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Design of a resonator-based metamaterial for broadband control of transverse cable vibration

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Abstract

Cables are widely used to bear loads, tether objects or transmit data over a distance. Vibration in cables can cause excessive wear, acoustic radiation and for sensing arrays, measurement error. Although a variety of control methods have been proposed, metamaterials potentially offer a high level of performance within limited weight constraints. This paper investigates the design of an elastic metamaterial (EMM) consisting of single-degree-of-freedom (SDOF) resonators modelled as mass-spring-dampers, evenly distributed along a one-dimensional string model. A systematic tuning approach for distributed tuned-vibration-absorbers (TVAs), is shown in simulations to significantly reduce the energy of the cable around the resonance frequencies. However, the effectiveness of this configuration is dependent on TVA position with respect to the cable mode shapes and so is not robust to changes in length of the cable. An EMM is designed using a systematic tuning approach and compared to an EMM optimised using a gradient descent approach. The EMM is shown to achieve a similar level of absorption to the TVAs but is more robust to location on the cable.

Keywords: Metamaterial, Vibration, Optimisation

1 INTRODUCTION

Cables are used in a wide variety of applications to tether objects, bear loads and transmit data over long distances. If subjected to vibration these cables can experience accelerated wear, requiring more frequent maintenance or potentially leading to catastrophic failure. In the case of towed sensor arrays, vibration in the towing cable leads to relative displacement of the sensors which can contribute to measurement errors. Research into reduction of cable vibration has been a large area of interest for many years and is continuing with modern materials and approaches.

A tuned-vibration-absorber (TVA) or dynamic-vibration-absorber (DVA) is a spring-mass system that can be used to absorb the kinetic energy of a system. A TVA is more commonly known as a tuned-mass-damper (TMD) if the absorbed energy is dissipated via damping. The optimisation of tuning frequency and damping for a single TVA to minimise maximum displacement of a single-degree-of-freedom (SDOF) was first derived by Ormondroyd and Den Hartog in 1928 (1) and has since been expanded to alternative optimisation criteria based on the ratio between TVA and primary system mass (2–6). For multiple-degree-of-freedom (MDOF) systems it has been shown that each mode can be considered individually for large ratios between adjacent modes, therefore, all modes can be controlled using multiple TMDs, with one tuned to each mode using the optimal approaches derived for the SDOF system (7). However, a TVA is tuned to a single frequency and therefore has limited robustness to changes in the frequency response of the primary system.

Multiple smaller TVAs/TMDs tuned to frequencies around the primary resonance have been shown to outperform a single TVA of the same total mass, both in attenuation of the response of the primary system and in robustness to changes in primary resonant frequency (8–11). The optimum tuning frequency distribution for multiple absorbers has been shown to be non-uniform (12). Multiple distributed TVAs could be applied to the control of multiple modes, however, according to the work done by Warburton in (7), these multiple TVAs must have tuning frequencies far enough away from the other resonant frequencies of the system such that each mode can still be considered individually.

An elastic metamaterial (EMM) is an artificial material that is able to interact with elastic waves in materials in a way that is not exhibited by natural materials. For example, an EMM might achieve effective negative mass or negative stiffness. Based on the same principles of operation as a TVA, an EMM consisting of an array of small, resonant sub-structures can be used to absorb vibration from a primary structure without significant additional mass. This behaviour has been demonstrated in multiple cases for different structures with varying degrees of freedom (13–18). A range of optimisation procedures has been used to optimise the individual parameters of the EMM substructures including genetic algorithms (19, 20) and topology optimisation (21, 22).

In this paper, a one dimensional (1D) EMM for vibration control of a taut cable is proposed. As an initial study, a simplified cable is modelled and the effect of distributed TVAs examined as an approach to multi-mode absorption. A metamaterial based on repeated cells of multiple SDOF resonators is then proposed, and resonant frequencies of the individual substructures are optimised to maximise the attenuation in kinetic energy using a gradient descent approach.

2 SIMULATION OF CABLE RESPONSE

Assuming the gravitational force acting on the cable is very small compared to the tension in the cable, the cable can be regarded as a simple, taut string. Single-degree-of-freedom (SDOF) mass-spring-damper models can be coupled to the string and represented as complex modal impedances. This section sets out the linearised modal model of a string and the modified model to include coupled SDOF resonators used to simulate the cable response.

