Sound field reproduction with exterior field cancellation using variable-directivity loudspeakers

Bokai DU¹; Michael KOHNEN²; Michael VORLÄNDER²; Xiangyang ZENG¹
¹ School of Marine Science and Technology, Northwestern Polytechnical University, China
² Institute of Technical Acoustics, RWTH-Aachen University, Germany

ABSTRACT
Sound field reproduction aims to create or reproduce a certain sound environment where not only the audio content, but also the spatial property of the sound field has to be preserved. For a real system which is usually placed in a “listening room”, the reproduction performance is affected or even destroyed by superimposing of reflections from walls. As a result, it is important to control the sound energy which is radiated to the space outside the loudspeaker array. Compared with omnidirectional loudspeakers, the use of variable-directivity loudspeakers can eliminate the exterior field according to the Kirchhoff-Helmholtz integral equation. This paper investigates the performance of variable-directivity arrays using pressure matching-acoustic contrast control (PM-ACC) and compares its performance with that of omnidirectional one.

Keywords: Variable-directivity loudspeakers, Pressure matching, Exterior cancellation

1. INTRODUCTION

Sound field reproduction (SFR) is the process of reproducing a desired sound field to make person or people in listening region of loudspeaker array experience a virtual sound environment. In this virtual environment, not only the content of the sound, but also the spatial property of the sound field needs to be recreated.

An overview of requirements and technical challenges related to audio immersive systems can be found in [1-2]. Generally speaking, there are three most popular approaches to SFR for modern reproduction system: Wave field synthesis (WFS) [4], Higher-order Ambisonics (HOA) [5] and the pressure matching method. Huygen's principle and the Kirchhoff-Helmholtz (K-H) boundary surface integral equation are theory foundations for WFS method. Reproduction using this method is usually based on a planar loudspeaker array. HOA method decomposes the primary sound field into spherical harmonics domain and Mode-matching is often applied to design the loudspeaker signals. For the pressure matching method proper loudspeaker signals are obtained by minimizing the difference between target sound pressures (sampled by microphone array) and reproduced sound pressures stirred by a loudspeaker array at sample points. Usually, a least-square (LS) criterion is used to design the loudspeaker signals and limit the loudspeaker energy.

However, the performance of a reproduction system placed in the ‘listening room’ suffers much from superimposing of reflections from walls. Active methods can be applied to compensate the effect caused by reflections [5], but this compensation requires measurement of the loudspeaker response at one or more positions for different ‘listening rooms’. Instead, Poletti proposed to use fixed or variable directivity loudspeakers in sound field reproduction systems to constrain the exterior sound field energy [6-7]. The loudspeaker signal design was achieved by HOA method. But the application of HOA method requires a spherical loudspeaker array whereas a more flexible geometry of loudspeaker array can be taken if we use a pressure matching method approach.

Considering works on multi-zone reproduction techniques [8-10], sound field reproduction with exterior sound field energy constrain method is proposed and the performance of variable-directivity loudspeaker array is investigated by simulation. At last, a subjective listening experiment is conducted to find the border of the reproduction ‘sweet point’ when primary sources are at different positions.
directions. This listening experiment provides some indication on primary sound field sample. The remaining part of this paper is outlined as follows. The preliminaries and problems are stated in Sec. 2. A proposed reproduction method is described in Sec. 3. Results of the simulation and subjective experiment are presented in Sec. 4 and conclusions drawn in Sec. 5.

2. Preliminaries and Problem Statements

2.1 Pressure Matching for SFR Problem

For a PM-based reproduction system, loudspeaker elements are at located at \( \mathbf{r}_m, m = 1, \ldots, M \), where \( M \) is the number of loudspeakers. Coordinates of control points of primary sound field are \( \mathbf{r}_{pm}, l = 1, \ldots, L \), where \( L \) is the number of control points. In the frequency domain, the sound pressure at \( m \)-th control point stirred by loudspeaker array can be expressed as the summation of the sound pressure stirred by each loudspeaker of the reproduction array:

\[
p_m = \sum_{l=1}^{L} w_l(\omega) g_{ml}(\omega) \tag{1}
\]

where \( w_l \) is \( l \)-th loudspeaker's driving signal and \( g_{ml} \) is the acoustic transfer function (ATF) between the \( l \)-th loudspeaker and \( m \)-th matching point. For the following part of this paper the methods are investigated in the frequency domain and the angular frequency \( \omega \) is no longer indicated.

