Design of a constant beamwidth beamformer for the parametric array loudspeaker

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ABSTRACT
The parametric array loudspeaker (PAL) is an unconventional type of directional sound source that transmits inaudible ultrasonic frequencies in air to generate the audible difference frequency based on the parametric array effect. The convolution model describes the beampattern of the PAL by convolving the product beampattern with the Westervelt beampattern. As a result, the beamwidth of the PAL becomes narrower when the difference frequency increases. A change of timbre is therefore perceived when people stand at an off-axis angle of the PAL. Hence, a constant beamwidth beamformer is required. This paper works on the beamformer design to result in the product beampattern that becomes broader when the difference frequency increases, in order for the Westervelt beampattern to be compensated. A constant half-power-beamwidth (HPBW) is successfully achieved for the typical frequency range of the PAL.

Keywords: Parametric array loudspeaker, Beamformer design, Constant beamwidth

1. INTRODUCTION
The principle of the PAL is explained by the parametric array effect (1, 2). When two ultrasonic frequencies are transmitted together, the acoustic energy of the ultrasonic frequencies is transferred into new frequencies by the nonlinearity of air. Among those new frequencies, the difference frequency can be made audible to human beings (3, 4). The difference frequency inherits the narrow beam of ultrasonic frequencies. Therefore, the PAL is known to be one of the most effective means to create a directional sound beam (5, 6). A typical design of the PAL consists of a driver circuit and an array of ultrasonic transducers. One of the two ultrasonic frequencies is set as the carrier frequency and the other is given by the sideband frequency.

PALS can be readily deployed in applications of sound field control, whereby tailored enhancement and attenuation of acoustic components are required (7, 8). With phased array technique, PALS possess controllable beampatterns. They are also known as steerable PALS (9, 10). There have been several attempts to develop sophisticated array signal processing methods for the steerable PAL (11, 12). However, the theoretical works are far from practical implementations, because of a lack of effective directivity models. Since 2015, the convolution model has been established, providing an accurate directivity model for the PAL (13). The convolution model has opened an opportunity in the beampattern design of the PAL. The desire for a constant beamwidth originates from the timbre variation perceived by people standing at an off-axis angle of the PAL. Therefore, this paper presents a constant HPBW beamformer specially designed for the PAL consisting of 100 ultrasonic transducers.

2. CONVOLUTION MODEL OF PALS
The well-known Westervelt’s directivity $D_W(\theta)$, or called the Westervelt beampattern in this paper, is expressed as

$$D_W(\theta) = \frac{1}{\sqrt{1 + \frac{\omega_d^2 \tan^4 \theta}{4c_0^2 (\alpha_1 + \alpha_2)^2}}} \quad (1)$$

where $\omega_d$ is the difference frequency; $c_0$ is the speed of sound; $\alpha_1$ and $\alpha_2$ are the absorption coefficients of the two ultrasonic frequencies. Sometimes, $\alpha_1 + \alpha_2 = 2\alpha_0$ is approximated, where
\( \alpha_0 \) is the absorption coefficient of the center frequency. The Westervelt beamwidth gets narrower when \( \omega_d \) increases or \( \alpha_0 \) decreases. Figure 1 shows the Westervelt beampattern and HPBW with respect to the difference frequency when the carrier frequency is 40 kHz.

![Westervelt Beampattern](image)

**Figure 1 – Westervelt beampattern and beamwidth for a typical PAL**

The convolution model explains that the difference frequency beampattern is estimated by the convolution between the Westervelt beampattern and the product beampattern of the two ultrasonic frequencies, i.e.

\[
D_d(\theta) = [D_1(\theta) \times D_2(\theta)] \ast W(\theta),
\]

(2)

where \( D_1(\theta) \) and \( D_2(\theta) \) are the beampatterns of the ultrasonic transducer array at the two ultrasonic frequencies; and \( \ast \) denotes the convolution operator. Considering the beampatterns of individual ultrasonic transducers, equation (2) is further extended to

\[
D_d(\theta) = [A_1(\theta) \times D_1(\theta) \times A_2(\theta) \times D_2(\theta)] \ast W(\theta),
\]

(3)

where \( A_1(\theta) \) and \( A_2(\theta) \) are the beampatterns of the ultrasonic transducers at the two ultrasonic frequencies. The ultrasonic transducers can be modeled as piston sources, whereby a larger radius leads to a narrower beampattern.

