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Lower bound on frequency validity of energy-stress tensor based diffuse sound field model

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Abstract

A lower bound on the frequency validity limit is established for an energetic wave equation derived from the energy-stress tensor, examined in the one-dimensional case [Dujourdy et al, Acta Acustica united with Acustica 103:480-491, 2017]. The method efficiently models diffuse sound fields that dominate reverberation at higher frequencies and larger distances. Initially noted in the course of an exhaustive search of the solution space of all valid model parameters, the low-frequency cutoff has implications for the utility of the method in a hybridization context. In practice, the bound is encountered when determining the absorption and diffusion coefficients by iteratively approaching the temporal and spatial decay of measured data. As the test frequency decreases, the ranges of coefficient combinations that result in less than 10% variation from each decay measure can diverge until the region where both measures are satisfactory (the intersection of the two domains) disappears. Further evidence for the bound is provided through comparison with measurements of a long hallway, and stability concerns in the cases where both coefficients are very small are addressed.

Keywords: Room acoustics, finite difference methods, diffuse field

1 INTRODUCTION

While geometric and wave equation based approaches to room acoustics have dominated theoretical research, in the realm of practical implementations, hybrid methods combining both strategies are increasingly popular. Since each approach has strengths and weaknesses depending on the frequency range that is being simulated, it has long been the goal of practicioners to leverage different types of simulations in a cohesive manner to create full-band predictions with efficiency and accuracy that would not be possible with a single type alone. For example, geometric acoustics simulations such as ray tracing (1) or the image source method (2) efficiently model the early part of an impulse response, even at high frequencies, but cannot account for diffraction in a physically meaningful way, an important part of the low frequency response, and become inefficient when considering complex enclosure geometry or when directly modeling the late reverberation of a space. On the other hand, wave equation based methods (such as finite element or volume methods in both the time and frequency domains) excel in the low frequency regime, since they simulate the sound field everywhere in the enclosure, including diffraction and directional effects, with relatively direct implementations of sources and receivers (3, 4). Extending wave modeling methods to higher frequencies, however, requires increasingly fine meshes that become computationally untenable for real-time applications, even with the advent of modern GPU implementations (5).

From this brief overview, it is not difficult to predict how hybrid approaches could combine these methods, utilizing each method in its most effective domain in order to create a full-band model. In practice, however, there are a number of difficulties that arise from combining separate methods in this fashion, not the least of which is the maintenance of machinery (including but not limited to numerical code, meshing tools, automatic matching of boundary conditions across simulation types, room geometry simplifications, and post-processing DSP) for each separate part of the simulation. In this paper, we consider how an energetic wave equation based method mitigates some of these concerns while considering its limitations for future hybridization.

This method, developed by Dujourdy et al. (6), has been found to accurately model the statistical late reverberation, or diffuse field, resulting from an initial perturbation, in both 1- and 2-dimensional contexts (7). Originating from Ollendorff and Picaut's work (8, 9), this method aims to take advantage of an additional degree of freedom given by the inclusion of a diffusion coefficient in addition to the traditional absorption coefficient. Previous work on this topic considered only the 1000 Hz frequency band, but in order to consider its future fitness for hybrid modeling, we will consider in greater detail the frequency ranges it is capable of modeling. Such information provides bounds on the crossover frequencies that would be required in a hybrid context, implying the necessary ranges that would need to be simulated with other methods in order to create a full-band model, and thus whether or not the diffuse field strategy can be useful in a practical architectural acoustics context.

2 ENERGETIC WAVE EQUATION METHOD

For this study, we directly used the theory and implementation of the numerical schemes in (6). For the sake of brevity, we will not reproduce those findings here, but encourage those desiring a full treatment to refer to the original work.

In brief, the main idea of the energetic wave equation is the development of two conservation laws, one each for energy density E and sound intensity J, as defined by Morse and Feshback in (10), rather than the typical energy balance based on Gauss's theorem found in typical finite difference or finite volume formulations. Using these two laws, it is possible to derive a system of coupled equations relating the energy density, the sound intensity, and the wave-stress symmetric tensor $\underline{\underline{E}}$ (11). In the case where one length dominates a space and the cross-sectional area is relatively constant (as is the case in a long hallway), this system of equations can be reduced to one dimension by introducing modified absorption and scattering coefficients and integrating (with further energy and momentum balance hypotheses) over the minor axis walls.