For the whole reproduction system, the reproduced sound pressure at matching points is expressed in matrix form:

\[
\mathbf{p}_{rep} = \mathbf{Gw} \tag{2}
\]

where \( \mathbf{p}_{rep} \) is a \( M \) by 1 vector of reproduced sound pressure at matching points, \( \mathbf{w} \) is an \( L \) by 1 vector of loudspeaker weights, \( \mathbf{G} \) is a \( M \) by \( L \) matrix whose \((m,l)\)-th entry is equal to \( g_{ml} \). Now, the LS-based SFR problem goes to solve the an optimization problem: minimizing the squared approximation error between the target sound pressure \( \mathbf{p}_{des} \) and reproduced sound pressure \( \mathbf{p}_{rep} \), namely:

\[
w: \arg \min_{\mathbf{w}} \| \mathbf{p}_{rep} - \mathbf{Gw} \|^2 + \lambda \| \mathbf{w} \|^2 \tag{3}
\]

where \( \lambda \) is a preselected regularization parameter, it is used to avoid obtaining some physical unrealizable solution (large magnitude), as a result, the driving signal can easily be obtained as:

\[
\mathbf{w} = \left( \mathbf{G}^H \mathbf{G} + \lambda \mathbf{I} \right)^{-1} \mathbf{G}^H \mathbf{p}_{des} \tag{4}
\]

2.2 Variable- Directivity Loudspeaker

In this paper, loudspeaker radiation is simulated using a variable-directivity approach. For example, the variable-directivity loudspeaker can be expressed as a weighted combination of monopole and dipole. Weights of monopole and dipole can be manipulated independently, which results in variable-directivity of a loudspeaker. Green functions of ideal monopole and dipole are reviewed as follows.

The sound field generated by an ideal monopole source in the free field can be expressed as:

\[
p(\mathbf{r}, \mathbf{r}_s) = G(\mathbf{r} | \mathbf{r}_s) = \frac{e^{-jk|\mathbf{r} - \mathbf{r}_s|}}{4\pi|\mathbf{r} - \mathbf{r}_s|} \tag{5}
\]

where \( \mathbf{r} = (x, y, z) \) and \( \mathbf{r}_s \) denotes the position of sound source, the wave number \( k = \omega / c \). For a dipole at position \( \mathbf{r}_d \) and oriented in direction \( \mathbf{v} \), sound pressure at any point is expressed as:

\[
p(\mathbf{r}, \mathbf{r}_d) = \frac{\partial G(\mathbf{r} | \mathbf{r}_d)}{\partial \mathbf{v}} = -jk \frac{e^{-jk|\mathbf{r} - \mathbf{r}_d|}}{4\pi|\mathbf{r} - \mathbf{r}_d|} \left[ 1 + \frac{i}{k|\mathbf{r} - \mathbf{r}_d|} \cos \gamma \right] \tag{6}
\]
where γ is the angle between \( \vec{v} \) and \( \vec{r} - \vec{r}_s \). And in order to ensure flat responses down to a frequency \( f_f \), we must keep a distance from any equalized dipole greater than \( c/(2\pi f_f) \).[7]

3. Sound Field Reproduction with Exterior Energy Constraint

Figure 1 illustrates a circular reproduction loudspeaker array. In order to achieve the reproduction and control to energy that radiates outwards the loudspeaker array simultaneously, the region outside the loudspeaker array should be taken into consideration, additional to the center region. These two regions are called dark zone and bright zone, respectively.

Figure 1 – A schematic of the sound field reproduction system with exterior energy constrain(red dots denote loudspeakers).

