

Investigation towards an active barrier for structure borne sound using structural intensity

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ABSTRACT

Vibrations of aircraft or vehicle engines, for instance, are often transmitted by structure borne sound, leading to a significant radiation of noise inside passenger cabins. Current active vibration control concepts use either velocity or acceleration as a control source. However, these only lead to a local reduction in vibration and not necessarily to the reduction of the vibration energy flow.

This study presents the implementation of current methods for structural intensity measurement with a real-time control. The work investigates one and two-dimensional structures. A reduction of energy flow in a beam structure is shown experimentally, as well as numerically for plates. The measurements are strongly influenced by theoretical simplifications concerning the composition of the structure borne waves and the quality of the sensor arrays used, i.e. the sensor spacing and the positioning accuracy. Though, sufficient accordance between numerically and experimentally estimated structural intensity can be found using methods with smaller sensor arrays. A barrier effect is shown by numerical investigations and is measured on a beam. Therefore, the control of vibration energy flow is a more effective method for a global reduction if vibration downstream the control area.

Keywords: active control, structural intensity

1. INTRODUCTION

Vibrating structures leading to noise radiation are often excited by sources distant from the relevant structural parts. In many cases structure borne sound is a major transmission factor for these vibrations. In order to reduce the share of structure borne noise transmission, in for instance passenger cabins, the control of the structural intensity (STI) as a measure for the energy transmitted proposes an effective method.

Hence, this paper addresses measuring techniques for the estimation of structural intensity in order to implement an active barrier for structure borne sound. First of all these methods are investigated numerically for an unstiffened plate. A control set up is furthermore examined. Experimental research towards measurement and control of structural intensity is done on a beam structure with different damping configurations. Furthermore the active control of structural intensity is investigated on an inhomogeneously damped plate in comparison to a velocity control.

2. MEASUREMENT OF POWER TRANSMISSION

2.1 State of the art methods

The measurement of structural intensity has already been addressed by NOISEUX in 1970 (1). In the beginning methods neglecting near field components of the bending waves due to inhomogeneity, i.e. waves decaying exponentially with their distance to the origin, were proposed, enabling the approximation of STI with two sensors for a beam. Based on this research more complex methods were established using more sensors and including one or more near field components (2, 3). For one dimensional STI estimation, a total of four sensors is necessary for the notionally exact estimation, respectively 8 sensors for the two dimensional case (2). All these methods use finite differences for approximation, being advantageous for a real-time implementation, as the calculation effort is reduced. Other methods for example use wave decomposition methods, leading to a higher demand for solving

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the equation systems (4, 5)

The following derivation is based on (2) using the internal forces of a plate element as described in figure 1.

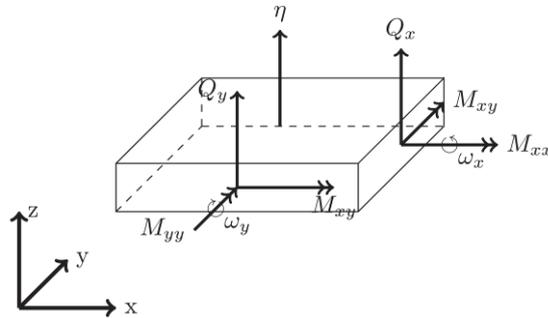


Figure 1 – Coordinate system and definition of displacements, rotations, internal forces of a plate element (positive influence line)

The structural intensity is a vector describing the vibratory energy flow through the cross section of the plate. In this case only the energy flow of bending waves is examined, as those are seen to largely contribute to sound radiation (6). Assuming a homogeneous thickness of the plate, normalizing the energy to the cross section may be omitted in the following formulas, so only the vibratory power is displayed. For plates, the bending wave power flow in x direction can be described as follows:

$$P_x = Q_x \dot{\eta} + M_{xx} \omega_x + M_{xy} \omega_y \quad (1)$$

The equivalent formula for the y direction can be derived by interchanging the subscripts x and y. The out of plane velocity $\dot{\eta}$ and angular rates ω , or their time derivatives respectively, can be measured by current technology sensors, whereas the direct estimation of the internal forces Q and moments M is impossible. One approach to calculate these measures is the application of constitutive equations of plate theory. Hence equation (1) can be written as follows