After dimensional reduction, the system resembles the telegrapher's equations, and by inspection can be transformed into a linear second-order hyperbolic equation with a single dependent variable (in this case, the energy density E). Finally, boundary conditions can be derived using the aforementioned hypotheses, and the entire wave propagation system can be discretized with common finite difference time domain strategies.

The main advantage of this approach is that the propagation of diffuse energy allows for very large spatial discretization. The reason for this is that the modulation frequency of the late energy decay that we are concerned about is very low. Typical acoustic wave simulations that are concerned with high frequency content have a very high modulation frequency (since they specifically want to resolve the individual pressure waves), but since we are primarily interested in the stochastic decay of the diffuse high frequency energy, we can assume that the decay itself is not changing very rapidly, and thus can accommodate relatively large spatial sampling. As an example, if we are only concerned with the diffuse energy level every tenth of a second, then the spatial sampling rate can be as coarse as 3 meters, with the temporal sampling rate chosen to satisfy the scheme's stability conditions.

3 FREQUENCY VALIDITY ANALYSIS

3.1 Room geometry

A similar corridor to the original was used for this study. The hallway had an overall length of 45 meters, with a width of 159 cm and a height of 237.5 cm. In the main narrow portion of the hallway, the ceiling height was measured to a fine metal grating suspended below the actual height of the corridor, which carried lighting and conduit. The hard ceiling was 326 cm, with a decrease every 1.5 meters for metal support beams to 280 cm.

In the recesses, all of which were of uniform length and depth (except for the doorway furthest on the right of the plan), the width increased to 239 cm and the height decreased to 220 cm, and ceiling was masonry rather than the grating mentioned above. In some of the recesses, there were glass display cases or small pieces of

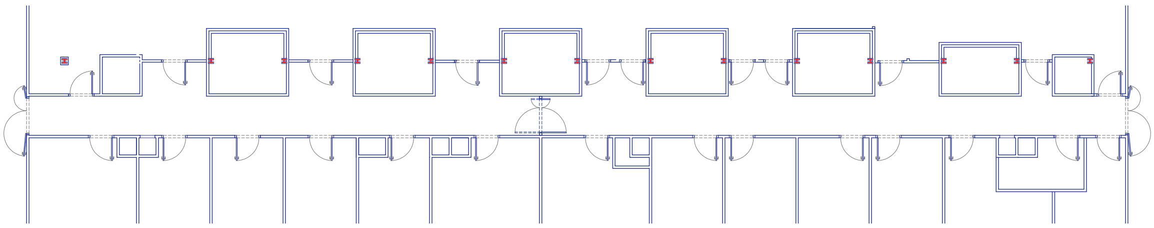

Figure 1. Floorplan for the corridor under consideration.

furniture that contributed to the diffusive effects of the recesses themselves. The entire floor was linoleum, and for the most part, the walls were wooden panel and masonry. There were occasional metal gratings on the flat wall for HVAC, which was audible but not distracting. All doors entering the hallway were closed, and the doors in the center of the hallway were fully open.

3.2 Measurements

Table 1. T_{60}s and Spatial Decays

Frequency [Hz]	62.5	125	250	500	1000	2000	4000	8000
T_{60} [S]	2.29	0.35	0.38	0.39	0.43	0.35	0.33	0.30
Spatial Decay [dB/m]	-0.63	-0.49	-0.88	-0.94	-0.66	-0.54	-0.66	-0.84

Impulse responses were collected using a *SoundField* ST250 microphone and an *Outline* GRS omnidirectional speaker, with a *MOTU* Traveler sound card. As before, the source was positioned 1 meter away from the end of the hall, 1.5 meters above the ground, and centered between the two walls. Beginning 1 meter from the source, measurements were collected with the microphone's X-axis aligned along the length of the hallway. A spacing of 1 meter was used out to 10 meters, which corresponds exactly to the discretization distance in the numerical simulations. Then, recordings were made every 2 meters until the end of the hallway for a total of 26 sampling locations. Recordings were made using the the swept sine method (12, 13) as implemented in the *Adobe Audition* plugin *Aurora*. The sweep length was 20 seconds, and the source level was adjusted digitally to maximize the signal-to-noise ratio without clipping as the microphone was moved further and further from the source. These gains were recorded in order to recover the true measured energy level for each measurement location. The sweep responses were then post-processed by convolution with the inverse sweep to recover impulse responses for each location.

