PROCEEDINGS of the
23rd International Congress on Acoustics

9 to 13 September 2019 in Aachen, Germany

Rotor Dynamics Analysis under Uncertainty in Lubricant Film

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ABSTRACT

Predicting vibrations in rotor dynamics under the impact of uncertainty is still a challenging issue. Research work published in this area is usually focused on the stochastic dynamics of rotor- bearing systems by considering the stiffness and damping coefficients of simplified bearing models as random parameters rather than the original characteristics of the oil film. In this paper, the characterizing parameters of the lubricant film such as the dynamic viscosity are considered as uncertain parameters to investigate their influences on the rotor dynamics response. To this end, the modified Jeffcott rotor model with offset disc is established considering the gyroscopic effect. At the same time, the parameters of the bearing are obtained by solving the dynamic characteristics of oil film. Then, the dynamic response of the rotor is numerically calculated. In order to investigate the influence of the parameters of the lubricant film, the response is modeled as a stochastic process with unknown coefficients by using the generalized polynomial chaos expansion. The coefficients are then determined from some realizations of the response on a set of collocation points. Finally, the accuracy of the method is compared with Monte Carlo Simulation.

Keywords: Uncertainty quantification, rotor dynamics, lubricant film

1. INTRODUCTION

In recent years, rotor dynamics problem has drawn a wide attention due to the constant increasing application of rotating machine in industry. As a consequence, numerous investigations about rotor dynamics have appeared, and most of them showed a great significance on improving the reliability of rotating machine.

Although many research work have been done for rotor dynamics, for example, the critical speed, unbalance response and even some nonlinear behavior of rotor have been investigated (1, 2), most of these researches are under deterministic analysis. However. there are some uncertainties in rotor system, such as the geometry parameters, material properties, and even some parameters in the related parts like bearing. These uncertain parameters can directly affect dynamics behavior, such as changing the critical speed which can affect the efficiency of rotating machine. Furthermore, some uncertain parameters can affect the stability problem, for instance, the oil whirl and whip phenomenon which can affect the stability of rotor may occur when considering the influence of oil-film bearing. Since the great influence on rotor dynamics caused by uncertainty has been commonly recognized, some research related to this problem has been done. For example, Sepahvand investigated the influence of random geometrical parameters of bladed disc on natural frequency of rotor system (3); Sinou chose the young modulus and stiffness of shaft to study their effect on dynamic response of rotor (4); Ma viewed the stiffness, damping coefficients of support as random parameters and studied their influence on dynamic response of rotor (5); Fu observed the influence of uncertainties from unbalance magnitude, crack depth, Young's modulus and bearing stiffness on dynamic response of a crack rotor (6). Although many work about rotor dynamics with uncertainty have been done, most of them only considered the random parameters of the rotor itself and ignored the influence of the connecting components. Even if someone considered the random parameters of connected components, such as Ma and Fu considered the parameters of support, but it should be noted that these parameters like stiffness in each direction cannot be changed independently in reality. Therefore, stochastic rotor dynamics considering the original properties of connecting parts, like viscosity of oil-film bearing, is still challenging.

As for the methods used for uncertainty problems, Monte Carlo (MC) method, a kind of sampling method, is widely used in many engineering problems. MC method can reach a better accuracy especially with more

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samples, but it usually causes a high computational cost. In order to deal with this problem, a non-sampling method called the generalized Polynomial Chaos (gPC) expansion has been introduced to address the uncertain problem (7, 8). Lots of research show that, compared with the traditional sampling-based method such as MC method, gPC expansion is more efficient.

In this paper, the random parameters of short oil-film bearing are considered to investigate their influence on rotor dynamics based on gPC expansion. Rotor-bearing model is explained in section 2, and the gPC expansion method is introduced in section 3, lastly, the numerical analysis is presented in section 4.

2. ROTOR-BEARING MODEL

2.1 Rotor Model

Jeffcott rotor with offset disc is usually described as a model including a massless shaft and a rigid disc, and its motion can be written as follows:

$$\mathbf{M}\ddot{\mathbf{U}} + (\mathbf{C} + \omega\mathbf{G})\dot{\mathbf{U}} + \mathbf{K}\mathbf{U} = \mathbf{F_f} \tag{1}$$

Where **M**, **C**, **G**, **K** are global mass, damping, gyroscopic and stiffness matrices; $\ddot{\mathbf{U}}, \dot{\mathbf{U}}, \mathbf{U}$ are acceleration, velocity and displacement vector, respectively; and $\mathbf{F_f}$ is external force, which include the unbalance force vector.

