Can I trust my TPA results?

Frank JÜRGENS¹; Christian NETTELBECK; Philipp SELLERBECK
HEAD acoustics GmbH, Germany (Consulting NVH)

ABSTRACT
In the automotive development process, Transfer Path Analysis (TPA) is a widely known tool for troubleshooting purposes of existing vehicles or for the prediction of vehicle interior noise based on test bench or CAE data. There are many different approaches, each having its pros and cons. Either determining interface forces while the source is disconnected or equivalent forces for in-situ condition; measured in the vehicle or at the test bench; calculated in frequency domain or in time domain. As different as the approaches are in detail, one question they all have in common: How reliable are their results?

Within this paper, the need of a validation and refinement process of the TPA model – independent of the specific TPA method - is discussed and illustrated by typical application examples. New tools like the Mosaic View for the visualization of the crosstalk and useful procedures like the Root Cause Analysis and the Model Robustness Check are exemplified. Furthermore, these technologies are evaluated to show their potential for the identification of errors in the TPA model and how to deal with them in order to increase the liability.

Keywords: Transfer Path Analysis, Robustness, Reliability

1. INTRODUCTION
The Transfer Path Analysis can be a very helpful tool within the complete product development process especially with respect to reaching defined sound quality targets. Starting with the early concept and design phase the knowledge of the source-transfer-receiver properties may effectively help to avoid later unwanted issues. But also during the following prototype phases until SOP the detailed knowledge of the excitation and transfer behavior is very helpful to support the optimization process.

The initial situation for all typical TPA projects is that the object to be examined is a coupled system with at least one source and the goal of the TPA is to separately quantify the contributions of noise and vibrations for each path and to additionally characterize the source. When applying experimental TPA to deal with this task, besides the operational data, the measurement of frequency-response-functions (FRF) is necessary. This leads unfortunately to a rather large effort and time consuming procedure. Once this effort has been made, the resulting TPA model can be used to analyze and determine relevant transfer paths with respect to all possible noise patterns in the time and frequency domain. Regardless of the analysis or a particular validation strategy, you will always achieve results for every noise pattern that you want to examine. But in order to be able to trust these results, and in particular to know their limits, the measured data and the calculated models must be extensively analyzed and validated. This includes the verification of the physical meaningfulness of the data used.

There is no general checklist which ensures correct results in every case and for all regarded systems. But the analysis of some key points in a structured procedure will provide valuable support with respect to the correct interpretation of the TPA results. This of course requires experience, but modern tools (like the Mosaic View) and fundamental procedures (like Root-Cause-Analysis and Robustness Check) support the engineer to gain a reliable TPA model within a reasonable timeframe.

2. KEY POINTS AND TOOLS FOR TPA MODEL REFINEMENT

2.1 Requirements on measured data

The basis for good results are good measurements of course. Especially the reproducibility is a big challenge. On the one hand it can be difficult to ensure the repeatability for the operating condition,
especially in transient cases (load changes, tip-in/tip-out, etc.). Effects of operating temperature (on oil or grease), control devices conditions (like ECU/TCU), regeneration process (like in diesel particulate filter) or battery SOC in EVs can have an immense impact on repeatability. On the other hand the FRF measurements are not trivial either. The point of impact or attachment point of shaker should be chosen carefully in order to represent the closest possible and most suitable interface point. The positioning of indicator sensors and selection of a suitable sensitivity depending on the position requires some experience. The transfer functions and coherences should be analyzed in the full indicator matrix (at least for random points) in order to ensure that there is enough energy in the complete system. Furthermore the signal-to-noise ratio should be analyzed, in order to gain information about the frequency limits of the later synthesis results. To handle all these variables is essential for reliable results.

2.2 Mosaic View

The Mosaic View is a tool for the analysis of transfer functions of the complete system in one diagram. It shows a cut through all FRFs at one frequency point, while the amplitude (or phase) of each path is represented by a color. It enables to visualize and detect the coupling of transfer paths, the identification of sub-structures and their interaction along the frequency axis. Typically the coupling decreases with rising frequency and each path becomes more and more independent. Furthermore the classification of resonances into local or global resonances can be done very easily. Also noisy indicator sensors can be identified very fast, in order to be eliminated from the matrix before the inversion. All in all it is a very useful tool to first understand the observed system and then to find the best matrix setup.