2.1 Modal Model of a String

The dynamic response of a string in one dimension can be represented by the sum of the response due to all of the modes of vibration (23). Considering a single element of the string of width δx, the sum of forces acting perpendicular to the cable can be linearised and simplified based on the assumption of small-amplitude, time-harmonic vibration. The response due to each mode can then be derived using a modal decomposition method as

$$\frac{L}{2}\left(\tau\left(\frac{n\pi}{L}\right)^2 + j\omega b_n - \omega^2\mu\right)\phi_n(t) = \int_L \mathrm{F}(x,t) w_{s,n}(x) dL, \tag{1}$$

where: L is cable length; τ is cable tension; n is the mode number; ω is the frequency in rads^{-1}; μ is the mass per unit length of the cable; b_n is the modal damping coefficient equal to $b_n = 2\zeta(\frac{n\pi}{L})\sqrt{\mu\tau}$; $\phi_n(t)$ is the response of the n-th mode; $\mathrm{F}(x,t)$ is the force distribution along the cable; and $w_{s,n}(x)$ is the modeshape function.

By assuming time-harmonic motion ($\phi_n(t) = A_n e^{j\omega t}$ and $\mathrm{F}(x,t) = Fe^{j\omega t}$) and approximating the force distribution along the cable as an array of I point forces at $x = x_i$ and with magnitude F_i, the response due to N modes at frequency ω can be expressed in matrix form as

$$diag\begin{bmatrix} B_1 & B_2 & \cdots & B_N \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ \vdots \\ A_N \end{bmatrix} = \begin{bmatrix} w_{s,1}(x_1) & w_{s,1}(x_2) & \cdots & w_{s,1}(x_I) \\ w_{s,2}(x_1) & w_{s,2}(x_2) & \cdots & w_{s,2}(x_I) \\ \vdots & \vdots & \ddots & \vdots \\ w_{s,N}(x_1) & w_{s,N}(x_2) & \cdots & w_{s,N}(x_I) \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \\ \cdots \\ F_I \end{bmatrix}, \tag{2}$$

where $B_n = \frac{L}{2}\left(\tau\left(\frac{n\pi}{L}\right)^2 + j\omega b_n - \omega^2\mu\right)$ and is the modal stiffness of the cable, A_n is the modal amplitude and $w_{s,n}(x_i)$ is the shape of mode n at the location of point force i, and

$$B_n = \frac{L}{2}\left(\tau\left(\frac{n\pi}{L}\right)^2 + j\omega b_n - \omega^2\mu\right). \tag{3}$$

Equation 2 can be represented symbolically as

$$\mathbf{Ba} = \mathbf{Wf}, \tag{4}$$

and $\mathbf{a}$ can then be solved at each frequency as

$$\mathbf{a} = \mathbf{B}^{-1}\mathbf{Wf}. \tag{5}$$

Differentiating equation 5 with respect to time, gives a vector of the cable velocity due to each mode at position x and frequency ω. The sum of the velocity due to each mode gives the total velocity at this point and frequency and can be expressed as

$$v(x, j\omega) = j\omega\mathbf{w_s}(x)\mathbf{a}(j\omega), \tag{6}$$

where $\mathbf{w_s}(x)$ is a vector of the amplitude of each mode shape at x. The spatial average velocity magnitude at each frequency can therefore be approximated by averaging equation 6 over discrete values of x as

$$|\bar{v}(j\omega)| = -\frac{1}{L}\sum_{x=0}^{L}\left|j\omega\mathbf{w_s}(x)\mathbf{a}(j\omega)\right| dL. \tag{7}$$

2.2 Modal Model with Coupled Resonators

A single-degree-of-freedom (SDOF) resonator can be approximated as a mass-spring-damper with mass m_r, stiffness k_r and damping coefficient b_r. Each resonator presents as a force acting on the cable, comprised of the force of the spring being compressed due to the motion of the cable, and the force due to the acceleration of the resonator mass. The vector of forces for R individual resonators can be derived and expressed in matrix form as

$$\begin{bmatrix}\hat{F}_{r1}\\ \hat{F}_{r2}\\ \vdots\\ \hat{F}_{R}\end{bmatrix} = diag\begin{bmatrix}Z_1 & Z_2 & \cdots & Z_R\end{bmatrix}\begin{bmatrix}\boldsymbol{\alpha}_1\\ \boldsymbol{\alpha}_2\\ \vdots\\ \boldsymbol{\alpha}_R\end{bmatrix}\begin{bmatrix}A_1\\ A_2\\ \vdots\\ A_N\end{bmatrix} \quad \text{or} \quad \hat{\mathbf{F}} = \mathbf{ZRa}. \tag{8}$$

where Z_r, the impedance corresponding to the r-th resonator, is

$$Z_r = (k_r + j\omega b_r)\left(1 - \frac{k_r + j\omega b_r}{k_r + j\omega b_r - \omega^2 m_r}\right) \quad \text{and} \quad \boldsymbol{\alpha}_r = \begin{bmatrix}w_{s,1}(x_r)\\ w_{s,2}(x_r)\\ \vdots\\ w_{s,N}(x_r)\end{bmatrix}^{\mathrm{T}}. \tag{9}$$

