Modeling and characterization challenges of multiple dynamics materials
(also known as metamaterials)

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Abstract
Passive acoustical materials exhibiting multiple dynamic behaviors (also known as acoustical metamaterials) bring new challenges, related either to the characterization of their elastic & acoustic intrinsic parameters or the development of models.
We present a brief summary of the physical phenomena used to enhance the absorption or insulation performances of passive materials. We show examples of challenges related to the characterizations of these materials. In particular, for acoustic characterizations, we show how the dynamic mass density & dynamic bulk modulus (or wavenumber & characteristic impedance) are impacted by the aforementioned physical phenomena. In a brief second part we discuss some modeling & characterization challenges related to these materials.
Keywords: materials, characterization, modeling

1 INTRODUCTION
The dynamic mass density \(\rho\) and dynamic bulk modulus \(K\) of materials are quantities describing the visco-inertial and thermal dissipation mechanisms, respectively, in a porous medium (see fig. 1). These quantities can be measured using an impedance tube (see e.g. \([1]\)). The examples reported in section 2 show the dynamic mass densities and dynamic bulk moduli for various passive acoustical materials exhibiting multiple dynamic behaviors (also known as acoustical metamaterials even if most of them are not made of multiple materials).

Note that this document focuses on acoustic excitation in air saturated porous materials or structures. Thus, we will not discuss about materials or structures with multiple mechanical behaviors nor active materials or structures. We will not discuss the “diode” effect which requires, at least, one flow, nor “black holes” (i.e. retardant structures) as their behaviors are not symmetrical while we assumed here homogeneous behaviors (at least at the scale of our impedance tubes).

The time convention used to obtained \(\rho\) and \(K\) is \(e^{+j\omega t}\) (this explains why the imaginary part of \(\rho\) is usually negative).

Finally, we can’t be exhaustive on references so bare with us if some terrific works you know of are not reported here and don’t hesitate to submit them by email (see address above).

2 PASSIVE ACOUSTICAL MATERIALS EXHIBITING MULTIPLE DYNAMICS
We can classify the recent developments on acoustic materials in a few number of categories : resonant and diffusive (even if diffusive can be considered as highly damped resonant systems). Resonant systems such as acoustic resonators (Helmholtz or quarter-wavelength) have localized effects in frequency while diffusive ones based on scattering or pressure diffusion effects have much broader frequency effects.

In the next pages, we briefly show (mainly due to the page limitation) some examples of these materials together with their influences on the acoustic properties (dynamic mass densities and dynamic bulk moduli).
Figure 1. Left: scheme of the 2 dynamic behaviors for a “usual” rigid-frame porous material based on Kirchhoff [-Langevin] theory [2, 3]. Right: mass density and bulk modulus normalized by the mass density of air at rest and the static pressure during measurements for a “usual” rigid-frame porous material.

Note that the periodicity of the material adds interferences but is not necessary to take advantage of the other physical phenomena mentioned above.

2.1 Materials with multiple scales of porosity
This first class is characterized by the addition of a pressure diffusion effects (see fig. 2) or \( n - 1 \) potential diffusion effect for a material with \( n \) scales of porosity.
Some examples of works involving such materials are discussed in references [4, 5, 6, 7, 8, 10, 9].

2.2 Materials with solid inclusions (scatterers)
These materials are characterized by the addition of a multiple scattering effect (which can be modeled as a diffusion effect). See fig. 3 as an example.
Such materials with solid inclusions of various sizes has been applied in the lower part of the audible frequency range [11] and at higher frequencies [12, 14] (upper audible frequency range and ultrasonic frequency range).

2.3 Materials with acoustic resonators
These materials are characterized by the addition of acoustic resonances mostly resulting from embedded Helmholtz resonators [15, 16, 17, 18, 19, 20] inside a host material (see example in fig. 4) but also from quarter-wavelength resonators [21].
Figure 2. Left: picture of a material with 2 scales of porosity as used in [6]. Right: normalized mass density and bulk modulus for such a material.

