Uncertainty analysis for the in situ sound power level determination using the substitution method

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Abstract
Sound power determination at different environments and using different techniques exhibits deviations especially at low frequencies. In previous studies the establishment of traceability has been introduced and the realisation and dissemination of the unit watt in airborne sound has been discussed. For the completion of traceability, the sound power level determination of realistic sources is required. The contribution presents the application of the substitution method for the in situ sound power determination of real sources. The substitution method includes both sound pressure and sound intensity measurements. For collecting experimental data, different measurement methods were applied in various environments. Following the concept of traceability, the sound power level is accompanied by the related uncertainty, which explicitly explains the contributing factors along with their extent of contribution. A comparison is made between the uncertainty levels using sound pressure and sound intensity. The proposed uncertainty covers a wider frequency range compared to the up-to-date standards.

Keywords: Sound power, Substitution, Uncertainty

1 INTRODUCTION
Sound power is a widely applicable acoustic quantity, since it characterizes a sound source based on its radiated sound energy. The current sound power determination exhibits to two main insufficiencies. The first is the low frequency limitation, where the results show deviations below 100 Hz depending on the measuring technique (sound pressure or sound intensity measurements) and on surrounding environment. The second is the applicability of the sound power determination methods to sources with broadband spectral characteristics. This excludes tonal sources. The determination is expected to improve with the establishment of traceability of the sound power unit in airborne sound (1). An additional advantage of traceability is the determination of a transparent uncertainty budget including the combined uncertainty along with its contributing factors.

The first part of traceability is the realization of the unit, which has been performed following the assembling of a number of primary sources (2). The second part is the dissemination of the unit where transfer sources are the key element (3). The last traceability stage is the application of the unit. At this stage the sound power of a sound source positioned in a real environment (in situ) is determined by applying the substitution method using a transfer source. The substitution method is the fundamental tool for the traceability, because it may enable the reference of the in situ sound power level of a sound source to its free field sound power level. The method is currently used based on sound pressure measurements (4, 5, 6, 7), but sound intensity levels have also been used for its application (8). According to the substitution method, the unknown sound power level of a sound source \( L_W, \text{unknown} \) can be determined by the known sound power level of another source \( L_W, \text{known} \) while measuring the time and surface averaged sound pressure or intensity level of both sources \( L_p, \text{unknown}, L_I, \text{unknown}, L_p, \text{known}, L_I, \text{known} \). This is described by

\[
L_W, \text{unknown} = L_W, \text{known} + L_p, \text{unknown} - L_p, \text{known} \\
L_W, \text{unknown} = L_W, \text{known} + L_I, \text{unknown} - L_I, \text{known}
\] (1)
2 MEASUREMENTS

The substitution method was applied for the determination of the in situ sound power level of three sources (a fan, an air compressor and a vacuum cleaner) to cover various spectral components. Sound pressure and sound intensity measurements were performed, while varying the measurement method in terms of measurement surface and surrounding environment. PTB’s scanning apparatus (9) was used for sound pressure and sound intensity measurements of various radii in PTB’s hemianechoic room. In the same room, measurements at discrete points were also performed at a single radius (12 points over a box shaped surface). The frequency analysis was performed in both broad (one-third octave) and narrow (FFT) bands (8). Discrete point measurements were performed at various surrounding environments of varying volume and absorption.

3 SOUND POWER DETERMINATION

The determination of the sound power level of the devices under test (DUT) requires the application of the substitution method twice. First, the sound power level of the transfer source (TS) is determined based on the sound power level of the primary source (PS). Then the sound power level of the DUT is determined based on the sound power level of the transfer source.

3.1 Measurements with different surface sampling

Following the analysis of the transfer source, the sound power level of the DUT is given by

\[ L_{W,TS,cal,p} = L_{W,PS} + L_{p,TS,cal} - L_{p,PS} + 10 \log \left( \frac{S_{TS,cal}}{S_{PS}} \right) \text{dB} - 10 \log \left( \frac{B_{TS,cal}}{B_{PS}} \right) \text{dB} + 5 \log \left( \frac{T_{TS,cal}}{T_{PS}} \right) \text{dB} \]

(2)

where \( S \) is the surface over which the sound power is determined, \( B \) is the atmospheric pressure, \( T \) the ambient temperature, \( C_{\text{noise,TS}} \) the background noise correction (10), \( C_{\text{scr,probe,p}} \) the correction for the use of windscreens, whose shape is different for sound pressure and sound intensity measurements, \( C_3 \) the correction for the air absorption (10) and \( C_{FFT} \) the correction for the FFT settings. The latter is imposed because the analysis of the primary source was performed with a uniform window, while the transfer source is analysed with a Hanning window, which adds energy due to the side lobes (11).