Adding this additional forcing term expressed by equation 9 to the right hand side of equation 4, the new modal amplitudes can be expressed as

$$\mathbf{a} = (\mathbf{B} - \mathbf{R}^{\mathrm{T}}\mathbf{ZR})^{-1}\mathbf{Wf}, \tag{10}$$

and the resulting spatial average velocity magnitude of the cable can again be calculated using equation 7.

3 Multiple Tuned-Vibration-Absorbers (TVAs)

Using the model described in the previous section, this section presents an investigation on the use of distributed TVAs to control the modes of vibration of the cable. A cable with length, $L = 1$ m; mass per unit length, $\mu = 0.0335$ kgm^{-1}; tension, $\tau = 500$ N; and damping ratio, $\zeta = 0.01$ is simulated with a force of 1 N applied equally to all modes (**Wf** is all-ones). As it is only practicable to treat a finite number of system modes, the frequency bandwidth is limited to 20-320 Hz, which includes the first five modal peaks. However, the model is evaluated using $N = 16$ modes, which provides convergence of the modelled response over the considered frequency range.

3.1 Systematic Tuning Method

The optimal tuning frequencies for TVA control of multiple modes on a cable has not been explicitly considered in the literature, therefore the near-optimal approach of tuning a TVA to each modal frequency and positioning at the first anti-node of the respective mode shape is taken as the 'systematic' tuning method. The TVA stiffness is kept constant to simplify the method and the masses are used to tune the TVAs. The stiffness is set such that the total mass of the TVAs is 20% of the total cable mass.

The systematic tuning approach depends on prior knowledge of the modal frequencies, however, changes to the parameters of the cable over time (e.g. change in length, added mass, variation in tension) will result in a shift in the modal frequencies and the performance of the TVAs will be affected. A change in cable length (for example, a cable partially wound in on a drum) would also displace the TVAs from the antinodes of their respective modes and in extreme cases one or more TVAs could be lost completely. The string is simulated with and without TVAs for the nominal length $L_0 = 1$ m and for a variety of significant changes in length in the range $0.9L_0 \leq L \leq 1.1L_0$, where the TVAs maintain the same distance from $x = 0$. The resulting spatial average velocity frequency response functions (FRFs) are shown in Figure 1. It can be seen that the five modes have been significantly attenuated at the nominal length, however, for other cable lengths the response can vary dramatically with amplification of higher order modal peaks. This can be attributed to the fact that at higher frequencies the wavelength of the vibration is comparable to the change in length.

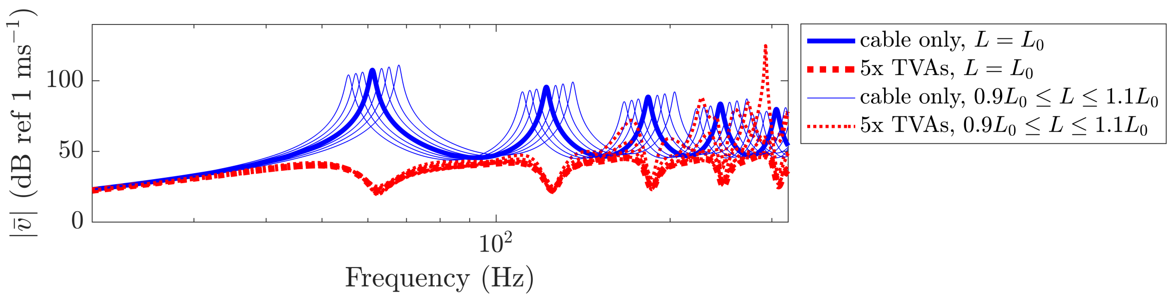

Figure 1. Spatial average velocity FRF for vibrating string simulated for different cable lengths, with and without 5x TVAs positioned and tuned to the first 5 modes of the cable at nominal length.

