

## Development of energy propagation analysis methods for low-frequency phenomena at BMW

Peter GROBA<sup>(1)</sup>, Johannes EBERT<sup>(1)</sup>, Torsten STOEWER<sup>(1)</sup>, Joachim BÖS<sup>(2)</sup>, Tobias MELZ<sup>(2)</sup>

<sup>(1)</sup>BMW Group, Department of Structural Dynamics, Structure-borne Sound and Vibrations, Germany, peter.groba@bmw.de

<sup>(2)</sup>TU Darmstadt, Research group System Reliability, Adaptive Structures, and Machine Acoustics SAM, Germany,  
boes@sam.tu-darmstadt.de

### ABSTRACT

In recent years, a rise in computing power and an enhanced understanding of energy propagation processes have led to an increased use of the numerical simulation of energetic quantities in vibroacoustical problems both including and excluding the coupled fluids. These energy-based quantities have proven themselves very useful in delivering information on the behavior of vibrating structures.

Over the last years the BMW Group has invested in research on energetic quantities to evaluate their potential to support the acoustic engineer in the early stages of the vehicle development process. This paper presents an overview of the development regarding the structural intensity (STI) and its usage as a tool for the analysis of low-frequency energy phenomena. It focuses on car structures in steady state vibration with and without fluid coupling.

Starting with the work of STOEWER, who conducted numerical simulations in the time-domain to perform design modifications, the overview continues with analyses conducted by EBERT who linked structure-borne with air-borne energy propagation. The paper closes by introducing the work of GROBA who analyzed energy propagation using power-based quantities calculated from the STI, allowing for a simplified quantification and description of energy propagation phenomena.

Keywords: Energy Flow, Power Balance, Structural Intensity, Vibratory Energy

### 1 INTRODUCTION

The car manufacturer's efforts to achieve an optimized design regarding noise, vibration, and harshness (NVH) of their products lead to a broad variety of approaches to understand a vehicle's vibrational behavior. The used methods include the estimation of local and global stiffness and the identification of global dynamics via modal analysis, transfer path analysis, laser-scanning vibrometry, use of acoustic cameras and so forth. While these methods deliver information on the structures' properties as well as a specific vibration behavior, they hold no information about the mechanisms that lead to the occurrence of a particular vibration phenomenon resulting from an existing excitation.

This paper focuses on the analysis of energy propagation based on acoustic intensities. While analyzing energy fields in structures via their mathematical representation by means of vector fields it is necessary to be aware of the topic's complexity. Though the information density is one of the major advantages of energetic examinations, it can be a disadvantage as well. The user faces the following difficulties:

- vector fields deliver information on the absolute value and the direction of a given quantity for every point of the field, i.e. the structure,
- within the vibrational energy flow in structures exists a distinction between flexural and longitudinal components,
- the high modal density in a given frequency band for structures with a high number of degrees of freedom (DOF),

- the intrinsic, complex patterns of vibration energy fields make their assessment time consuming and design modifications are difficult to compare against one another and
- so far no consistent relations between design modifications and their influence on the acoustic intensities could be specified.

The present research projects aim to achieve a better understanding of the acoustic chain of effects, as depicted in Figure 1. Most of the current methods used to describe the acoustic behavior of a structure solely connect different points of the acoustic chain without delivering further information about the processes that lead to the establishment of a specific vibration state. In order to gain further insight into the processes that lead to an acoustic phenomenon in a structure the introduced research aims at finding the links from the excitation (that ultimately leads to a noticeable sound event) to the coupling points between subsystems and finally to the recipient. The chosen tools for all analysis derived from the works of STOEWER (1), EBERT (2; 3; 4) and GROBA (5; 6; 7) are the acoustic intensities, i.e the structural intensity (STI) in solids and the sound intensity (SI) in fluids, respectively. They are introduced in Section 2.