The relevant frequency bands were selected by filtering each impulse response with a standard octave-band filterbank, and then calculating the desired metrics for each resulting bandlimited response. The T_{60} (a measure of temporal decay) was calculated by a linear fit to the Energy Decay Curve in dB for each frequency band and at each receiver location. This is sometimes known as Schroeder's reverse integration (14). Next, each receiver location was averaged to arrive at a single T_{60} for the entire hallway in each band. Finally, spatial decays (similar to strength of sound, or G) were calculated by a linear fit to the sum of energy for each band-limited response across each receiver location. Both sets of calculated values are presented in Table 1.

3.3 Simulations

In order to determine the frequency bands the model was able to represent, we ran simulations for a sampled subset of all combinations of absorption and scattering coefficients α and β. In a typical scenario, a scheme to converge on the desired spatial and temporal characteristics would be used to match a single frequency band, but here, we want to visualize the entire problem space.

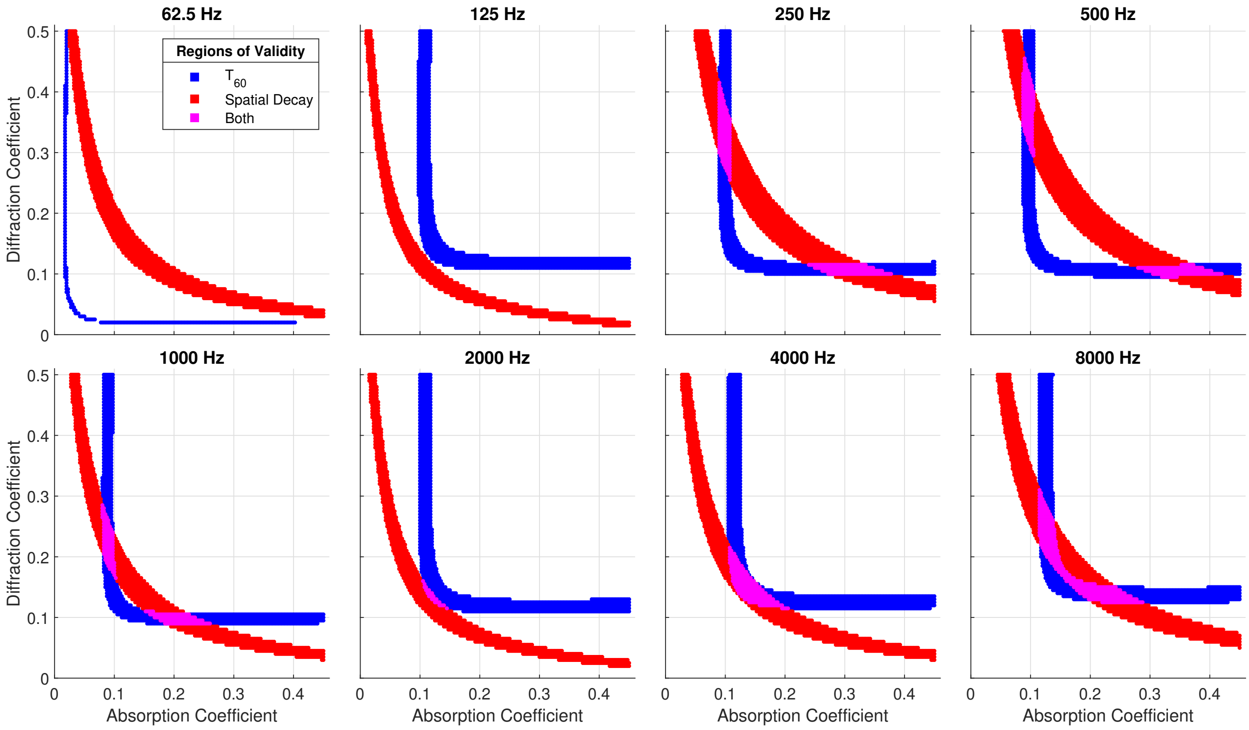

Figure 2. Agreement between simulated and measured data for the hallway under consideration.