$$P_x = B \left(\left(\frac{\partial^3 \eta}{\partial x^3} + \frac{\partial^3 \eta}{\partial x \partial y^2} \right) \dot{\eta} - (1 - \nu) \frac{\partial^2 \eta}{\partial x \partial y} \frac{\partial \dot{\eta}}{\partial y} - \left(\frac{\partial^2 \eta}{\partial x^2} + \nu \frac{\partial^2 \eta}{\partial y^2} \right) \frac{\partial \dot{\eta}}{\partial x} \right) \quad (2)$$

only using the flexural stiffness B of the plate, the Poisson ratio ν and the normal displacement of the plate element η and its spatial and time derivatives. The first may be obtained by integration of time signals from accelerometers. The latter require approximation by finite differences, where local derivatives are estimated by the difference of measures of an array of accelerometers. For example the third spatial derivative of the local displacement as well as the mixed derivative is shown below, using an array of twelve equally spaced sensors (figure 2) based on (2).

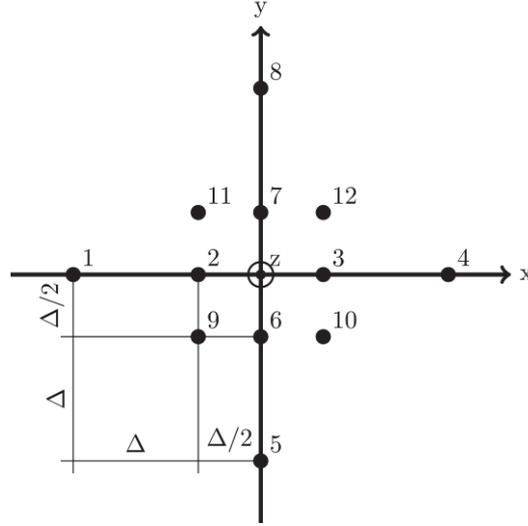


Figure 2 – Sensor array for two dimensional STI estimation of a plate (sensor spacing Δ)

$$\frac{\partial^2 \eta}{\partial x \partial y} \approx \frac{\eta_9 - \eta_{10} - \eta_{11} + \eta_{12}}{\Delta^2} \quad (3.1)$$

$$\frac{\partial^3 \eta}{\partial x^3} \approx \frac{\eta_4 - 3\eta_3 + 3\eta_2 - \eta_1}{\Delta^3} \quad (3.2)$$

Applying finite differences to equation (2) and omitting derivatives in y-direction (i.e. assuming a beam), the following simplified equation can be derived for the STI calculation for a beam with the four point method (4FD):

$$P_{4FD} = \frac{B}{\Delta^3} [\dot{\eta}_2(2\eta_4 - 4\eta_3 + 2\eta_2) + \dot{\eta}_3(-2\eta_3 + 4\eta_2 - 2\eta_1)] \quad (4)$$

However even these relatively simple equations show the importance of the exact sensor spacing for a sufficient approximation, as errors in phase and amplitude rise with the order of the derivative. Furthermore a time synchronous data acquisition is necessary, avoiding phase errors between the sensors. Since finite differences are quite vulnerable to these kinds of errors, simplifications may be made to reduce the order of finite differences needed and therefore the amplification of errors. As already introduced, another approach is proposed by NOISEUX's method for one dimensional STI calculation, assuming a far field (1). By separating the wave equation and omitting the time part as well as the near field components, two sensors are sufficient to estimate the STI. The derivation of the two point method (2FD) results in the following equation, using the wave number k :

$$P_{2FD} = \frac{Bk^2}{\Delta} [(\eta_1 + \eta_2) - (\dot{\eta}_2 - \dot{\eta}_1)] \quad (5)$$

2.2 Comparison of methods

Numerical and experimental investigations towards the accuracy and robustness of the two and four sensor measurement were already done for the STI estimation on a beam (one dimensional) in (7). One major issue of finite difference approaches is the sensitivity to sensor spacing. The theoretically exact estimation of spatial derivatives demands an infinitesimal spacing between the sensors. The size of this systematic error caused by enlarging the spacing is dependent on the wavelength, i.e. larger wavelengths with slower changes in amplitude over distance allow larger sensor distances.