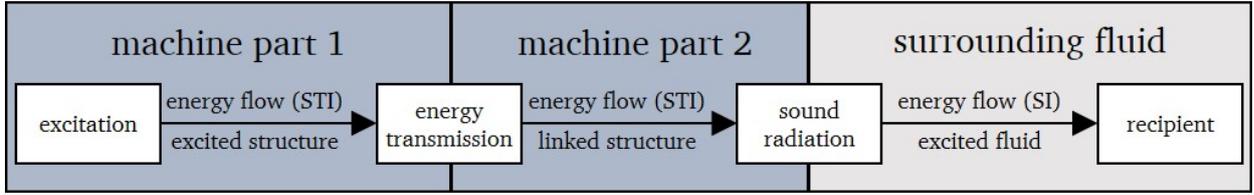


Figure 1. Acoustic chain of effects, from structural excitation to noise input at the recipient's position

## 2 ENERGY FLOW

The first law of thermodynamics is the starting point for energy conservation and propagation analysis. For an elastic medium in a closed system and negligible heat transfer in a steady-state, i.e. with constant energy density, the first law of thermodynamics yields the relation

$$\iint_{A_s} \mathbf{I}_s(f) \mathbf{n}_s dA = P_{s,in}(f) - P_{s,diss}(f) \quad (1)$$

between the structural intensity,  $\mathbf{I}_s$ , and the input power,  $P_{s,in}$ , as well as the dissipated power,  $P_{s,diss}$ , (8; 4). Equation (1) does not take a connected fluid into account, which would have the effect of an additional damping on the structure. In this case it would be necessary to extend Equation (1) by a fluid related term (4). In steady-state dynamics it is feasible that all field quantities are described in the frequency and thus the complex domain. The active part of the complex intensity, in the frequency domain,

$$\mathbf{I}_{s,a}(f) = \text{Re}(\mathbf{I}_s(f)) = \langle \mathbf{I}_s(t) \rangle = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{t=0}^T \mathbf{I}_s(t) dt \quad (2)$$

is related to the time-averaged value of the STI in the time domain,  $\langle \mathbf{I}_s(t) \rangle$ . For three dimensional structures in steady-state, the cross-spectral density of the complex stress tensor,  $\underline{\mathbf{S}}$ , and the conjugate of the complex velocity vector,  $\underline{\mathbf{v}}^*$ , yields the complex STI

$$\mathbf{I}_s(f) = -\frac{1}{2} \underline{\mathbf{S}}(f) \cdot \underline{\mathbf{v}}^*(f). \quad (3)$$

In thin-walled structures the complex STI in shell elements,  $\underline{\mathbf{I}}'(f)$ , is calculated by means of section forces  $\underline{N}$ ,  $\underline{Q}$  and section moments  $\underline{M}$  and their multiplication with the respective translational and rotational velocities  $\underline{v}$ ,  $\underline{\phi}$  (8)

$$\underline{\mathbf{I}}'(f) = -\frac{1}{2} \begin{bmatrix} N_x v_x^* + N_{xy} v_y^* + M_x \phi_y^* - M_{xy} \phi_x^* + Q_x v_z^* \\ N_y v_y^* + N_{yx} v_x^* - M_y \phi_x^* + M_{yx} \phi_y^* + Q_y v_z^* \end{bmatrix}. \quad (4)$$

The connection between Equation (3) and (4) is given through the integration of the STI in the local  $z$ -direction

$$\underline{\mathbf{I}}'(f) = \int_{-\frac{h}{2}}^{\frac{h}{2}} \mathbf{I}_s(f) dz. \quad (5)$$

This yields the energy flow over the thickness,  $h$ , of the plate. A simplification is made for thin-walled structures. Under the assumption of a negligible energy flow in the  $z$ -direction the two dimensional problem in Equation (4) is obtained.

In order to conduct the analogous calculation of the energy flow in a structure the complex pressure,  $\underline{p}$ , is linked to the complex sound particle velocity,  $\underline{v}$ , as follows:

$$\underline{I}_f(f) = \frac{1}{2} \underline{p}(f) \cdot \underline{v}^*(f). \quad (6)$$

Using Equation (6), the sound intensity (SI) is obtained, which constitutes the equivalent quantity to the STI in a fluid.