For the bright zone \( S_b \), which is corresponding to the so called ‘listening area’, reproduced sound field in this circular area should be the same to the desired sound field. On the other hand, sound field energy in the dark region should be as low as possible. In practice, the listening and dark zone can be sampled at discrete points, and sound pressure in these two zones can be expressed as vectors:

\[
P_b(\omega) = \left[ P(r_1^{(1)}; \omega) P(r_2^{(2)}; \omega) \ldots P(r_{M_b}^{(M_b)}; \omega) \right]^T
\]

\[
P_d(\omega) = \left[ P(r_1^{(1)}; \omega) P(r_2^{(2)}; \omega) \ldots P(r_{M_d}^{(M_d)}; \omega) \right]^T
\]

(7)

Combine what was introduced in Sec 2, the sound filed reproduction and the exterior sound field energy control turn to two optimization problems:

\[
w : \arg\min_{\alpha} \| P_{des} - G_s w \|_2^2
\]

\[
w : \arg\min_{\alpha} \| 0 - G_s w \|_2^2
\]

(8)

where \( G_s \) and \( G_d \) are transfer function matrix between loudspeaker and sample points of bright zone and dark zone. And \( w \) denotes the loudspeakers' weights vector.

3.1 Omnidirectional Loudspeaker

Loudspeaker weights can be solved as weighted combination of these two optimization problems and finally goes to find the minimum \( J \):

\[
J = (1-a) \| P_{des} - G_b w \|_2^2 + a \| 0 - G_d w \|_2^2
\]

\[
= w^H \left[ aG_b^H G_b + (1-a)G_d^H G_d \right] w
\]

\[
+ (1-a) \left( P_{des}^H - P_{des}^H G_d w - w^H G_d^H P_{des} \right)
\]

(9)

where parameter \( a \) is the weighting factor that determines the relative extent of less mean square error in bright area and less outwards energy. \( H \) denotes the conjugate transpose. This optimization is a convex problem and therefore \( J \) has global minimum and can be obtained using coordinated descend algorithm. As a result, the optimal solution is :
\[ w = \left[ a G_{d}^H G_{d} + (1-a) G_{b}^H G_{b} \right]^{-1} (1-a) G_{b}^H P_{\text{des}} \]  

(10)

### 3.2 Variable-Directivity Loudspeaker

As mentioned in Sec. 2.2, variable-directivity loudspeaker is a combination of monopole and dipole loudspeaker where amplitude and phase of these two loudspeakers can be manipulated independently. As a result, for every variable-directivity loudspeaker, there are two unknown weights to design. The solution for variable-directivity loudspeaker reproduction system is:

\[
\begin{bmatrix}
    u \\
    v
\end{bmatrix} = \left[ a \begin{bmatrix} G_{du} & G_{dv} \end{bmatrix}^H + (1-a) \begin{bmatrix} G_{bu} & G_{bv} \end{bmatrix}^H + \lambda \mathbf{I} \right]^{-1} (1-a) \begin{bmatrix} G_{bu} & G_{bv} \end{bmatrix}^H P_{\text{des}}
\]  

(11)

where \( u \) and \( v \) are two \( L \times 1 \) vectors for monopole weights and dipole weights, respectively. \( G_{m} \) is the acoustic transfer function matrix from the monopole to bright zone control points. \( G_{d} \) is the acoustic transfer function from the dipole to bright zone control points. Similarly, \( G_{md} \) and \( G_{bd} \) are transfer function matrix from monopole and dipole to dark zone control points. And \( \lambda \) is the regularization parameter to limit the total energy of loudspeakers.

### 4. Simulation and Subjective Listening Test

#### 4.1 Objective Evaluation

We now present simulations of sound field reproduction using pressure matching with variable loudspeakers in free field conditions. We use a circular array of 25 loudspeakers which are uniformly arranged at a radius 1 m in the horizontal plane. The reproduction will be implemented in the central area of the array. For comparison, omnidirectional loudspeaker array simulation will also be investigated.