2.3 Matrix Condition Number

The Matrix Condition Number (MCN) is one of the most mysterious things in the world of matrix inverse TPA techniques. It describes the influence of errors in measured data on the result of the matrix inversion, if everything is calculated correctly. Everybody has heard about it and knows “the lower the number the better”. But there is no fixed OK/not OK limit and depending on the kind of data, the size of matrix and use of regularization techniques it is very easy to get lost in a blind optimization loop for gaining lower condition numbers. However, the MCN should be evaluated in context with the data and the subsequent synthesis result instead of interpreting it independently. By doing this it really helps to identify weak points in the matrix setup and optimize the complete TPA model correctly. Nevertheless it is just one indicator of the quality of the results, but not the only one.

2.4 Mean error vs. mean magnitude

Another useful indicator for evaluation of the result quality is the analysis of the mean error of synthesis result compared to the mean magnitude of excitation. The mean error is defined by the difference between the measured reference (like interior noise or target vibration) and the synthesis result (sum of all transfer paths). It can be calculated either by using just the difference of the FFT spectra or by calculating the transfer function between reference and synthesis. The mean magnitude is calculated by the fraction of the summed level of all operational excitation signals (required for the synthesis) to the number of signals. If now both values are analyzed together, it becomes clear in which frequency range the error occurs and if the error occurs at low or high magnitudes, i.e. the error relevance. Just in case the biggest error occurs at low magnitudes it can be an indicator for either a low relevance of the error or for a missing transfer path. Conversely, if the biggest error occurs at high magnitudes it can be an indicator for errors in the TPA model. Hence this comparison is helpful for judgement of overall synthesis quality.

2.5 Root-Cause-Analysis

The Root-Cause-Analysis is essential for system understanding. In principle it shows the source-transfer and receiver properties in a clearly arranged summary: Starting from the active side acceleration in case of a mounted transfer path, via the effectiveness of the mount to the passive side forces and local stiffness (a/F) through the receiver sensitivity (p/F) to the noise or vibration share of the single transfer path. In that way the weak point of the system or the errors in the TPA model become clearly visible. The combination of the Root-Cause-Analysis and a TPA database – which is reflected by scatter bands of comparable systems (e.g., mid-size passenger vehicle) – makes it very easy to pinpoint the weak point of the system. Furthermore it can be used for benchmarking of different products or even to visualize the load dependency of a transfer path.
2.6 Approaches for robust model design

One of the most difficult steps in model refinement is to gain a robust TPA model. Once a model setup is favored for a certain condition it needs to be proved for other conditions. In case of vehicles often part load and full load run-ups are measured. Consequently both operational data should be used for receiver synthesis by using the same matrix setup (same paths, regularization, etc.). Besides the comparison of the resulting synthesis to the measured reference, also the Root-Cause-Analysis should be done in order to check the results for physical meaningfulness (like load dependency, force levels, etc.). If there is only one test condition, repetition measurements should be used to check if the results are comparable to the initial one. In that case the responsible transfer paths should be the same. The comparison of the synthesis and path contributions of different models for one operating condition or even the comparison of the results of one model for different operating conditions can help to find a robust model setup. Nevertheless this is one of the steps with the highest effort in TPA model refinement.

3. Exemplified model refinement process

3.1 Case 1: How to analyze the TPA model?

First example is an insight into an investigation of a passenger vehicle (3-cylinder transverse engine, manual transmission) regarding powertrain noise (several patterns between 30 Hz to 2 kHz). The vehicle was equipped to determine the paths of the three powertrain mounts and the transfer paths of front wheel suspension (six bushings in total) as well as the airborne paths from the engine compartment, the intake and the exhaust orifice. After the measurements were done (or even partly during the measurements), the data was checked regarding the reciprocity and coherences as well as the signal to noise ratio. The results showed that the focused frequency range can be analyzed faithfully by using this measurement data.

In a first approach all transfer paths were collected in a full matrix (separate for airborne and structure borne). The synthesis result shows a good agreement to the measured reference in a broad frequency range for several noise patterns (see Figure 1 left). Also the comparison of several psychoacoustic analyses (like the modulation spectrum vs. engine speed) shows that the synthesis model looks like a good representation of the vehicle for this operating condition.