### 3 ENERGY FLOW IN VEHICLE STRUCTURES

STOEWER evaluates whether or not the STI can be used as a tool in an effective structural design process by assessing the energetic transfer paths from excitation to radiation, i.e. the energy flow in the machine parts as depicted in Figure 1. In (1) the analysis of the STI is conducted in time and frequency domain. Through his assessment of the energy flow in the time domain STOEWER shows that the inclusion of in-plane-waves delivers further information on the energy propagation in structures. He is able to explain the occurrence of out-of-plane vibrations at a structure's discontinuities in a measurement set up, even before the initial wave is able to propagate into the specified region. An additional simulation verifies that the in-plane-waves transport energy, which is converted into out-of-plane waves at the discontinuity and thus create a 'new' vibration source. Via examinations of test structures STOEWER shows the various benefits of analysis conducted in time and frequency domain, herein either for singular frequencies or frequency ranges.

In a next step, a more realistic structure is analyzed. Figure 2 shows the result for the energy flow in three different set-ups for the excitation frequency of 120Hz. The structure is excited on its right hand side, as indicated by the red dot. This analysis' aim is the reduction of the vibration on the left side via suppression of the energy flow over the tunnel. By fixating all translatory DOFs and the in-plane DOFs of the marked nodes (pink) in Figure 2 b) and c), respectively, it is possible to reduce the magnitude of the displacement on the left side by a factor of 2 and 100, respectively. The changes have been chosen to affect the paths with the highest energy density. However, while it is viable to completely block the energy from propagating to the left side for state c) (connected to an higher magnitude of the vibration on the ride side) the energy chooses a new path in case of modification b). This is a general observation when manipulating energy paths. Although it is possible to efficiently influence the energy flow by imposing an impedance discontinuity, the energy often finds new paths to propagate on, thus making the effect of a structural change difficult to predict.

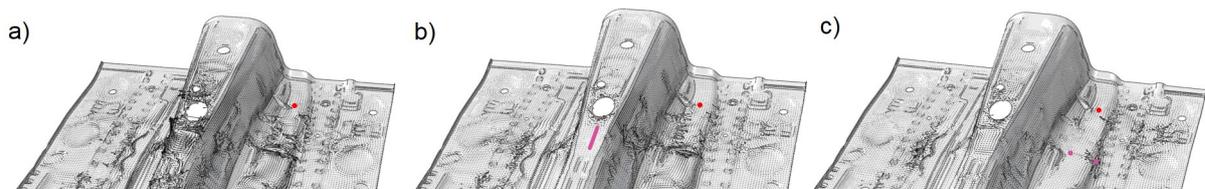


Figure 2. Underbody panel in different states at an excitation frequency of 120Hz: a) original state, fixed nodes at b) top side of tunnel and c) at right hand side of the structure; excitation marked by red dot, fixed nodes via pink dots (1)

Via analysis of the depicted body in white in Figure 3 and a number of variations the following observations are derived:

- components with high, homogeneous stiffness tend to be the main paths of the energy flow,
- increased stiffness in energy conducting components lead to an increased energy density at the interior confining panels, i.e. the roof, windows, underbody and so forth, whereas increased stiffness of the confining panels leads to a decreased energy density and
- the absolute value of the STI is not related to the structural sound level.

In the presented case the highest reduction of the structural sound level is obtained via increasing the thickness of the car's roof as shown in Figure 3, which constitutes an increased stiffness at an interior confining

panel. STOEWER concludes that the STI is a valuable quantity in describing the energetic behavior of complex structures and in understanding the formation of a certain state energy-wise. However, the effect of a certain design variation on the energy flow has proven to be predictable with difficulties and even then, since there is no relationship between the STI and the sound level, it is difficult to evaluate such a measure in terms of its acoustic impact.