On the other hand, small sensor spacing leads to small differences in amplitudes. This occurs to be a problem when the signal to noise ratio is small, as mainly the local derivative of the noise is calculated instead of the signal, leading to a random error in STI estimation.

Further numerical investigations on an aluminum plate model were done to estimate the error sensitivity of the two and four point STI estimation method as well as the theoretically exact eight position method (assuming infinitesimal sensor spacing and no additional noise). The system was modelled in ANSYS® using SHELL281 elements.

A virtual minimum sensor spacing of 5mm was initially assumed to minimize the systematic error of finite differencing. However, the sensor spacing in a practical implementation will be larger. For estimating the systematic error, the STI was calculated with twelve equally distanced virtual sensors for a frequency range between $20..1000\text{Hz}$. The following figure 3 shows the error due to finite differencing for the twelve sensor method as an example. The error level was calculated in respect to the calculated STI using the internal forces of the finite elements.

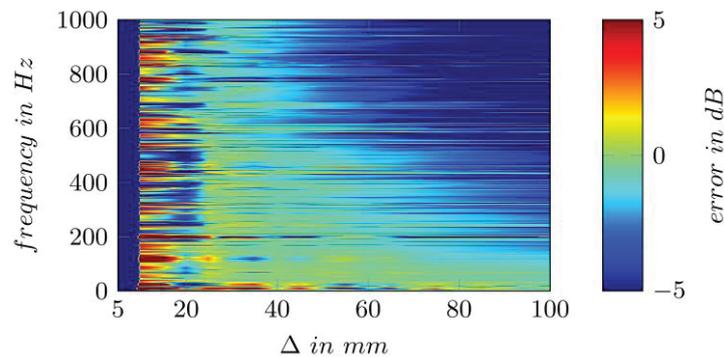


Figure 3 – Finite differencing error for approximating shear forces (i.e. spatial derivative of third order) with variable sensor spacing

As already presumed, the twelve position method is highly sensitive towards the increase of sensor spacing. With decreasing wavelengths the sensor distance has to decrease too in order to avoid a large systematic error.

This error was also found to be less when using finite differences of lower order. The two point method appeared to be the most robust. Nevertheless, the simplified methods include systematic errors due to neglecting parts of the energy transmission (e.g. twisting moments or near field). These errors, however, are potentially smaller than the finite differencing errors with large spacing, depending on the distance to structural discontinuities.

Apart from the systematic error at larger sensor distances, a random error occurs with decreasing values. As already presumed, a small sensor distance leads to a very small change in amplitude between the signals. Hence even numerical noise leads to a severe error in approximating the deviations.

Furthermore the sensitivity towards phase errors of the array sensors was investigated. Those can occur for example due to different signal transit times or misplaced sensors. It was observed that the use of finite differences of higher order for the twelve and four point method led to larger amplification of phase errors in comparison to the two point method, what makes it vulnerable for a real time control application.

The numerical investigations were examined experimentally on a beam. It could be shown that the two position method delivers adequate results even in the near field, as the four point method lacked accuracy due to noise and phase errors. Therefore the two point method proposes to be preferable for a practical implementation of a real time control, as the main part of the structural intensity can be addressed.

3. ACTIVE CONTROL OF POWER TRANSMISSION

3.1 Beam

Investigations towards the control of structural intensity were done for a cantilever in the beginning. A steel beam of $1000 \times 40 \times 3 \text{ (mm)}^3$ was investigated, being excited on the free end by a single point force. An optimal control was calculated according to (8), using the force excitation as reference signal. The secondary controlling force was also assumed to be a point force at 750 mm and a measuring point 600 mm from the bearing.