Robustness to a change in frequency response could be achieved using additional TVAs, however, this does not address the issue of a shift of the TVAs away from the antinodes if there is a change in cable length. Additional distributed mass would reduce kinetic energy but a substantial change in mass would be required for significant attenuation. A distributed array of TVAs on a small scale however, can be designed with little additional mass and is less dependent on mode shapes and therefore cable length. If the scale becomes small compared to the wavelengths involved the system can be considered homogeneous and is known as an elastic metamaterial (EMM). An EMM could also be designed so that the cable can still be run through pulleys or stored on a drum, whereas larger TVAs may present a problem.

4 Elastic Metamaterial (EMM)

An optimised EMM presents a possible light-weight solution to the presented problem of controlling the response of a cable with low sensitivity to the position of the treatment on the cable. An EMM consisting of an array of small, SDOF TVAs is proposed, as shown in Figure 2. Based on achieving a small size compared to the wavelength of the fifth mode shape of the 1 m string model described in the previous section a unit cell of 0.2 m has been used. In order to make the proposed EMM practicable for future experimental validation, each resonator within the EMM is spaced at 0.01 m intervals allowing a total of 20 resonators per unit cell. The unit-cell is repeated end-to-end along the length of the simulated cable, giving 5 unit cells in the case of the 1 m cable.

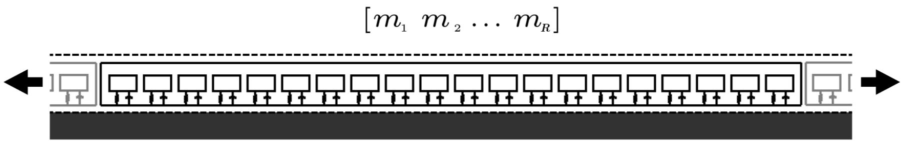

Figure 2. Proposed EMM design.

4.1 Systematic Tuning

Based on the systematic tuning approach for distributed TVAs, a systematically tuned EMM is first described. Consecutive individual resonators within the cell are tuned sequentially to the first five ($N = 5$) modal frequencies of the cable, such that for $R = 20$ there are four repeats. To allow comparison with the distributed TVA approach, the fixed stiffness is set so that the total added mass, due to the 100 resonators (20 resonators in each of the 5 unit cells), is equal to 20% of the mass of the cable.

4.2 Optimisation Procedure

Although it is expected that the systematic tuning approach proposed will achieve a significant level of performance, it may not be optimal and, therefore, a procedure to optimise the masses of the resonators within the unit cell has also been considered. This optimisation procedure could also be extended to consider more complex objective functions that, for example, take into account the need for robustness. The optimisation is carried out using the interior-point algorithm described in (24), implemented using `fmincon` in MATLAB. It is assumed that more mass will give higher attenuation and, therefore, to limit the search area non-linear constraints are imposed to limit the total added mass to between 15 and 20 % of the mass of the cable.

The cost function to be minimised in this case is defined as the total gain in kinetic energy of the cable at the nominal tension τ_0, after application of the EMM, thus maximising attenuation, approximated by

$$J = E_{k,gain} = \sum^{\omega} \left(\frac{|v(j\omega)|}{|v_0(j\omega)|} \right)^2, \tag{11}$$

where $|v_0(j\omega)|$ is the spatial average velocity magnitude of the cable alone (with no vibration control).

The optimisation procedure is initially carried out using the systematic EMM masses as the start point of the algorithm, and then repeated using multiple randomly generated start points to help ensure that the global minimum is obtained.

4.3 Results and Discussion

In the optimisation procedure the minimum was achieved from the systematic start point. Figure 3 compares the optimised EMM with the systematic EMM in terms of the resonant frequencies of individual masses. From this plot it can seen that for the optimised EMM the masses still remain close to the systematic approach but with some small deviations.

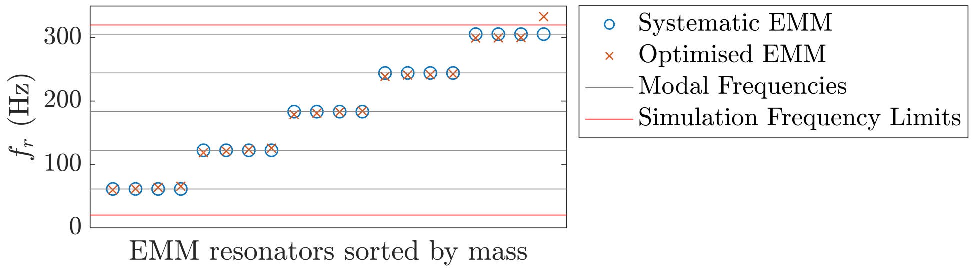

Figure 3. Comparison between the resonant frequencies of unit cell resonators in the systematic and optimised EMM.