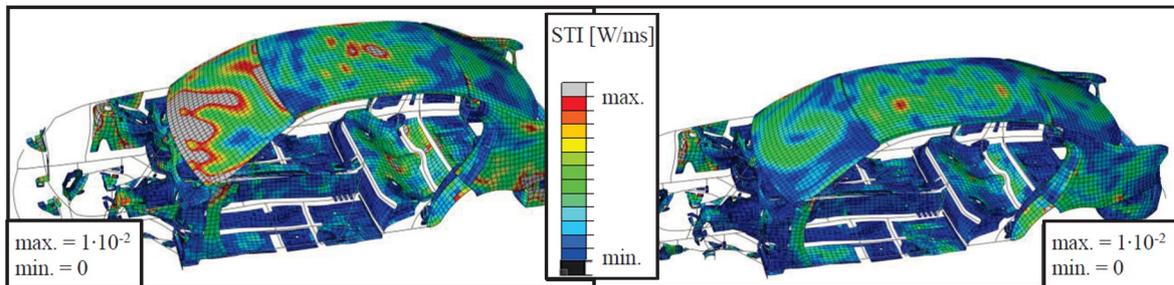


Figure 3. Energy distribution of a body in white and a modified version with increased thickness of the roof (right side) (1)

#### 4 INSIGHTS INTO FLUID-STRUCTURE INTERACTION

EBERT starts his considerations by tackling the matter of time and memory requirements necessary for energy flow calculations. Tied to frequency band analysis conducted by STOEWER, an approach is introduced to reduce the number of frequencies necessary to adequately describe a frequency band solution of a structure. Therefore, in (2) three frequency selection criteria are derived and compared. The most reliable criterion is a local selection criterion based on the excitability of the structure. With this, it is possible to show that in order to accurately describe the energy flow in a complex structure in a given frequency band, only 10% of the contained resonance frequencies need to be superimposed. One conclusion is, that the greater the complexity of a structure, the less the number of frequencies necessary for the calculation of the frequency band solution.

EBERT extends the research on energy flow by taking the surrounding fluid into account in order to create a more holistic understanding of the acoustic chain (Figure 1). For a simple coupled system as depicted in Figure 4 he concludes, that the wave propagation depends mainly on the dominant subsystem. In the upper pictures the system is dominated by the behavior of the structure. In these cases a closed loop of active energy exists and a transport of energy between the structure and the fluid and vice versa takes place, i.e. the fluid establishes a connection between different parts of the structure and contributes to their active energy supply. In the pictures in the bottom row, showing the energy flow for the sixth excitation frequency, the system's behavior is dominated by the fluid. The energy flow in the fluid clearly varies from the energy flow in the structure. There is no clear connection between either the SI in the coupling surface and the STI or the SI in the whole fluid and the STI. The structure is nonetheless responsible for the energy supply of the fluid, but the re transfer from fluid to structure cannot be detected.

Combining the ideas behind the approaches in (2) and (3), the efficient handling of energy flow simulation and the description of fluid-structure interaction, EBERT develops a method for determining the main supply path of energy for a given problem in (4). In this approach the energy flow is analyzed in a way that allows for the estimation of the most important frequency in terms of energy supply for the sound radiation in a target region. This idea relates to the well known principle that it is most efficient to tackle the highest peaks in a sum spectrum. Here, the aim is to reduce or eliminate the main energy path between excitation and energy radiation by applying local design changes. The first steps of the approach are depicted in Figure 5. For the given problem that consists of a plate and a coupled fluid it is estimated that the 11th and the 55th frequency are the main contributors in terms of energy supply to the sound radiation in the system. By deriving a structural design variation based on the energy flow it is possible to reduce the STI by a factor of 30 and the SI at the coupling surface by a factor of 87.

The approach aims at reducing the energy flow between structure and fluid, which results in reduced energy and sound radiation. An occurring problems is the complex energy flow. The energy tends to split up into several sub-paths, thus making the effective application of structural modifications complex. Tackling only one sub-path will lead to a shift in the energy flow towards other sub-paths making the outcome of a design modification hardly predictable.