The structural intensity according to equations (4) and (5) however cannot be used as a cost function for the control, as the relationship between excitation and STI is non-linear. Hence the intensity is split into its multipliers, delivering two components linear dependent from the reference for the two sensor method and four components for the four sensor method, as proposed in (9). For the four sensor method for example, the components can be written as

$$A = \dot{\eta}_2 \quad (6.1)$$

$$B = 2\eta_4 - 4\eta_3 + 2\eta_2 \quad (6.2)$$

$$C = \dot{\eta}_3 \quad (6.3)$$

$$D = -2\eta_3 + 4\eta_2 - 2\eta_1 \quad (6.4)$$

and equation (4) can be abbreviated to $P_{4FD} = \frac{B}{2\Delta^3} [A \cdot B + C \cdot D]$. Preliminary numerical investigations were done using tonal and random excitation. For the tonal excitation, four frequencies were chosen: Two eigenfrequencies (57 Hz , 620 Hz) and two frequencies between resonances (76 Hz , 676 Hz). These two types of frequencies were used to examine the efficiency with standing waves on the one hand and a higher ratio of travelling waves on the other hand. In order to compare the effectiveness of the methods, a simple control using the velocity at measuring point as error signal was calculated.

As an example, the structural intensity and velocity of the beam elements are shown in figure 4 for the uncontrolled vibration, a velocity control (VEL) and the four point STI control (4FD) at 76 Hz .

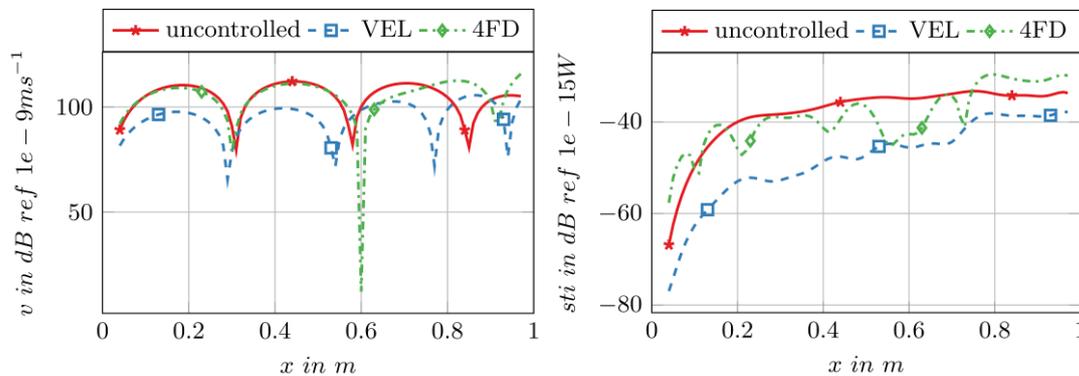


Figure 4 – velocity (left) and STI (right) versus beam position at 76 Hz

The velocity control achieves a nearly total attenuation of velocity at the measuring point, however there is nearly no effect on the downstream structure. This can also be seen in the structural intensity, as there is only a local reduction. Using the STI (or its components respectively) as error signals shows advantages in the global attenuation of travelling waves. The STI itself is reduced by about 10 dB globally, also leading to a global reduction of velocity. The same effect could be observed for the second non-resonant frequency.

In the resonant cases on the other hand, the STI control showed no advantage compared to a one point velocity control, as there is no change in the phase of the beam elements when purely standing waves are observed.

Based on the numerical investigations, a beam structure, clamped on one side, was investigated experimentally. To estimate the difference between the control of standing and travelling waves, two configurations of boundary conditions were set, as seen in figure 5.

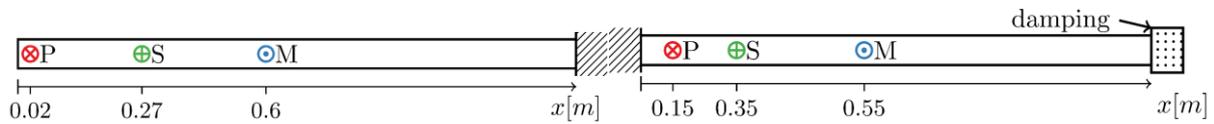


Figure 5 – experimental setup for control of flexural waves, cantilever (left, configuration I) and cantilever with inhomogeneous damping (right, configuration II); disturbance (P), control force (S) and measuring point (M); sensor spacing $\Delta = 2cm$ for all arrays

The second beam setup is terminated in a sand box to achieve inhomogeneous damping and a virtually non-reflecting boundary. According to the pre-investigations, an optimal control was calculated using measured transfer paths and reconstructing intensity time signals. The beam was excited by an electrodynamic shaker. The control force was induced by an inertial shaker. The STI was measured using four PCB 352A24/NC accelerometers, enabling the calculation using the two and four position method with the same sensor spacing. The velocity of the beam along its length was measured with a laser scanning vibrometer (POLYTEC OFV 055), also providing an error signal for the one point velocity control. The beam in configuration I was excited in a resonance frequency (127,5Hz) and a non-resonant frequency (210Hz). Configuration II was excited using a frequency with mainly travelling waves (763,5Hz).