The desired sound field is radiated by a point source placed on the x-axis at \([10,0,0]\). Due to the uniformly arrangement of loudspeakers, the error performance was found to be similar for point source at different angles. Taking the size of the head of a listener, the radius of the bright zone \( r_{b} \) is selected to be 0.25 m. The bright and dark zones are sampled with discrete points which distribute uniformly with distance 6 cm. In order to achieve outwards energy radiation limitation, the dark zone should be the half space out of the array which is not feasible to sample. In this paper, the dark zone is only sampled at those points \( 2.5 \text{m} \leq r_{d} \leq 3 \text{m} \). The corresponding Nyquist frequency is more than 2800 Hz which covers our interest frequency range (up to 900 Hz) well.

#### 4.1.1 Reproduction error

To evaluate the reproduction performance of the system in the listening area, the relative reproduction error of every point in listening area is defined as mean energy error between primary sound field \( p_{\text{pre}} \) stirred by the point source and reproduced sound field \( p_{\text{re}} \). The final relative error is expressed in dB, and the exterior sound field relative energy (ESL) is also defined in Eq. (13), where \( p_{\text{pre}} \) is primary sound pressure at the point \((0,0)\).

\[ E(dB) = 10 \log_{10} \frac{\| p_{\text{re}} - p_{\text{pre}} \|_{2}^2}{\| p_{\text{pre}} \|_{2}^2} \]  

(12)

\[ ESL = 10 \log_{10} \frac{\| p_{\text{re}} \|_{2}^2}{\| p_{\text{pre}} \|_{2}^2} \]  

(13)
4.1.2 Simulation Verification

As mentioned before, a circular array with 25 loudspeakers on the horizontal plane is used as the reproduction system. The minimum distance of adjacent loudspeakers is 0.26 m. The corresponding Nyquist frequency is 660 Hz. Figure 1 shows the simulation result about reproduction error and relative exterior sound field energy for variable-directivity loudspeaker array, reproduced sound fields on the horizontal plane, the energy and directivity for each variable-directivity loudspeaker and relative sound field energy of at 200 Hz. The weighting parameter in Eq. 11 is set to be $a=0.5$.

![Simulation](image1.png)

Figure 2 – Simulation for variable-directivity loudspeaker array at 200 Hz. (a) Reproduction error ($r<1$ m) and relative exterior sound field energy ($r>1$ m), the red line denotes variable-directivity loudspeaker array and the blue line denotes monopole loudspeaker array. (b) Reproduced sound field (real part) (c) Directivity and energy of variable-directivity loudspeakers. (d) Exterior and interior sound field relative energy.

As what can be observed in figure 2 (a), compared with monopole array, the variable-directivity loudspeaker array achieves both sound field reproduction in the bright zone and exterior sound field energy control in the dark zone at 200 Hz. In the bright zone, the reproduction error is below -30 dB but the number for monopole one is around -8 dB. And when we move on the exterior sound field relative energy, the value of variable one goes down to -30 dB in 1 m away from the array but the monopole one decreases to around -20 dB at 1.5 m and goes down continue but slowly. This means, for the variable-directivity sound field reproduction system, the reproduction performance in the bright zone of the area will be less affected by reflections from the walls (if walls are in sampled dark region) when this array is placed in a ‘listening room’. The energy of the sound field is constrained to be in the array rather than propagates outwards (refer to figure 2(d)).
Figure 3 shows the related reproduction performance of the array at 900 Hz which is higher than the Nyquist frequency (660 Hz) of the array.

Figure 3 – Simulation for variable-directivity loudspeaker array at 900 Hz. (a) Reproduction error (r<1 m) and relative exterior sound field energy (r>1 m), the red line denotes variable-directivity loudspeaker array and the blue line denotes monopole loudspeaker one. (b) Reproduced sound field (real part). (c) Directivity and energy of variable-directivity loudspeaker. (d) Exterior and interior sound field relative energy.