For this study, as was the case before, the spatial sampling step was chosen to be $\Delta x = 1$ meter.

The initial conditions were chosen to be a temporal Gaussian, centered at the same location as the source, in order to minimize spurious numerical oscillations. Afterward, the same acoustical indices as with the *in situ* measurements were extracted from each result.

3.4 Valid domains

Finally, we compared the measured and simulated results for each index. For each frequency band (presented here in ascending order), we classified which simulations were within 10% of each desired metric, the spatial and temporal decay rates. To preserve legibility, we have represented in Figure 2 only the region with the most relevant combinations of coefficients for this hallway. The absorption coefficient α ranges from 0.01 to 0.45, and the diffusion coefficient β ranges from 0.01 to 0.5. This region corresponds well to the typical ranges where both T_{60} and spatial energy decays are valid.

As expected, the two coefficients have different effects on the simulations, given that the validity patterns are not symmetric, confirming the observation in the original paper. Furthermore, the regions of validity for each individual measure appear to be smoothly varying. Perhaps most importantly, for some low frequencies, there is no region where both indices are valid, implying a lower bound on the frequency that can be represented by the model for this hallway.

At the lowest frequency band, 62.5 Hz, there are regions of validity outside of the chosen region, but we have chosen to exclude these as either their absorption or diffusion coefficients are unrealistically small or large. Since the regions of validity appear to smoothly vary, it may be the case that with a particular frequency in between this band and 125 Hz, there is a point where the spatial decay and T_{60} curves "switch" places and become valid. We also note the similarity of the 125 Hz band and the 2000 Hz band, since the region of validity is determined by the 10% threshold. In other bands, the regions cross, and therefore can be uniquely determined to match both measures to arbitrary precision without the need of a threshold. Both of these topics related to a more continuous perspective on the methodology for determining validity deserve closer examination in future work.

The case where the individual metric regions cross resulting in two disconnected regions of validity, up to the lack of symmetry discussed above, can be explained by the observation by Dujourdy et al. (6) that the two coefficients can typically be exchanged, where the smaller of the two acts as the absorption coefficient. While in general this situation is preferable, it implies that the optimization problem, at least in its current formulation, is not convex, and furthermore, more information about the problem will be required in order to uniquely determine a preferred combination of coefficients.

One possible explanation for the lack of a valid region at the lowest frequencies is the mismatch between long T_{60} and relatively low spatial decays. At low frequencies, it is common to have more reverberation, and similarly, the energy may be more evenly distributed as a result of diffusion. At higher frequencies, the opposite is the case, where increased absorption of high-frequency energy results in shorter reverberation times and steeper spatial decay, both of which appear to be more readily represented by this model.

In the case where either coefficient is very small, generally speaking, the simulations become closer and closer to the lossless case, extending the response times and magnifying small perturbations. As a result, simulations with some combinations of α and β can veer into instability. This would further constrain the region of validity, but in practice, instability only occurs when approaching the most extreme values of T_{60} or spatial decay. Thus, such instability can simply serve as a indicator that careful attention should be paid to the physical measurements that are being matched. It appears that since the region of validity at the lowest frequencies is reasonably well balanced between the two coefficients, for this type of hallway, instability should not present a major concern in the context of hybridization since the most dangerous simulations can simply be discarded.

3.5 Implications

Since the lowest frequency that contains a convincing overlap is 250 Hz, for this corridor, that is the lowest bound on the validity of the model. This makes sense, seeing as the late reverberation is more statistical at higher frequencies. For small concert halls, 250 - 500 Hz is a very convenient cutoff frequency for a potential hybridization since a complementary pressure wave time domain approach would be capable of modeling the region in real time (5). Thus, at least for this case, it is worthwhile to consider the model as a possible candidate for accelerating the synthesis of the late tail in a full-band audio simulation.