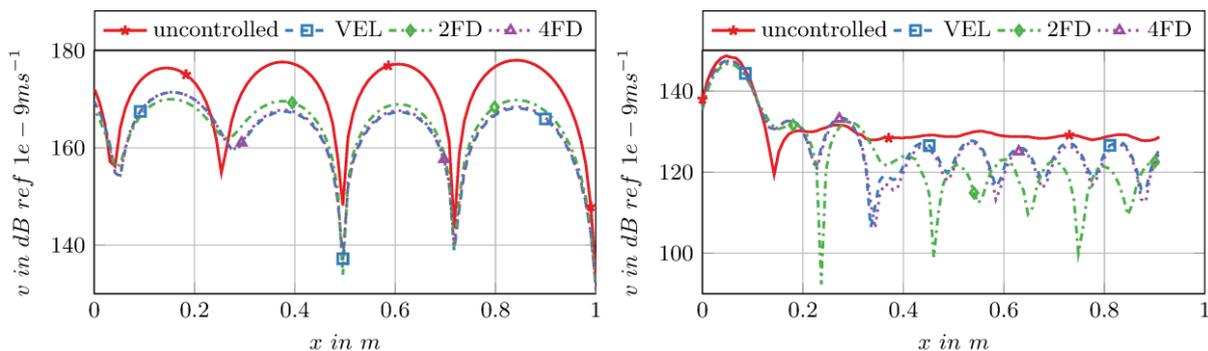


Figure 6 – measured velocity versus beam position for configuration I (left, 127,5Hz) and configuration II (right, 763,5Hz)

For the resonant excitation there is no remarkable benefit using the structural intensity as error signal. A single point velocity control is also able to reduce to overall vibration by about 9dB. When observing the control of travelling waves in the second configuration, again a global reduction of velocity can be achieved by a simple velocity control. A four point intensity control shows no benefit in attenuation. The two point intensity control however shows slightly better results compared to the four point method, probably due to the higher robustness against finite differencing errors.

3.2 Plate

A numerical plate model with inhomogeneously damped boundaries was used to estimate the attenuation of an intensity control using different methods. The aluminum plate with the dimensions 1200mmx1000mmx3mm was modeled in ANSYS® using SHELL281 elements (element length 10mm) with free boundaries. The setup conditions can be obtained from figure 7.

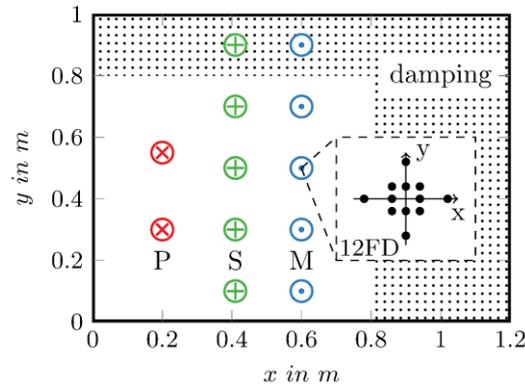


Figure 7 – model of inhomogeneously damped plate for intensity control; disturbance (P), control force (S) and measuring point (M); sensor spacing $\Delta = 2cm$ for all arrays

The disturbance is modeled as single point forces, the secondary forces are independently driven force pairs. The measuring points represent the locations of virtual measuring arrays. For this research, four measuring concepts were chosen:

- velocity control (single point per array)
- two point STI control (x direction)
- four point STI control (x direction)
- twelve point intensity control (two dimensional)

The velocity control is again used for comparison. The two and four point method are used under the assumption that firstly the energy flow in y-direction does not contribute to vibrations downstream the plate and secondly twisting moments can be omitted, so the plate is represented by beam segments. The twelve point method is derived from (2), virtually combining the two eight sensor arrays for the calculation of the additional twisting moments. Any of the above arrays is used at five evenly spaces measuring points at $x = 0,6m$.