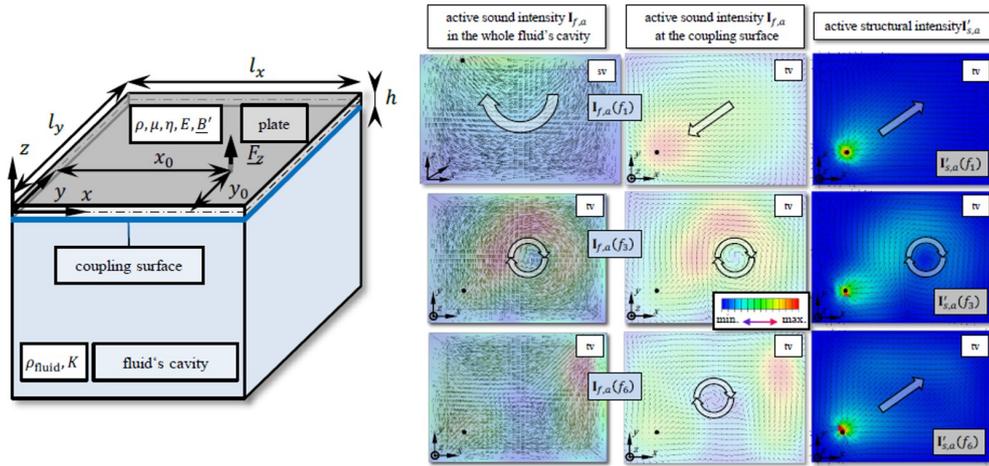


Figure 4. Coupled system consisting of a plate and a fluid cavity and the respective energy flows for various resonance frequencies of the system as viewed from the top (tv) and the side (sv) (3)

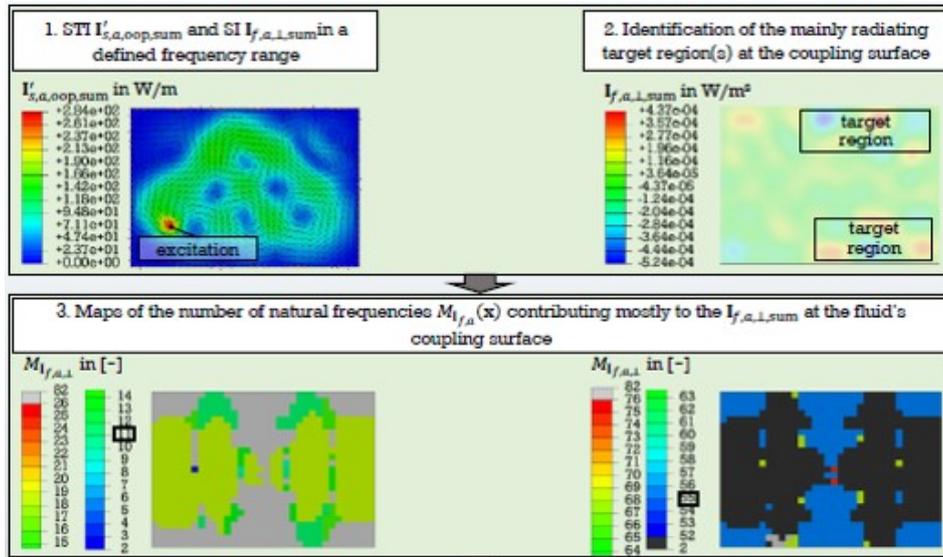


Figure 5. Analysis steps to identify the main contributing resonance frequencies to the energy flow between a structure and the coupled fluid (4)

## 5 Energy propagation analysis by means of power balance calculations

Thus far, the analyses conducted have concerned the energy flow between linking points in the chain of effects (see Figure 1). To gain further insight into the energetic behavior of structures a method to quantify the energy transmission at the linking points (excitation, energy transmission and sound radiation) is derived. (5) introduces an approach to identify whether a linking point is an energy source or sink based on the divergence of the STI's active part

$$\text{div}(\mathbf{I}_s) = \nabla \cdot \mathbf{I}_s = \frac{\partial}{\partial x_i} I_i. \quad (7)$$