When sound intensity is used, the sound power level is given by

\[ L_{W,TS,cal,I} = L_{W,PS} + L_{I,TS,cal} - L_{I,PS} + 10 \log \left( \frac{S_{TS,cal}}{S_{PS}} \right) \text{dB} - 10 \log \left( \frac{B_{TS,cal}}{B_{PS}} \right) \text{dB} + 5 \log \left( \frac{T_{TS,cal}}{T_{PS}} \right) \text{dB} \]

(3)

The sound power level of the primary source was determined from vibration velocity measurements using a laser scanning vibrometer and was kindly provided for the present study. In equations (2) and (3) there are corrections for the changes in atmospheric pressure and ambient temperature. The factor -0.5 was chosen for the changes in temperature because of the difference in the source order (primary source is a monopole, the transfer source a dipole).

Following the analysis of the transfer source, the sound power level of the DUT is given by

\[ L_{W,DUT,cal,p} = L_{W,TS,cal,p} + L_{p,DUT,cal} - L_{p,TS,cal} + 10 \log \left( \frac{S_{DUT,cal}}{S_{TS,cal}} \right) \text{dB} - 10 \log \left( \frac{B_{DUT,cal}}{B_{TS,cal}} \right) \text{dB} + C_{\text{noise,DUT,cal}} - C_{\text{noise,TS,cal}} + C_3 \text{DUT,cal} - C_3 \text{TS,cal} \]

(4)
and

\[ L_{W,DUT,cal,I} = L_{W,TS,cal,I} + L_{I,DUT,cal} - L_{I,TS,cal} + 10\lg \left( \frac{S_{DUT,cal}}{S_{TS,cal}} \right) \ dB \]

\[ -10\lg \left( \frac{B_{DUT,cal}}{B_{TS,cal}} \right) dB + 25\lg \left( \frac{T_{DUT,cal}}{T_{TS,cal}} \right) dB + C_{3,DUT,cal} - C_{3,TS,cal} \]

(5)

The temperature correction for this case is -2.5, which corresponds to a dipole, since both transfer source and DUTs are of similar order as spherical harmonics decomposition revealed (12).

Concerning the measurement surface, for the primary source only scanning measurements were performed at 0.70, 0.80, 0.90, 1.45, 1.70, 2.34 & 2.75 m. The transfer source was also measured by scanning at the same radii and at discrete points. The DUTs were measured by scanning only at 1.45 m and at discrete points. For the measurement surface correction of equations (2) and (3) the combination of the same radii were only used, because this way the sound power determination corresponds more to realistic measurements.

The uncertainty analysis of GUM (13) was applied for the calculation of the related uncertainty. The uncertainty of equation (2) is

\[ u^2(L_{W,TS,cal,p}) = u^2(L_{W,PS}) + u^2\left[ L_{p,TS,cal} - L_{p,PS} + 10\lg \left( \frac{S_{TS,cal}}{S_{PS}} \right) dB \right] + u^2\left[ L_{p,PS} + 10\lg \left( \frac{S_{PS}}{S_{PS,ref}} \right) dB \right] \]

\[ + u^2(C_{noise,PS}) + u^2(C_{noise,TS}) + u^2(C_{scr,probe,p}) + u^2(C_{3,TS,cal} - C_{3,PS}) + u^2(C_{FFT}) \]

\[ u^2(L_{W,TS,cal,I}) = u^2(L_{W,PS}) + u^2\left[ L_{I,TS,cal} - L_{I,PS} + 10\lg \left( \frac{S_{TS,cal}}{S_{PS,ref}} \right) dB \right] \]

\[ + u^2\left[ L_{I,PS} + 10\lg \left( \frac{S_{PS}}{S_{PS,ref}} \right) dB \right] \]

\[ + u^2(C_{scr,probe,I}) + u^2(C_{3,TS,cal} - C_{3,PS}) + u^2(C_{FFT}) \]

(6)

The uncertainty of the sound power level of the transfer standard based on sound intensity is

\[ u^2(L_{W,DUT,cal,p})_{scan} = u^2(L_{W,TS,cal,p}) + u^2\left[ L_{p,DUT,cal} - L_{p,TS,cal} + 10\lg \left( \frac{S_{DUT,cal}}{S_{TS,cal}} \right) dB \right]_{scan} \]

\[ + u^2\left( L_{p,DUT,cal} \right)_{scan} + u^2\left[ L_{p,TS,cal} + 10\lg \left( \frac{S_{TS,cal}}{S_{TS,cal,ref}} \right) dB \right]_{scan} \]

\[ + u^2(C_{noise,DUT,cal}) + u^2(C_{noise,TS,cal}) + u^2(C_{3,DUT,cal} - C_{3,TS,cal}) \]

(7)

The uncertainty of the environmental conditions was found to be negligible and was apparently omitted from the uncertainty determination. The measurement radius of 1.45 m was used as reference value for the uncertainty of the time and surface averaged sound pressure and sound intensity level.