![Figure 1 - Comparison of Reference and Synthesis as well as noise contribution for whine](image)

Nevertheless the result needs to be checked in order to gain a robust and satisfying result. In a first step the responsible transfer paths for the specific noise pattern need to be identified. This should be done not only based on analysis of levels, also the phase information needs to be considered. By the analysis of the transfer path influence (related to the total noise) this can be done in a well arranged way. The right diagram in Figure 1 shows this kind of analysis for the whine noise of the investigated vehicle. It is clearly visible that the “Path 8” has the strongest influence on the whine. This can be done for all relevant noise patterns in a similar way.

In a next step the responsible transfer paths have to be checked by the Root-Cause-Analysis. In Figure 2 it can be seen exemplified for “Path 8” (black curve is described). Focusing on the higher frequency range the whine order becomes clearly visible in the active side acceleration (indicator for source property). The mount isolation is in a typical range for a powertrain mount. Around approx. 700 Hz the isolation decreases while the local apparent mass increases at the same frequency range. It shows a constant offset to the typical slope (40 dB per decade) above 700 Hz. Nevertheless it is no weak point of the mount, it just indicates that the excitation level is high. In the opposite case (decrease
of apparent mass) the lower isolation values would be a clear indicator for ineffective interaction of a rubber mount and the supporting structure. The resulting passive side force (at the attachment point) clearly shows the strong amplitude for the whine order around 1 kHz. Following the transfer path to the acoustic sensitivity transfer function, the high values around 400 Hz and 1 kHz are eye-catching. Especially in the comparison to the competitors’ scatter band it becomes obvious that the sensitivity is at the upper limit (and partly above it). In consequence the starting point for improvement of the transfer path could be the excitation or the sensitivity. Following this procedure for the relevant transfer paths helps to understand the vehicle as a coupled system and highlights potential errors of the TPA model.

As already mentioned the Matrix Condition Number is one of the indicators for result quality. The MCN of the current example is shown in black color in the Figure 3 (right diagram). The matrix condition shows unacceptable high values and peaks (based on experience of comparable applications), especially in mid frequency range \( f < 2 \text{ kHz} \). Seeing this, the trust in the results is reduced and a solution for lowering the MCN needs to be found. Consequently the results are calculated again by use of the same matrix setup (size, transfer functions, windows, etc.) with the only difference that this time regularization is used to improve the condition (just one of several possibilities). Regularization can improve the results but also removes information especially at low signal levels. Accordingly there is always a trade-off between ignorance of information for gaining higher numerical stability and to be able to reflect the investigated noise patterns. So experience is required for finding suitable parameters.

The resulting MCN is shown in red and is much lower and more constant than the initial one. Consequently the results (forces and noise shares) should be dramatically different. Comparing now the force of “Path 8” (strongest influence on whine noise pattern) for both matrix inversions shows, that there is almost no difference except the lower noise floor (see Figure 3 left and middle diagram, both have the same scaling).

When comparing now all forces for both inversion results, only very few transfer paths with different forces can be found. The impact on the total synthesis result is almost unnoticeable except the lower background noise. However, this does not allow the conclusion that this is always the case for matrix inverse TPA. Each other system can behave differently.

A different approach instead of using regularization is the change of the matrix setup from full matrix to sub-matrices. In order to find suitable sub-matrices, the Mosaic View helps for the identification. In the current example there can be identified several sub-systems for the frequency
range of the whine noise. A part of the matrix is shown in Figure 4 at the right side. The diagram shows five separate sub-matrices (indicated from A to E) within the full matrix representation. This setup is now used for the calculation of the operational forces by the prior separate inversion of each sub-system. The resulting path contributions are shown in Figure 4 at the left side, compared to the previous matrix setups. Setup 1 is the full matrix approach without regularization, setup 2 is the full matrix with use of regularization and setup 3 is the sub-matrix result (partwise matrices). The Path 8 is still the strongest contributor in all models what indicates a stable result. If the main contributor would change, the results would be judged as less reliable. In that case the reason for differences needs to be identified. This can be done by using again the Root-Cause-Analysis and the Mosaic View.