For a quantitative description of the energy transmission it is necessary to derive scalar quantities. This is done by calculating power balance values based on the first law of thermodynamics. Therefore, in (6) it is shown that the integration of the STI's divergence in shell elements

$$\iint_{A_s} \text{div}(\mathbf{I}(f)) dA = P_{s,\text{in}}(f) - P_{s,\text{diss}}(f) \quad (8)$$

is equal to the integration of the STI in Equation (1). In a first series of analyses different calculation approaches have been tested and compared with one another. An example is depicted in Figure 6. For a simply

supported rectangular plate a surface area has been defined so that the smallest area has twice the size of two elements adjoining the concentrated nodal force. The largest area is when the balance area reaches the boundaries of the structure. At the right side of the picture the curve progression of the power level over the balanced surface is given as a direct integration of the STI for each element in the defined area, the calculation of the divergence from the STI followed by the surface integration, and a reference solution calculated via the volume integration of the strain energy density obtained from the solver itself. The tests show that in the direct vicinity of the concentrated load the power values obtained based on the STI either over or underestimate the real power value. This changes with growing distance to the excitation force, where the solutions reach comparable values for each of the methods. The comparison with the reference value shows a difference of 0.2dB and is negligible.

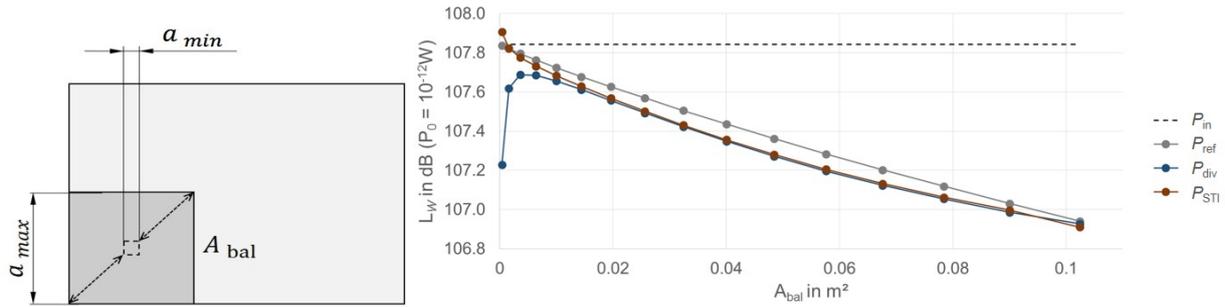


Figure 6. Simply supported rectangular plate and curve progression of estimated input power values for different calculation approaches over balanced surface area, power values are averaged over all occurring eigenfrequencies in a frequency band from 0 to 1000Hz

Via the analysis of test structures such as the rectangular plate, it is shown that the power calculations based on acoustic intensities deliver consistent results. Furthermore, this is valid for the calculation of power balances at the boundaries of a specified region, showing that it is not always necessary to know the behavior of an entire enclosed area. This is an important outcome because it shows that the description of the coupling points in the acoustic chain of effects delivers valuable information about the whole system. In order to show the usefulness of the introduced approach a further quantity, the effective radiated power (ERP), is introduced (1):

$$\text{ERP}(f) = \rho_a c_a \overline{S \bar{v}^2(f)}. \quad (9)$$

The ERP is a widely used quantity since it delivers an approximation for the sound radiation of a structure. As already stated, there are (so far) no known correlations between the STI and the ERP. Figure 7 shows the result of an analysis on the behavior of the ERP and power balance values. The structure resembles the rectangular plate depicted in Figure 6 containing gaps to achieve a more concentrated energy flow. The structure is excited by a concentrated load in the area marked by 'exc'. The ERP values are calculated for the overall structure, the input power,  $P$ , for the excitation area and both values for the right side as the target area. The power values have been calculated by integrating the STI at the boundaries of the excitation area (marked by the dark blue line) and the boundary connecting the left and right side (marked by the light blue lines). The results show that the curve progression over the eigenmodes is similar for all quantities. Furthermore, both values depict that the sound radiation of the analyzed structure is dominated by the right side, since the differences between  $\text{ERP}_{\text{all}}$  and  $\text{ERP}_{\text{right}}$  as well as  $P_{\text{exc}}$  and  $P_{\text{right}}$  are small.

