Kims model, respectively. An outer hair cells model is also included as the source of cochlear nonlinearities. Cochlear responses to air-conducted (AC) tones can generally be simulated through stapes vibration. However, in this study, sound pressure was employed to affect the cochlear ducts outer wall for simulating cochlear responses to BC tones. Both frequency- and time-domain solutions can solve the proposed model. Through simulation, a forward traveling wave in the cochlea can be observed for both AC and BC tones in the frequency-domain solution. The time-domain solution indicates compressive nonlinearity for both AC and BC tones. However, the degree of compressive nonlinearity is the primary differentiating factor for AC and BC tones. For lower and higher input levels, the degree of compressive nonlinearity for BC tones is respectively higher and lower than that for AC tones. The obtained simulation results thus corroborate that compared to AC tones, BC tones generate a cochlear traveling wave response with marginally different compressive nonlinearity.

Keywords: Sound, Insulation, Transmission

1 INTRODUCTION
In general, sounds propagating in an external field can be heard. In this situation, the sounds are collected by the outer ear and transmitted into the cochlea. This path is known as air conduction (AC). However, a different path exists in addition to this. Bone conduction (BC) hearing is an alternative to AC hearing, and refers to the transmission of vibrations through the skull bone to the cochlea. However, the mechanism of BC hearing is still unclear. Toward solving this problem, five pathways are suggested [1] as follows: sound pressure in the ear-canal, compression and expansion of the cochlear shell, transmission of pressure from the cerebrospinal fluid, inertia of the middle ear ossicles, and inertia of the cochlear fluid. Therefore, one can hear the BC tone by the combination of all these pathways.

The contribution of the BC propagating pathways depends on the frequency of the incoming sound. In a real situation, however, it is impossible to consider them separately owing to technical difficulties in investigating a complicated hearing system. Meanwhile, a modeling study can facilitate understanding the contribution of the BC pathways.

Linear cochlear models simulating BC excitation have been proposed and demonstrated different responses between AC and BC tones [2, 3]. These results may suggest different cochlear mechanics excited by AC and BC pathways in a linear domain. However, it is well known that cochlear responses are nonlinear. A perception study has indicated compressive curves in AC and BC loudness functions owing to nonlinear cochlear processes [4]. This process can be caused by nonlinear and active elements [5].

The aim of this study is to investigate the effects of nonlinear and active elements on the input–output (IO) property of the cochlea, develop a two-dimensional (2D) cochlear model based on our previously proposed model [6], and estimate the BM responses for the vibrations of the stapes and cochlear shell caused by AC and BC tones, respectively.
2 COCHLEAR MODEL WITH BONE CONDUCTION

2.1 Macrocochlear model

The cochlea is a coiled duct containing fluid. As shown in Fig. 1, in the longitudinal direction of this duct, the cochlear partition (CP) divides two chambers that are the scala vestibuli and scala tympani. Incoming wave through the middle ear transmits in the cochlear fluid, and generates a pressure difference between two chambers. Particularly, this pressure difference vibrates the basilar membrane (BM), which is included in the CP, to face the scala tympani. The vibration pattern of the BM depends on frequency of the incoming sound. At the base and apex sides, higher and lower frequencies of the sounds vibrate the BM, separately. This resonant frequency is called the characteristic frequency (CF). The BM contains numerous sensor cells such as inner and outer hair cells (IHCs and OHCs). The IHC transforms the incoming sound from mechanical to neural information at different locations. Meanwhile, the OHC amplifies the BM motion by 60 dB.

In this study, we develop a two-dimensional (2D) cochlear model based on our previously proposed model [6]. Fluid contained in the cochlear duct is assumed as an ideal fluid. Hence, the pressure difference \( p \) affecting the CP can be expressed using the Laplace equation as follows [7]:

\[
\frac{\partial^2 p(t)}{\partial x^2} + \frac{\partial^2 p(t)}{\partial y^2} = 0, 0 < x < L, 0 < y < H, \tag{1}
\]
where $t$, $x$, and $y$ represent time, and the location toward the length and height, respectively; $L$ and $H$ are the length and height of the cochlear duct, respectively. In the proposed model, we assume that AC and BC tones vibrate the stapes and cochlear shell, respectively, as shown in Fig. ?? Therefore, the boundary conditions in this model are as follows:

$$\frac{\partial p(t)}{\partial x} \bigg|_{x=0} - 2\rho \ddot{w}_{SR}(t) = 2\rho \ddot{w}_{SO}(t), \quad (2)$$

$$p(t) \bigg|_{x=L} = 0, \quad (3)$$

$$\frac{\partial p(t)}{\partial y} \bigg|_{y=0} = 2\rho \ddot{w}(t), \quad (4)$$

$$\frac{\partial p(t)}{\partial y} \bigg|_{y=H} = 2\rho \ddot{w}_{CS}, \quad (5)$$

where $\ddot{w}_{SR}$ and $\ddot{w}_{SO}$ are the accelerations of the stapes caused by the incoming sound and reflection from the cochlea, respectively; $\ddot{w}$ and $\ddot{w}_{CS}$ are accelerations of the cochlear partition and cochlear shell, respectively; $\rho$ represents the fluid density.

2.2 Microcochlear model

In this study, we use the Neely and Kim model [8] as the micromechanical model of the cochlea, as shown in Fig. 3. In the micromechanical system, the tectrial membrane (TM) and OHC exist on the BM. The displacement of the BM and TM are represented by $\xi_1$ and $\xi_2$, respectively. The Displacement vector is denoted as $\xi = [\xi_1 \, \xi_2]^T$. Neely and Kim assumed that the discrete CP can be modeled in two mass models as the following equation:

$$M\dddot{\xi} + C\dot{\xi} + K\xi = P + P_a, \quad (6)$$
where the matrices $M$, $C$, and $K$ represent mass, resistance, and stiffness, respectively, and are defined as

$$
M = \begin{pmatrix}
m_1 & 0 \\
0 & m_2
\end{pmatrix},
$$

(7)

$$
C = \begin{pmatrix}
c_1 + c_3 & -c_3 \\
-c_3 & c_2 + c_3
\end{pmatrix},
$$

(8)

$$
K = \begin{pmatrix}
k_1 + k_3 & -k_3 \\
-k_3 & k_2 + k_3
\end{pmatrix}.
$$

(9)

The external pressure $P$ and $P_a$ affect the BM; the former and latter are the pressure difference between two chambers and the active pressure through the OHC, respectively.

$$
P = \begin{pmatrix}
p(x, 0, t) \\
0
\end{pmatrix},
$$

(10)

$$
P_a = \begin{pmatrix}
p_a(x, t) \\
0
\end{pmatrix}.
$$

(11)

(12)

2.3 Simulation condition

To simulate human cochlear responses, we used the values of the parameters listed in Table 1.
Table 1. List of parameters in the cochlear model to simulate the human’s cochlea

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
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<tbody>
<tr>
<td>l</td>
<td>0.035</td>
<td>m</td>
</tr>
<tr>
<td>N</td>
<td>300</td>
<td></td>
</tr>
<tr>
<td>M</td>
<td>10</td>
<td></td>
</tr>
<tr>
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<td>1000</td>
<td>kg/m³</td>
</tr>
<tr>
<td>A</td>
<td>10⁻⁶</td>
<td>m²</td>
</tr>
<tr>
<td>b</td>
<td>10⁻³</td>
<td>m</td>
</tr>
<tr>
<td>m₁</td>
<td>3 x 10⁻²</td>
<td>kg/m²</td>
</tr>
<tr>
<td>c₁</td>
<td>120 + 13400e¹⁵₀₅ₙ</td>
<td>N·s/m³</td>
</tr>
<tr>
<td>k₁</td>
<td>2.2 x 10⁸e⁻³₀₀ₙ</td>
<td>N/m³</td>
</tr>
<tr>
<td>m₂</td>
<td>5 x 10⁻³</td>
<td>kg/m²</td>
</tr>
<tr>
<td>c₂</td>
<td>88e⁻¹₆₅ₙ</td>
<td>N·s/m³</td>
</tr>
<tr>
<td>m₃</td>
<td>1.4 x 10⁷e⁻⁻³₀₀ₙ</td>
<td>N/m³</td>
</tr>
<tr>
<td>c₃</td>
<td>16e⁻⁶₀ₙ</td>
<td>N·s/m³</td>
</tr>
<tr>
<td>k₃</td>
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<td>N/m³</td>
</tr>
<tr>
<td>c₄</td>
<td>8800e⁻¹₅₀ₙ</td>
<td>N·s/m³</td>
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<tr>
<td>k₄</td>
<td>1.15 x 10⁹e⁻⁻³₀₀ₙ</td>
<td>N/m³</td>
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<tr>
<td>γ</td>
<td>0.5</td>
<td></td>
</tr>
</tbody>
</table>