3.6 Other hallways

In order to confirm these results, the procedure was followed using the recordings from the previous hallway. In the interest of space, we will not fully reproduce the measurements as we have with the current study, but the findings were consistent with the conclusions we have drawn. The corridor from the initial work exhibited regions of validity down to 125 Hz. Given that the two were relatively similar in terms of shape and length, this is perhaps unsurprising, but a useful confirmation nonetheless.

A third unrelated hallway of a similar length but with fewer diffusing surfaces was also measured and was only representable (with “crossed” validity regions) by the model in the 125 Hz and 1000 Hz bands. Upon initial review of these results, it appears that the reason for the lack of validity is primarily due to the reduced spatial decays across all bands. Because the hallway did not have as many absorptive surfaces, the sound energy was much more evenly distributed throughout or even modal, and therefore presented difficulty for the model, which is predicated on the assumption of spatial decay. One possible interpretation is that this incompatibility between temporal and spatial decays is a manifestation of a truly uniform reverberant field in regions far from the source, where the so-called diffuse field theory is valid. Thus, the case where the energy-based model fails may actually be a criterion for spaces where the Sabine equation holds.

Figures displaying the regions of validity for both of these corridors can be found in the appendix.

4 CONCLUSIONS AND FUTURE WORK

In this paper, we have discussed our findings regarding the lower frequency bound for which an energy-based late-reverberation model can accurately represent the stochastic soundfield. The current study focused on a one-dimensional model of a long hallway for which the lower bound was 250 Hz. This means that above this cutoff frequency, the model was able to recreate a diffuse soundfield with acoustical indices consistent with measurements made in a real hallway within 10% error.

The investigation of this lower frequency bound brings to mind a number of questions for future research. In this case, we have examined a single hallway with a given length and certain acoustical indices, but it should be possible to extend the analysis to one dimensional problems of varying lengths, T_{60}, and spatial decays, such that any problem dominated by its length with a sufficient amount of diffusion could be verifiably modeled above a certain frequency. Furthermore, given the discovery that in some frequency bands there are multiple regions of validity, it should be possible to examine more optimal methods for converging on desired coefficients for a given physical measurement in an interative fashion. In such a fashion, it may be possible to better characterize the contours of the underlying search space in order to accelerate convergence.

In terms of bringing the theoretical model into practical application, it is imperative to begin experimenting with a sonification procedure. While the theoretical aspects of the method are interesting in and of themselves, the real value of such an approach is the additional accuracy and efficiency that would be enjoyed upon integration with a full-band hybrid auralization system. To that end, the next step in developing the model is extending the current 1- and 2-dimensional models to 3 dimensions.

Finally, given the large spatial discretization step in this method, it may be the case that the energy model and a pressure wave approach could use the same grid, reducing the meshing that must be completed before simulations can be performed. Verifying that meshes (or subsets thereof) could be trivially reused for both methods in a hybrid context would be especially relevant in the cases where room geometry is time varying, a possibility that deserves closer inspection.