This analysis is also applied to more complex structures. Figure 8 represents the power distributions in a car's underbody panel as the mean level for all eigenmodes in a range between 0 and 150Hz. The power balance values,  $P_{\text{bal}}$ , are calculated at the boundaries between each region. The ERP and  $P_{\text{bal}}$  values are normalized to the overall structure and input power values, respectively. The results show that both quantities predict the same normalized values for each of the identified areas within the structure. The difference between the two quantities results from the inclusion of the in-plane power in the  $P_{\text{bal}}$  value calculation.

The presented examples show that both, ERP and  $P_{\text{bal}}$  deliver comparable results regarding the radiation behavior of defined areas in a structure. This principle can be extended to more complex structures, in that the radiation tendencies of the parts within those structures can be compared solely by describing the power balance relations at the part boundaries, and to the evaluation of structural changes based indirectly on the energy flow by deriving power quantities.

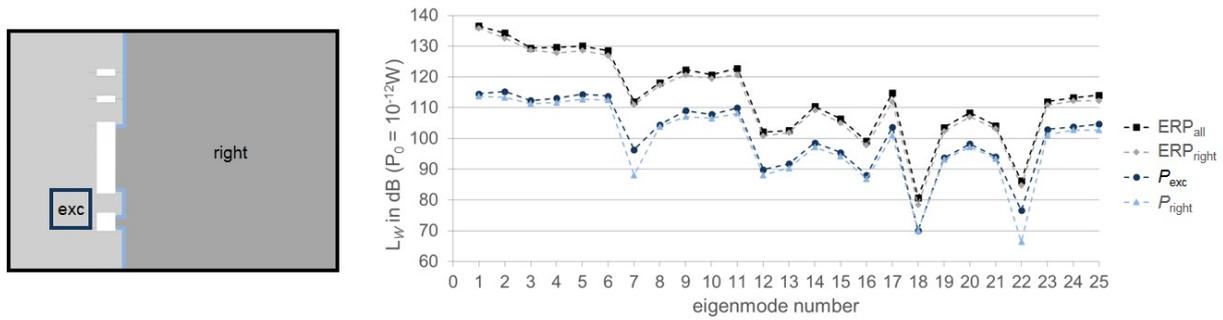


Figure 7. Test structure and results for the ERP and power balance for the first 25 eigenmodes of the structure

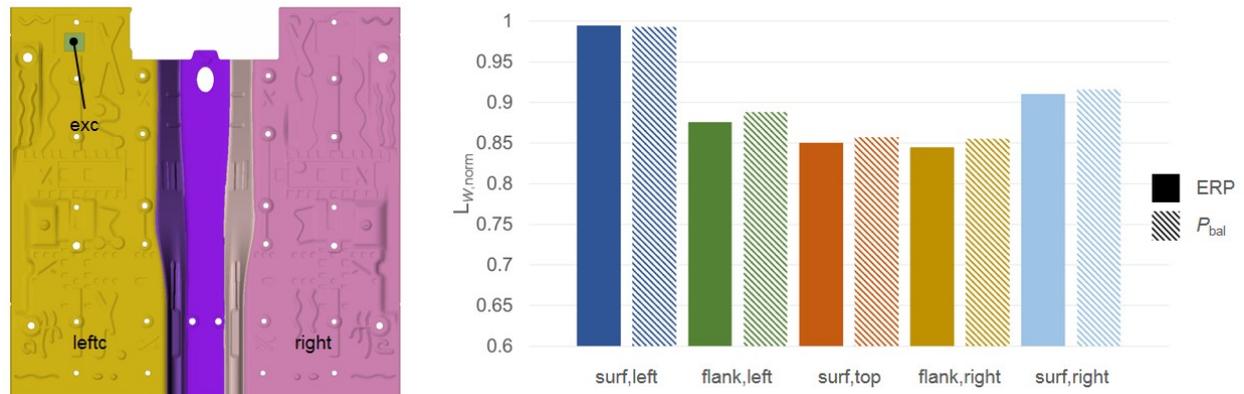


Figure 8. Underbody panel divided into balance regions (color reproduction) and their classification in terms of normalized power values