The natural frequencies and the response of the system can be calculated by assuming the solution of **U** as the form of $\mathbf{U} = \mathbf{U}_0 e^{\lambda t}$. In addition, since the rotor is supported by bearings, the dynamics characteristics of the rotor system will be affected by the properties of bearing. Therefore, the bearing is usually taken into consideration of rotor dynamics.

2.2 Bearing Model

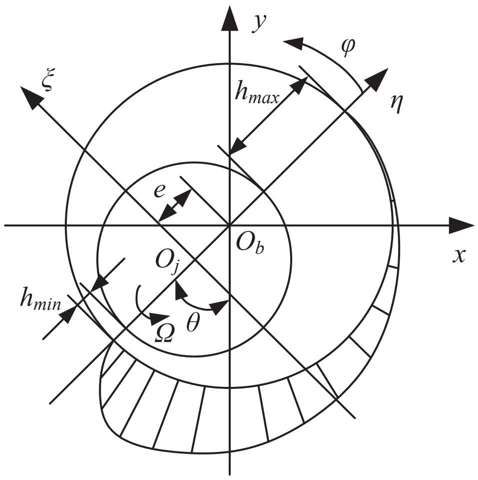

Figure 1 –Scheme of a hydrodynamic bearing

The scheme of hydrodynamic bearing can be seen in Figure 1. It can be seen that the thickness of oil-film h is a function of eccentricity e:

$$h = c + e\cos(\varphi) \tag{2}$$

Where c is the clearance between journal and bearing shell, and φ represent the coordinate in cylindrical direction.

Reynolds' Equation of cylindrical oil-film bearing can be written as:

$$\frac{1}{R^2}\frac{\partial}{\partial\varphi}\left(h^3\frac{\partial p}{\partial\varphi}\right) + \frac{\partial}{\partial z}\left(h^3\frac{\partial p}{\partial z}\right) = 6\mu\left[\Omega\frac{\partial h}{\partial\varphi} + 2\left(\dot{e}\cos\varphi + e\dot{\theta}\sin\varphi\right)\right] \tag{3}$$

Where R is radius, p is pressure, μ is viscosity, Ω is rotating speed. For a relative short bearing, the first term in the left side of Eq. (3) can be ignored (9). Thus, the analytical solution of oil-film pressure can be described by integration considering the boundary condition:

$$p = -\frac{3}{2}(\frac{l}{D})^2\frac{\varepsilon\sin\varphi}{(1+\varepsilon\cos\varphi)^3}\left(z^2 - 1\right) \tag{4}$$

In which ε is the ratio of eccentricity to clearance. By integrating Eq. (4) on the length direction, the oil-film force can be obtained. And then, this force can be expressed as the form of vertical and horizontal force applied to shaft:

$$F_x = F_\varepsilon \cos\theta - F_\theta \sin\theta \tag{5}$$

$$F_y = F_\varepsilon \sin\theta + F_\theta \cos\theta \tag{6}$$

$$F_\varepsilon = F_\eta \left[\left(1 - \frac{2\dot{\theta}}{\Omega}\right) \frac{2\varepsilon^2}{\left(1-\varepsilon^2\right)^2} + \pi \frac{\dot{\varepsilon}}{\Omega} \frac{1+2\varepsilon^2}{\left(1-\varepsilon^2\right)^{5/2}} \right] \tag{7}$$

$$F_\theta = F_\eta \left[-\frac{\pi}{2} \left(1 - \frac{2\dot{\theta}}{\Omega}\right) \frac{\varepsilon}{\left(1-\varepsilon^2\right)^{3/2}} - \frac{\dot{\varepsilon}}{\Omega} \frac{4\varepsilon}{\left(1-\varepsilon^2\right)^2} \right] \tag{8}$$