The path contributions of setup 3 further show that the less relevant paths have stronger variances compared to both full matrix setups. Especially Path 1, 9 and 10 show significant stronger noise shares compared to model 1 and model 2. This affects the total synthesis for the third model which is reflected in an overestimation of the whine noise compared to the measured reference.

![Figure 4 - Path contribution for different TPA models (left), Mosaic View for high frequency range (right)](image)

Having a closer look to the mosaic view the reason for the overestimation can be found. The strong crosstalk of matrix C and matrix E becomes visible in the non-diagonal area of the diagram. In consequence there is a strong interaction between these paths which has to be considered in the TPA model. The consideration of this crosstalk (combine sub-matrix C and E) within the matrix setup now leads to similar path contributions and similar forces for the responsible transfer paths in the frequency range of the whine noise (not shown here).

To return briefly to the Matrix Condition Number, the matrix setup 3 (sub-matrices) has the lowest MCN values but overestimated forces and path contributions. Once there is every time only one matrix condition number characterizing the underlying matrix, the MCNs of the sub-matrices now can also be used to identify ill-conditioned parts inside the full matrix (e.g.; for setup 1). The setup 1 has highest condition numbers and setup 2 has low condition numbers. But both show a suitable match of the total synthesis to the measured reference and reasonable forces. As mentioned in section 2, the condition number is just one indicator and without the relation to the synthesis quality and plausibility of the results it is worthless. There is always a trade-off between low condition numbers, a good match of the synthesis to the reference as well as the plausibility of the source-transfer-receiver properties. Furthermore, experience helps to accelerate the setup and analysis process, finding the suitable regularization settings and the judgment of the results.

At the end, the final TPA model needs to be verified in order to ensure its robustness. This can be done by using “unknown” data, which can be operational repetition measurements or even different operating conditions. In case of different operating conditions, the Root-Cause-Analysis again helps in the validation process. Different operating conditions should be reflected by reasonably different forces and meaningful load dependent transfer functions, e.g. the mount isolation.

### 3.2 Case 2: What is right?

The second example is an insight into an investigation of a powertrain (4-cylinder transverse engine, automatic transmission), which was operated at a test bench and used for vehicle interior noise and vibration estimation. The focus was set on structure borne transfer in the frequency range of typical powertrain noise patterns from 50 Hz to 1 kHz. The powertrain was equipped to determine the transfer via the three powertrain mounts. After the measurements, the data was analyzed regarding the
reciprocity and coherences as well as the signal to noise ratio. According to that, the focused frequency range can be analyzed faithfully by using this measurement data.

In order to get a quick first overview of the results, a reference vibration point anywhere at the test bench was chosen as a temporary receiver point for the TPA synthesis. Also in this example the first model was set up with a full matrix. Again the matrix inversion was repeated by use of regularization techniques, because the Matrix Condition Number showed unexpected high values. In the comparison of both Condition Numbers in Figure 5 (right diagram) the extreme difference is clearly visible. In the left diagram the analysis of the “mean error vs. mean magnitude” is shown. The black (without regularization) and red (with regularization) curves show the error function between the measured acceleration at the receiver point and the result of the TPA Synthesis.

![Figure 5 - Comparison of two TPA models](image)

Comparing both curves, above approximately 900 Hz the difference between synthesis and measured acceleration becomes noticeable. In the same diagram, the blue curve shows the mean magnitude of all accelerometers used for the operational force synthesis. Now the combination of both information – in which frequency range errors occur and where the excitation is significant – leads to valuable conclusions. The biggest error occurs at small amplitudes while at high amplitudes there is almost no error.

As introduced before, the next step is to identify relevant paths regarding the noise patterns of interest. In this investigation, one focus was set on the mid frequency range around 320 Hz (engine roughness). The relative path contribution (influence of each path to the total synthesis) shows significant differences (see Figure 6 left diagram). While in the model without regularization (model 1) the y and z directions of all three mounts have strong contributions, the same paths are not relevant in the model with regularization (model 2). Furthermore, the analysis of the phase relation clearly shows an opposite phase of the y and z directions in model 1 (Figure 6 middle diagram). The right diagram from model 2 has the same scaling and shows the same result for the total vibration (black phasor), but now it results mainly from the x directions of the three mounts.