## 6 CONCLUSIONS

The presented research cases show that analyzing the STI results in an improved understanding of the acoustic chain of effects since it delivers precise information on the energy propagation and the mechanisms that lead to the occurrence of an acoustic phenomenon. There are ways to influence the energy propagation in structures and design variations have shown to be more effective when positioned in areas with a high energy density. The introduced approaches show methods that make the calculation of intensity values more efficient and the assessment of the results less time consuming. Furthermore, since there are now methods focusing either on the estimation of energy paths in a structure or the quantification of energy transmission in terms of singular power values, the user is able to choose the most feasible method for their purpose. Regarding the goal to achieve a better understanding of the energetic chain of effects, the introduced methods are feasible to describe certain points in the chain.

Future research should concern the sound radiation, which already can be estimated by power balance values of the structure, and its link to the power input at the recipients position. Here an improved understanding is achievable when comparing the ratio of the dissipated power of the structure with the energy input into the fluid in order to identify the area where a fluid is inclined to absorb the energy. Furthermore for complex systems, where energy is always given the possibility to choose new paths to flow through, the outcome of design changes is not accurately predictable when design variations are manually introduced. In these cases it is necessary to apply algorithm based approaches that reduce the power input in the sub-structures, i.e. optimization based on the power input as target value, to further increase the ability to handle the complexity of energy propagation phenomena.

## REFERENCES

1. Stoewer, T. Berechnung der Strukturintensität von Fahrzeugstrukturen. PhD thesis, Technische Universität Darmstadt (Germany), 2016.
2. Ebert, J.; Stoewer, T.; Schaal, C.; Bös, J.; Melz, T. Efficient simulation of the active vibratory energy flow

- of structures in a given frequency band. Proceedings of ISMA 2014 International Conference on Noise and Vibration Engineering, Leuven (Belgium), September 15-17, 2014. pp 3663–3676.
3. Ebert, J.; Stoewer, T.; Schaal, C.; Bös, J.; Melz, T. NVH development of car structures by means of numerical simulation of energetic quantities. Proceedings of AAC 2014 Aachen Acoustic Colloquium, Aachen (Germany), November 24-26, 2015. pp 117–126.
  4. Ebert, J.; Stoewer, T.; Schaal, C.; Bös, J.; Melz, T. Opportunities and limitations on vibro-acoustic design of vehicle structures by means of energy flow-based numerical simulations. Proceedings of NOVEM 2015 Noise and Vibration - Emerging Technologies, Dubrovnik (Croatia), April 13-15, 2015. pp 49798-1-49798-16.
  5. Groba, P.; Ebert, J.; Stoewer, T.; Schaal, C.; Bös, J.; Melz, T. Detection of energy sinks and sources in the active vibratory energy field of coupled structures and systems with fluid-structure interaction. Proceedings of ISMA 2016 International Conference on Noise and Vibration Engineering, Leuven (Belgium), September 19-21, 2016. pp 3945–3956.
  6. Groba, P.; Ebert, J.; Stoewer, C.; Bös, J.; Melz, T. Analysis of the energy exchange between coupled sub-systems by means of the divergence in the vibratory energy field. Proceedings of Fortschritte der Akustik - DAGA 2017 - 43. Deutsche Jahrestagung für Akustik, Kiel (Germany), March 06-09, 2017. pp 371-374.
  7. Groba, P.; Ebert, J.; Stoewer, C.; Bös, J.; Melz, T. Estimation of power input and output of structures using structural intensity measurements. Proceedings of Fortschritte der Akustik - DAGA 2018 - 44. Deutsche Jahrestagung für Akustik, Munich (Germany), March 19-22, 2018. pp 1231-1234.
  8. Hering, T. Strukturintensitätsanalyse als Werkzeug der Maschinenakustik. PhD thesis, Technische Universität Darmstadt (Germany), 2012.