Figure 3. Left: scheme of a material A with scatterers made of material B as depicted in [12]. Right: normalized mass density and bulk modulus for such a material.
2.4 Materials with soft membranes
These materials include elastic resonances inside their pores. While some studies exist in active sound control, only a few works have been published in passive sound control [22, 23, 24] (see example at fig. 5).
A commercial product was manufactured by Dow Chemical in the 1990’s and the early 2000’s : the “Quash”. The acoustic absorption of the material together with is impervious properties to liquids was the main advantage of this material for which the manufacturing might start again.

3 MATERIAL MODELING
3.1 Some issues
One modeling issue lies in the fact that analytical solutions are yet limited to simple problems (usually involving simple geometries and simple excitations). Other cases require “heavy” computations. Another key point is that dissipation should be accounted for to accurately predict the behaviors of the materials presented above. This point can be seen in the evolution, over time, of the works, on any of the materials mentioned above. Finally, the vibration of the skeletons is partially or not accounted for in the models today while essential for applications to transmission.

3.2 Some developments
In addition to the ongoing works on specific classes of materials, Lafarge’s non local theory seems promising to offer a general solution to the rigid-skeleton modeling issue. To sum up, Lafarge’s theory can be seen as an extension of Zwicker & Kosten work to space dispersion : $\rho(\omega) \rightarrow \rho(\omega,k)$ and $K(\omega) \rightarrow K(\omega,k)$ where $k$ is the wavenumber of the acoustic wave inside the material.
Concerning the skeleton motion and its coupling with the fluid phase, examples of works in progress are [28,
Figure 5. Left: picture of a material with membranes on pore connections as used in [24]. Right: normalized mass density and bulk modulus for such a material

Note that “usual” porous materials with elastic skeletons exhibit 3 dynamic behaviors, the frame resonances adding 1 more dynamic to the system as seen in fig. 6

4 CHARACTERIZATION

4.1 The major issue

When testing multiple dynamics materials, it appears the most important issue is related to the size of the samples: too big or too small. Materials with multiple scales of porosity, scatterers or inner resonators are usually too large for most impedance tubes. These usually “hand-made” materials are however too small for diffuse sound field measurements usually (or at least in a too few number to be tested in diffuse field). Some nano-scale materials are too small for any standard measurements [32].

4.2 Some developments

One solution to overcome the major issue reported above is to use dedicated tubes or horn-style extensions [33, 34, 35].

As it can be seen in the previous pages, for each material, a dynamic mass density and a dynamic bulk modulus can be obtained. For most of the material classes, these dynamic quantities have been measured using dedicated tubes. Note that measurements have been done by the authors for materials with solid inclusions but it was “clearer” here to show simulations (which can be compared to the theoretical work by D. Lafarge).

Other developments, in particular for multiple-scale porous materials, involve the identification of the different
Figure 6. $\rho$ & $K$ as functions of the frequency for a porous material (a foam) with a visco-elastic skeleton. A third dynamic is added to the visco-inertial & thermal ones due to the skeleton mechanical resonances.

open-pore networks [36, 6, 7]. This point is of great importance as printed materials as well as most natural fibers & some minerals exhibits an internal scale of porosity different from the “acoustic scale” (see e.g. the PhD works by Rodolfo Venegas[?], Gaelle Benoit[?] or Philippe Glé[39] dedicated to this topic).

REFERENCES


[3] In the original work by G. Kirchhoff, the air is supposed to behave as an ideal gas and the bulk viscosity is neglected. P. Langevin has later extended Kirchhoff’s work by considering the complete Navier–Stokes–Fourier linear equations.


[18] Lafarge D., Nemati N., Nonlocal Maxwellian theory of sound propagation in fluid-saturated rigid-framed porous media, Wave Motion 50 (6), 2013, pp. 1016-1035, 10.1016/j.wavemoti.2013.04.007