![Figure 6 - relative path contributions of 2 TPA models and corresponding vector diagrams](image)

Now the question is, what result is right? Again the root cause analysis can help to find answers. In this special case, the sensitivity transfer functions (relation of force to receiver vibration) show no unexpected high values or strong peaks. But there are very strong forces in case of model 1. The operational deflection shape analysis could not provide further hints about the origin of these forces. These high forces resulting from model 1 are very unusual in this frequency range (factor 15 times higher than for model 2). Additionally the forces are out of phase and the Matrix Condition Number shows very high values. As a conclusion one should better not trust this model.
Philippen (1, 2) reported about a similar phenomenon in the OTPA (operational transfer path analysis). He mentioned that an ill-conditioned matrix (highly correlated signals and almost linear dependent rows) amplifies small measurement errors leading to bigger errors in the transfer function estimation, like strongly overestimated transfer functions with opposite phase. In case of OTPA this minimizes the synthesis error but leads to wrong path contributions. In the current investigation the transfer functions were measured but the effect is the same without the regularization. In consequence the model 2 was selected as basis for further investigation.

3.3 Case 3: How to validate?

In a third example a further procedure for gaining a robust model is shown. The investigation was about powertrain noise patterns in a broad frequency range for different operating conditions of a vehicle with front transverse engine and automatic transmission. The investigated transfer paths were the powertrain mounts and the front suspension as well as the airborne paths from engine compartment, intake orifice and exhaust orifice. In this example the focus is on the idle condition.

Except the difficulty to ensure a good measurement repeatability of the idle state (same engine speed, similar temperature of oil and parts, electrical load) the idle condition is also difficult to reflect in a TPA model because the excitation energy (orders of the powertrain) is concentrated in narrow band peaks. If there is any drift of engine speed or temperature during operation (e.g.; within a measurement campaign) the noise patterns can shift slightly in frequency range and strength. This leads to the effect that peaky transfer functions like sensitivities (p/F or p/Q) can either fit or not. In order to make the TPA model more robust against these effects the transfer functions should be smoothed in a suitable way.

Once the forces and volume velocities are calculated, again the relevant transfer paths should be identified for the focused noise patterns. The left diagram in Figure 7 shows the result of this investigation which looks quite accurate (black is measured reference in interior, red is total synthesis result). But as shown in the previous cases, this result alone is not sufficient without questioning the robustness of the results. First, the Matrix Condition Numbers of airborne and structure borne matrices are suitably low, which is a good indicator combined with the good match to the reference.

Focusing on the main combustion order around 30 Hz the contributions of airborne (blue) and structure borne (green) are higher than the total noise. In consequence, both shares are out of phase. This result stayed stable when using repetition measurements as well as results for different matrix setups.

Nevertheless in order to gain more confidence, a verification measurement was done. The exhaust orifice noise has the strongest influence in the airborne share of this investigation. Consequently it should be the starting point for a modification, because modifying the structure borne paths is much more difficult in this frequency range. Accordingly additional mufflers were attached to the exhaust orifices to lower the excitation level (see middle picture in Figure 7). The comparison of the measured interior noise levels for original and modified state are shown in the right diagram of Figure 7. Clearly the main combustion order rises about 10 dB fully in line with the prediction of the TPA model. In consequence model accuracy including phase relations is verified for this noise pattern.

In a similar way, modification measurements can help to verify the TPA model in other conditions and frequency ranges. Attention should be paid on keeping the number of variables as low as possible and to keep in mind the measurement repeatability for the investigated noise patterns.
4. CONCLUSIONS

Transfer path analysis based on matrix inverse techniques is no black-box procedure. It is advisable to question the TPA results in depth after the first synthesis is done. Many model configurations are possible, and transfer functions are not free of errors. There will be a result anyway, and more than one could even match a measured reference. The actual engineering task is to screen the TPA model regarding plausible path contributions and root-causes in order to achieve a robust model describing the investigated system as close as possible. Not till then the TPA results can lead to correct conclusions for optimizing the system e.g. the vehicle. The presented key points and procedures help to not get lost in the high amount of data during the model refinement process. However, appropriate tools or software and, of course, experience can help making the right decisions and shorten the refinement process.

REFERENCES