3 SIMULATION RESULTS

3.1 Linear responses

To calculate the BM response in the frequency domain, in this simulation, we define the active pressure $p_a$ according to the original Neely–Kim model as follows:

$$p_a(x,t) = γ \left( c_4 \xi_c + k_4 \xi_c^\prime \right),$$

(13)

where $γ$, $c_4$, and $k_4$ represent the gain factor of the feedback, and the damping and stiffness of the OHC, respectively; $ξ_c$ denotes the gap between the BM and TM as follows:

$$ξ_c(x,t) = ξ_1(x,t) - ξ_2.$$  

(14)

Figure 4 shows the BM responses for BC and AC tones at three different frequencies. The amplitude in the BM responses exhibit peaks depending on the sound frequency. Additionally, the locations of these peaks match with those of both BC and AC tones. Furthermore, the angular in the BM responses is decayed from the base to apex, and this trend is clear at the peak site of the amplitude in the BM responses. Therefore, these results indicate that the BC tone can generate the traveling wave of the BM, as obtained by the AC tone.

3.2 Nonlinear responses

In the simulation above, we calculated the BM responses in the linear domain. However, the cochlear responses demonstrated nonlinearities that were caused by nonlinear transductions in the OHC processing [5]. In this simulation, we denote the active pressure $p_a$ containing the nonlinearity as follows:

$$p_a(x,t) = γ \left( c_4 ξ_c^{nl} + k_4 ξ_c^{nl} \right),$$

(15)

where $ξ_c^{nl}$ is the saturating property that is calculated from the distance of the gap $ξ_c$ as follows:

$$ξ_c^{nl}(t) = α_r \tanh \left( \frac{ξ_c}{α_r} \right),$$

(16)
where \( \alpha_{phatr} \) is determined to be \( x_i c = x_i m \) when \( x_i c \) is sufficiently small at 1 nm or less. The following initial conditions are used to calculate the output from the model in the time domain.

\[
\begin{align*}
\xi(x,t)|_{t=0} &= \xi(x) |_{t=0} = 0, \\
p_a(x,t)|_{t=0} &= 0.
\end{align*}
\]

(17)  

(18)

Figure 5 shows the IO properties of the BM responses for BC and AC tones at frequencies of 0.25, 1, and 4 kHz. The amplitude in the BM responses is varied with the input level. For the lower input level, the IO curves are linear for both the BC and AC tones. However, at moderate and higher input levels, the IO curves exhibit different degrees of compressive nonlinearity. For the BC tone, this degree is higher than that for the AC tone among the moderate input levels, but disappears at higher input levels.

4 DISCUSSION

The vibrations of the stapes and cochlear shell are related the AC and BC pathways that generate the BM traveling wave, respectively, as shown in Fig. 4. In Ref. [2], this trend was reported. Furthermore, at the CF site, the proposed model demonstrated cochlear amplification, i.e., a basic feature in cochlear mechanics, as reported in Ref. [5]. Thus, these results suggest that the cochlear responses are similar for both AC and BC tones.
Figure 5. IO property of BM responses obtained from the nonlinear cochlear model. Solid and dashed lines indicate the BM responses for BC and AC tones, respectively.

In comparison to the above consideration in the linear domain, however, real cochlear responses are nonlinear. Hence, we proposed a nonlinear cochlear model excited by BC tones. As shown in Fig. 5, the IO property of the model shows nonlinear curves. Particularly, it is known that the growth of the IO on a moderate input level is less than that at a lower input level [9]. This phenomenon is known as compression and is key for understanding cochlear mechanics [5]. Comparing the IO curves of AC and BC tones, at moderate sound pressure levels, the degree of compression caused by the BC tone is large. However, at higher sound pressure levels, the degree of compression caused by the AC tone tends to increase. Hence, the IO curves derived from the loudness function demonstrated the similar result [4]. Therefore, these results indicate that the AC and BC pathways can be divided by focusing on the difference in the degree of compression.

5 CONCLUSIONS
Nonlinear and active elements that affect the IO property of the cochlea were investigated in this study. Additionally, a 2D cochlear model excited by both AC and BC tones was developed. The proposed model could
exhibit the BM traveling waves for both tones. However, the degree of compression was different between the two pathways.

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