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APPENDIX

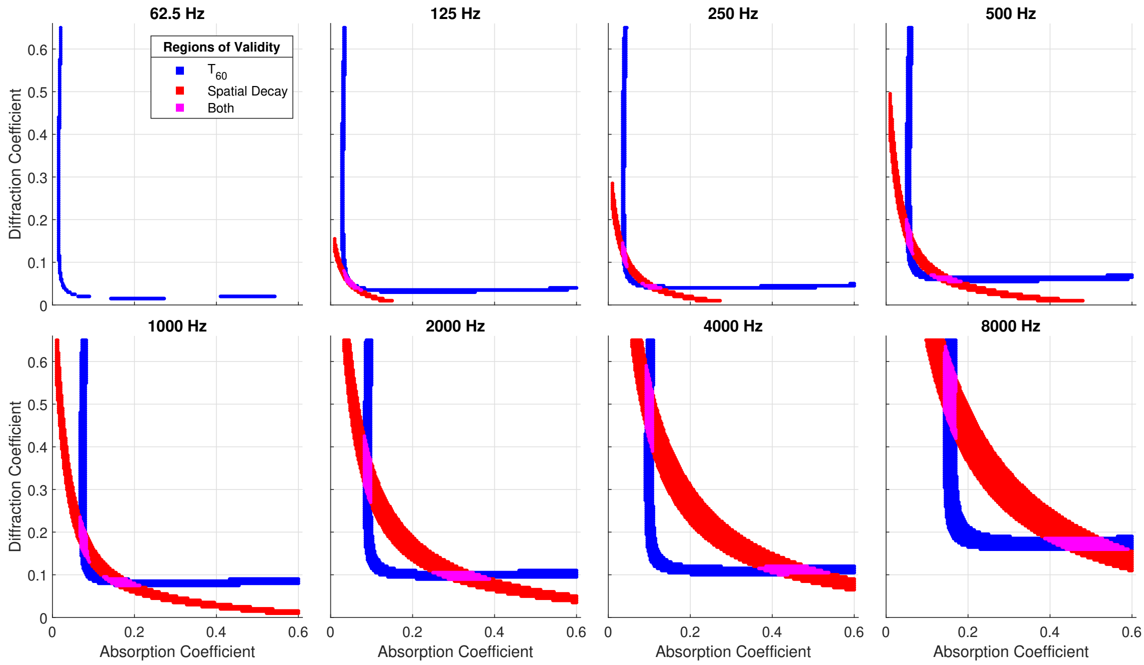

Figure 3. Agreement between simulated and measured data for the hallway studied in Dujourdy et al.

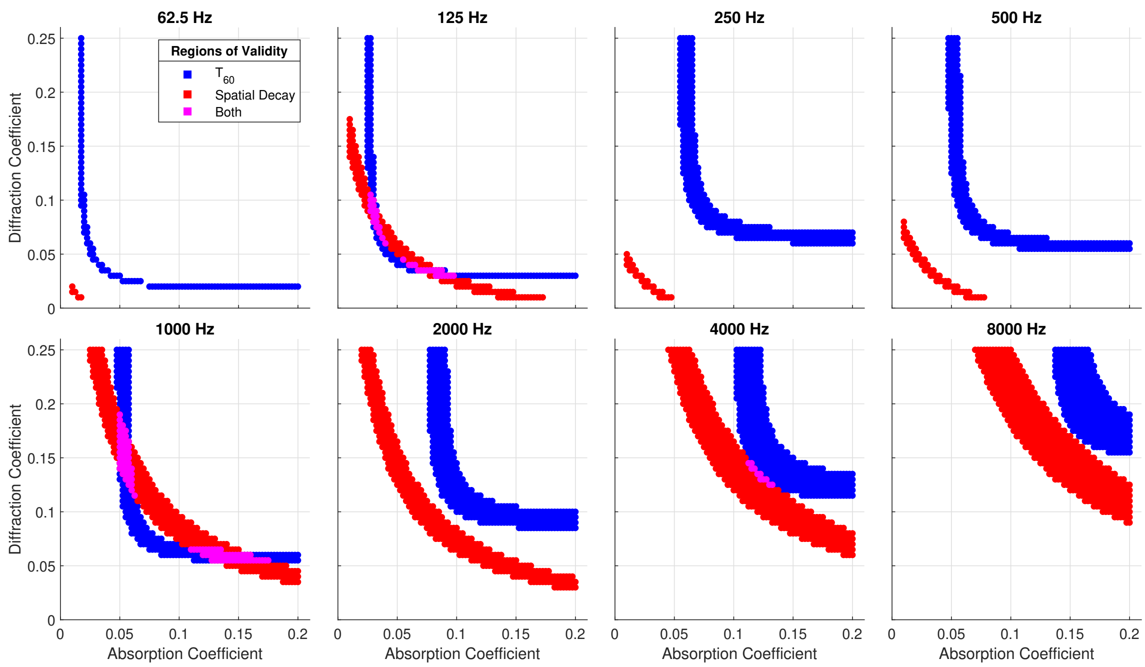

Figure 4. Agreement between simulated and measured data for a hallway with fewer diffusing surfaces.