In the same sense, the uncertainty of the sound power level of the DUTs in case of scanning measurements is described by

\[ u^2(L_{W,DUT,cal,p})_{discrete} = u^2(L_{W,TS,cal,p}) + u^2\left[ L_{p,DUT,cal} - L_{p,TS,cal} + 10\lg \left( \frac{S_{DUT,cal}}{S_{TS,cal}} \right) dB \right]_{scan} \]

\[ + u^2\left( L_{p,DUT,cal} \right)_{discrete} + u^2\left( L_{p,TS,cal} \right)_{discrete} \]

\[ + u^2(C_{noise,DUT,cal}) + u^2(C_{noise,TS,cal}) + u^2(C_{3,DUT,cal} - C_{3,TS,cal}) \]

(8)

For measurements at discrete points the uncertainty is

\[ u^2(L_{W,DUT,cal,p})_{discrete} = u^2(L_{W,TS,cal,p}) + u^2\left[ L_{p,DUT,cal} - L_{p,TS,cal} + 10\lg \left( \frac{S_{DUT,cal}}{S_{TS,cal}} \right) dB \right]_{scan} \]

\[ + u^2\left( L_{p,DUT,cal} \right)_{discrete} + u^2\left( L_{p,TS,cal} \right)_{discrete} \]

\[ + u^2(C_{noise,DUT,cal}) + u^2(C_{noise,TS,cal}) + u^2(C_{3,DUT,cal} - C_{3,TS,cal}) \]

(9)
Corresponding equations may be derived for sound intensity measurements by omitting the uncertainty related to the background noise.

\[ u(L_{W,T}) + u(\Delta L_p) + u(L'_{p,DUT,cal}) + u(L'_{p,TS,cal}) + u(C_{noise,DUT,cal}) + u(C_{noise,TS,cal}) + u(C_3) + u(L_{W,DUT,cal,p}) \] (11)

Figure 1. Total and contributing uncertainties of the air compressor sound power level for sound pressure measurements under calibration conditions using the scanning method. Top: one-third octave bands. Bottom: FFT bands.

### 3.2 Measurements at different surrounding environments

If the sound pressure or sound intensity measurements are performed in situ, equation (4) is modified to

\[ L_{W,DUT,\text{in situ},p} = L_{W,TS,\text{in situ}} + L'_{p,DUT,\text{in situ}} - L'_{p,TS,\text{in situ}} + 10\log\left( \frac{S_{DUT,\text{in situ}}}{S_{TS,\text{in situ}}} \right) \text{ dB} \] (10)

The environmental corrections along with the corrections for the background noise and the air absorption vanish due to the identical conditions during the measurements. A similar equation applies for sound intensity measurements. The uncertainty of equation (10) is provided by

\[ u^2(L_{W,DUT,\text{in situ},p}) = u^2(L_{W,TS,\text{in situ},p}) + u^2(L'_{p,DUT,\text{in situ}} - L'_{p,TS,\text{in situ}}) + u^2(L'_{p,DUT,\text{in situ}}) + u^2(L'_{p,TS,\text{in situ}}) + u^2(C_{noise,DUT,\text{in situ}}) + u^2(C_{noise,TS,\text{in situ}}) \] (11)
The uncertainty for the sound intensity measurements is similar to equation (11) while omitting the background noise uncertainty.

![Graph](image1)

![Graph](image2)

Figure 2. Sound power level difference compared to existing and proposed uncertainty. Top: one-third octave bands. Bottom: FFT bands.

4 RESULTS

Figure 1 provides an example of the uncertainty analysis for the determination of the sound power level of the air compressor by scanning sound pressure measurements in the hemianechoic room. As it may be observed, there are highly influential factors such as the uncertainty of the sound pressure level difference, the uncertainty of each sound pressure level and the uncertainty of the transfer source sound power level. On the other hand the uncertainties related to background noise, air absorption and FFT windowing is not influential to the overall uncertainty. The uncertainties related to the sound intensity levels and their difference are also the most influential for the case of sound intensity measurements.

For the validation of the method, the sound power levels after the substitution method were compared to directly determined levels (14, 15, 16) by

\[
\Delta L_W = L_{W, sub} - L_{W, dir} \tag{12}
\]
The sound power level differences were then compared to the existing (14, 15, 16) and to the proposed uncertainty levels. For the FFT analysis the current one-third octave band uncertainty was used. Figure 2 shows the sound power level differences along with both uncertainties for the case of sound intensity measurements at discrete points in the hemianechoic room. As it is shown the sound power level differences are sufficiently covered by the proposed uncertainty, especially at low frequencies where the existing uncertainty does not apply. At some high frequencies, the existing uncertainty covers the sound power level differences with lower uncertainty values.

5 CONCLUSIONS

The sound power determination of realistic sound sources was performed using the substitution method while varying the measurement surface and the surrounding environment. Apart from the sound power, the related uncertainty was determined in a transparent approach, which allows the decomposition of the combined uncertainty to the contributing factors. The proposed uncertainty was compared to the existing one. The uncertainty following the sound power dissemination widens the frequency range of the sound power determination. Low frequency effects are sufficiently covered by the proposed uncertainty. The combination of the proposed and existing uncertainty may provide an updated and improved uncertainty budget for the in situ sound power level determination.

REFERENCES