The velocity response at the nominal length, $L_0 = 1$ m, and the attenuation of kinetic energy (shown as a negative gain) for variations in the length are shown in Figure 4 for the systematic and optimised EMMs, alongside the results for a 20% increase in the mass of the cable, in each case the fixed-size unit cell is repeated along the full length of the cable. Comparing the systematic EMM to the optimised version shows improved attenuation in kinetic energy at the nominal tension and a 'centering' of the performance over length. Both metamaterials perform significantly better than simply increasing the mass of the cable. It can also be seen that the EMM has limited robustness to changes in the frequency response, offering a similar level of robustness to that of the TVAs in the previous section. An optimisation procedure based on the robust response could improve on this.

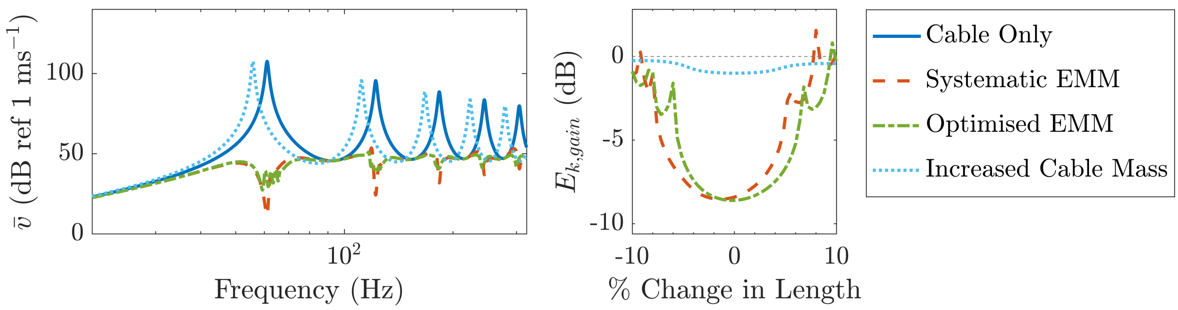

Figure 4. Spatial average velocity FRF of the cable at nominal length, L_0 (left), and gain in the kinetic energy of the cable over a range of cable length (right), for cable with systematic and optimised EMMs.

Figure 5 compares the response over length for different geometrical configurations of the optimised resonators within the EMM unit cell. Although there are variations between different configurations at non-nominal lengths, the general trend is fairly consistent and at the nominal length all achieve similar attenuation in the kinetic energy. This suggests that the EMM can be considered as near-homogeneous for the nominal frequency response.

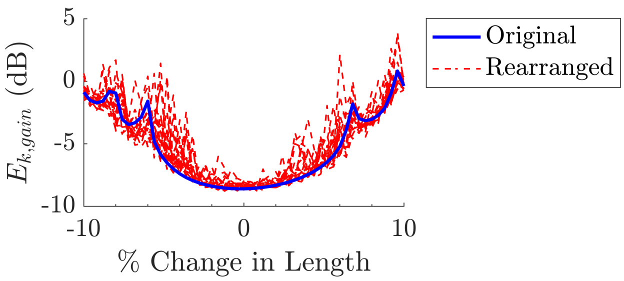

Figure 5. Gain in the kinetic energy for different length cables when treated with the optimised metamaterial for various rearrangement of the unit cell masses.

5 Conclusions

In this paper it has been demonstrated how systematically tuned TVAs can provide excellent attenuation in the kinetic energy of a simulated vibrating cable by reducing the amplitude of the modal peaks, however, this approach is not robust to changes in the frequency response or cable length. An elastic metamaterial (EMM) has been proposed with 20 individually tuned resonators in each unit cell with the aim of being independent of mode shapes such that it achieves good absorption independent of location along the cable. Using a constrained gradient descent algorithm, the tuning frequencies of the unit cell resonators have been optimised to minimise the kinetic energy at the nominal length and it has been seen that increased attenuation in the kinetic energy at the nominal length is achieved for the optimised masses. It has also been shown that when the masses are rearranged within the unit cell, although there is variation between the responses when the length is changed, the attenuation at the nominal length is quite consistent. This suggests that the EMM is close to acting as a homogeneous material, independent of the mode shapes. There is still, however, a lack of robustness to the change in frequency response, and using a robust optimisation process has been proposed.

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