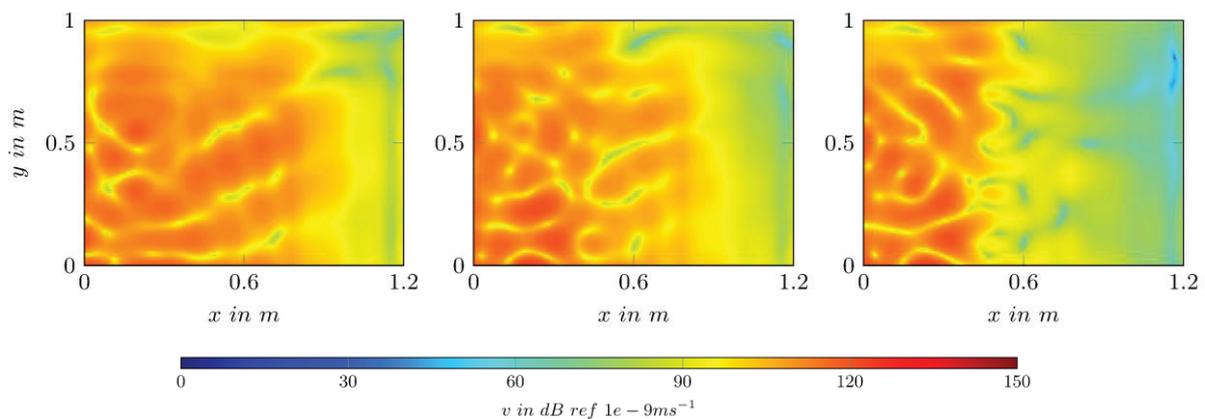


Figure 8 – numerical velocity distribution of inhomogeneously damped plate at $792Hz$; from left to right: uncontrolled reference, velocity control, 12 point intensity control

Figure 8 shows the velocity distribution for a simple velocity control and, exemplarily, a twelve point structural intensity control in the corresponding measuring points in comparison to the uncontrolled reference. A tonal excitation with $792Hz$ has been chosen, which occurred to be a frequency with mainly travelling waves. A velocity control leads to a merely local reduction of velocity in the corresponding measuring points, but only to a comparably smaller global reduction in the barrier area at $x > 0,6m$. The intensity control on the other hand reduces the velocity globally.

Using the 2D intensity as error signal, the root mean square reduction of velocity in the barrier area is -30dB , whilst using a velocity control the achievable attenuation is -18db . Further investigations using different frequencies have shown a higher performance in terms of global velocity attenuation for mainly travelling waves. As well as in the one dimensional case, an intensity control shows no advantage for resonant excitation. Furthermore, simplifying the plate to beam segments using four or two sensors per measuring point appeared to be less performant. However, noise related errors have not been taken into consideration yet, so the performance of the twelve point intensity control is expected to be less in practice.

4. CONCLUSIONS

The experimental investigation towards the active control of structural intensity on a beam partially confirmed the preliminary numerical calculations. There is a small advantage in global attenuation of vibration for systems with dominant travelling waves, if the structural intensity is used as a control target. A significant improvement compared to a single point velocity control could not be shown clearly, especially due to the investigated error sensitivity of the four point method. Generally a global attenuation could also be achieved using a single velocity sensor.

The numerical results of an actively controlled inhomogeneously damped plate could also show a reduction in energy transmission and hence a global reduction of velocity in the barrier area, especially for travelling waves. A velocity control on the other hand could only achieve a local reduction. Further investigations have also shown the importance of measuring the energy components contributed by twisting moments. In order to make a practical implementation feasible, especially regarding robustness with respect to phase errors and noise, simplifications have to be taken into account.

Subsequently to this research, the measurement of structural intensity using finite differencing schemes shall be evaluated experimentally. In this case, especially the influence of sensor spacing in regards to measuring noise shall be investigated numerically and experimentally. Furthermore these concepts have to be applied for more complex structures (supported, curved). For an active barrier for structure borne sound, further investigations towards sensor and actuator placing and numbers have to be carried out.

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