### 3. CONSTANT BEAMWIDTH BEAMFORMER DESIGN

Table 1 lists the parameters of the PAL considered in the constant HPBW beamformer design.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter of ultrasonic transducers</td>
<td>0.08 meter</td>
</tr>
<tr>
<td>Spacing between ultrasonic transducers</td>
<td>0.10 meter</td>
</tr>
<tr>
<td>Number of ultrasonic transducers</td>
<td>100</td>
</tr>
<tr>
<td>Carrier frequency</td>
<td>40000 Hz</td>
</tr>
</tbody>
</table>

![Table 1 – Parameters in the constant HPBW beamformer design](image)

In order to compensate for the Westervelt beampattern, the constant HPBW beamformer has to also include an adjustable parameter with a large dynamic range. Therefore, the Dolph-Chebyshev weights are adopted. The product beampattern of the two ultrasonic frequencies can be adjusted largely by the sidelobe attenuation.

Firstly, the sidelobe attenuation of the carrier frequency is fixed at 6 dB. The lower and upper single sideband modulation methods are considered. They have only one sideband. The sidelobe attenuation of the sideband frequency varies from 6 dB to 50 dB. The contour plot of the resultant HPBW of the difference frequency based on the convolution model is shown in Figure 2. With exception for the line representing 25 degrees of HPBW, no other line is able to cover the typical frequency range of the PAL.
which is from 500 Hz to 8000 Hz. Moreover, the difference between the contour plots of the lower and upper single sideband modulation methods is trivial. This suggests that 25 degrees of HPBW is also applicable to the double sideband modulation method.

Figure 2 – Difference frequency HPBW with respect to the sidelobe attenuation of the sideband frequency

Secondly, the sidelobe attenuations resulting in 25 degrees of HPBW are extracted for difference frequencies. After which, they are used to obtain the constant beamwidth beampatterns. The results are almost the same for the single and double sideband modulation methods. To maintain presentation clarity, only the result of the double sideband modulation method is shown in Figure 3. To illustrate the constant beamwidth, the magnitude above -3dB are colored with yellow in the subplot below.

Figure 3 – Constant beamwidth beampattern of the double sideband modulation method

Furthermore, the second harmonic beampattern is obtained by the convolution model, whereby the Westervelt beampattern is computed at the second harmonic frequency and beampatterns of the two sideband frequencies in the double sideband modulation method are multiplied. The second harmonic distortion is calculated by the ratio of the difference frequency beampattern and the second harmonic beampattern. The results are shown in Figure 4. The HPBW of the second harmonic beampattern is narrower than that of the difference frequency beampattern. This results in a rapid decay of the second harmonic distortion at off-axis angles of the PAL, especially for relatively low difference frequencies.
4. CONCLUSIONS

In this paper, a constant HPBW beamformer is designed for the PAL, which has been constructed using 100 ultrasonic transducers. A constant HPBW is achieved from 500 Hz to 8000 Hz, when the spacing between the ultrasonic transducers is 0.01 meter. The Dolph-Chebyshev weights are adopted. An initial testing shows that only 25 degrees of HPBW can be designed to cover the typical frequency range of the PAL. It has been observed that because of the Westervelt beampattern, several strong constraints have been incurred in the constant HPBW beamformer design. Therefore, a systematic approach is yet to develop. Furthermore, when the double sideband modulation method is applied, the constant HPBW beamformer is achieved for the first time. Since the second harmonic beampattern is narrower, it is advantageous to listen to the PAL on a slightly off-axis angle for reduced second harmonic distortion.

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REFERENCES