Similar to the result at 200 Hz, variable-directivity loudspeaker array still shows better performance than the monopole one. Its reproduction error is less than -20 dB in the bright zone and exterior sound field relative energy is -30 dB at 2.5 m, but the corresponding number for monopole one is -7 dB and -22 dB. But compared with the performance at 200 Hz, this variable-directivity loudspeaker system's performance at 900 Hz faded. Taking a careful observation to the energy distribution and the directivity of loudspeakers, at high frequency, this algorithm fails to design a proper loudspeaker directivity and make a good energy allocation.

4.2 Subjective Evaluation

Reproduction system is often used for creating a virtual environment, so people’s subjective perception to the reproduced sound field is the most important thing. And for a real system, a larger effective region (sweet point) means more people can share the best reproduction result. Taking this into consideration, a subjective experiment was conducted to find the border of the effective region of reproduced sound field.

In this subjective experiment, 12 virtual variable-directivity loudspeakers are combined together
following the loudspeaker setup in the VR laboratory at the Institute of Technical Acoustic (ITA), RWTH-Aachen. This virtual array is placed in an anechoic chamber (weighting parameter is set to be a=0) simulated by RAVEN [11]. Sound fields stirred by 5 primary point sources on different directions are sampled respectively with 72 microphones which uniformly distribute in a 0.25 m spherical space in the center of loudspeaker array. Primary sources are 7 m away from the center of the loudspeaker array. The HRTF (head related transfer function) of a dummy head was used to get the binaural signals. Directions of primary sources and the dummy head are shown in figure 4. In this reproduction, the up-limit frequency of reproduction is 900 Hz. Twenty-eight people (8 people working at ITA, Germany and 20 students at NWPU, China) participated in this listening experiment. In this subjective experiment, the dummy head moved along the axis towards ‘front virtual source’, people were asked to try to find the closest position to the coordinate origin where they hear the noticeable difference compared with sound signal in the array center. In order to improve the speed of this process, the Bayesian adaptive psychometric method was used[12].

![Diagram showing directions of primary sources and the dummy head](image)

Figure 4 –Directions of primary sources and the dummy head used in this paper.

As shown in figure 5 (a), the sweet point border varies with the change of the direction of primary source. Taking the symmetrical setup of the array into consideration(the physical reproduction performance should be same), this means that on these five directions people are most sensitive to the change of the sound field which comes from the front of them, by contrast, they are not so much sensitive to the change of the sound field from the left. This result provides us with some indication about the sample of the primary sound field and the design of loudspeaker array. Uniformly distributed sample points are not a bad option, but it seems that there should be more sample points when the primary sound field comes front of people or place more loudspeakers in the front of people compared with other directions

![Box plots showing sweet point borders for different directions](image)

Figure 5 –Sweet point border: (a) Different primary sound directions. (b) Primary source on the front reproduced by monopole array and variable-directivity loudspeaker array.

The figure 5(b) shows the result of sweet point borders with the same primary source direction but different loudspeakers. As what we expected, variable-directivity loudspeaker shows better performance than monopole.
5. CONCLUSIONS
In this paper, the performance of variable-directivity loudspeakers array using pressure matching method acoustic contrast control has been checked. Compared with the monopole loudspeaker array, variable-directivity loudspeaker system can reproduce the target sound field and constrain the energy radiating outwards simultaneously, especially when the frequency is lower than Nyquist frequency. At frequency higher than the Nyquist frequency, this method with variable loudspeakers still shows better performance than reproduction with omnidirectional ones. On the other hand, a subjective experiment to find the border of sweet point is conducted. The result shows that people's sensitivity to sound fields from the different directions is not constant. This provides us with some indication on decision of sample points distribution and loudspeaker array design.

The result in this paper is based on the simulation and a subjective experiment, and the practical performance of this kind of loudspeaker array in a virtual or a real listening room has not been investigated. In the future work, the performance evaluation, both subjective and objective ones, in virtual rooms with different reverberation time and its practical performance in the listening room will be discussed.

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