The journal bearing connects the support structure and rotating shaft, and it is usually simplified as a model with stiffness and damping coefficients, as shown in Figure 2. The stiffness and damping coefficients of bearing in this model can be obtained by partially differentiating the vertical and horizontal oil-film forces by displacements (x, y) and velocities $(\dot{x}, \dot{y})$:

$$k_{ij} = \frac{\partial F_i}{\partial j} = \frac{\partial F_i}{\partial \varepsilon} \frac{\partial \varepsilon}{\partial j} + \frac{\partial F_i}{\partial \theta} \frac{\partial \theta}{\partial j} \qquad (i, j = x, y) \tag{9}$$

$$d_{ij} = \frac{\partial F_i}{\partial j} = \frac{\partial F_i}{\partial \dot{\varepsilon}} \frac{\partial \dot{\varepsilon}}{\partial j} + \frac{\partial F_i}{\partial \dot{\theta}} \frac{\partial \dot{\theta}}{\partial j} \qquad (i, j = \dot{x}, \dot{y}) \tag{10}$$

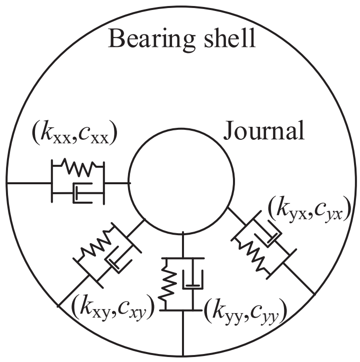

Figure 2 – Simplified model of journal bearing

For the global stiffness matrix **K** in Eq. (1), it only includes the stiffness of rotating shaft when using rigid supports, but when considering the support as flexible one, this stiffness matrix will be reconstructed by adding the parameters of supports (10).

3. UNCERTAINTY ANALYSIS

3.1 Polynomial Chaos Expansion Based on Collocation Method

The basic idea of gPC expansion is to project the variables of system onto a stochastic space spanned by a set of complete orthogonal polynomials which are the function of random variable, for example, an uncertain parameter χ: $\Omega \rightarrow \mathbb{R}$ can be represented as:

$$\chi = x_0 \Psi_0 + \sum_{i_1=1}^{\infty} x_{i_1} \Psi_1(\xi_{i_1}) + \sum_{i_1=1}^{\infty} \sum_{i_2=1}^{i_1} x_{i_1 i_2} \Psi_2(\xi_{i_1}, \xi_{i_2}) + \sum_{i_1=1}^{\infty} \sum_{i_2=1}^{i_1} \sum_{i_3=1}^{i_2} x_{i_1 i_2 i_3} \Psi_3(\xi_{i_1}, \xi_{i_2}, \xi_{i_3}) + \cdots \tag{11}$$

Usually, it can be written as:

$$\chi = \sum_{i=1}^{\infty} x_i \Psi_1(\xi_i) \tag{12}$$

In which x_i are unknown coefficients of polynomial Ψ_i, while Ψ_i are a set of multidimensional polynomials with respect to the multidimensional random variables $\boldsymbol{\xi}$ with the orthogonality relationship, and in practical simulation, the series in Eq. (6) is commonly truncated to a finite number of terms and denoted by N. In addition, the selection of polynomial Ψ_i is depend on the distribution type of random variable $\boldsymbol{\xi}$, for example, the Legendre orthogonal polynomial are the optimal basis for the random variable with uniform distribution.

In some cases, the stochastic process of system response can be represented by directly substituting the random parameters into the specific expression between input and output. However, the expressions of some systems are very complex, and even some systems can not be described by a series of analytical equations. In such situation, the stochastic process of system can be constructed by collocation method, which directly represent the output parameters as the gPC expansion.

3.2 Coefficients of Polynomials

In gPC expansion based on collocation method, the polynomials of output are determined by the distribution type of random variables $\boldsymbol{\xi}$. For example, if $\boldsymbol{\xi}=\{\xi_1,\xi_2\}$, and $\xi_1 \sim N(0,1)$ which best polynomial is Hermite polynomial $H_j(\xi_1)$ while $\xi_2 \sim U(-1,1)$ which best polynomial is Legendre polynomial $L_j(\xi_2)$, then polynomial of output can be assumed as:

$$\Psi_j(\boldsymbol{\xi})=H_j(\xi_1)\otimes L_j(\xi_2) \tag{13}$$

After that, the gPC expansion of output can be viewed as the combination of polynomials in Eq. (13) with several unknown coefficients. These coefficients can be solved by the least square method, which need a number of sample points, and the number of points should be more than the number of unknown coefficients. Meanwhile, the process of solving these points can be viewed as a black-box.

Details of gPC expansion based on collocation method can be seen from Figure 3:

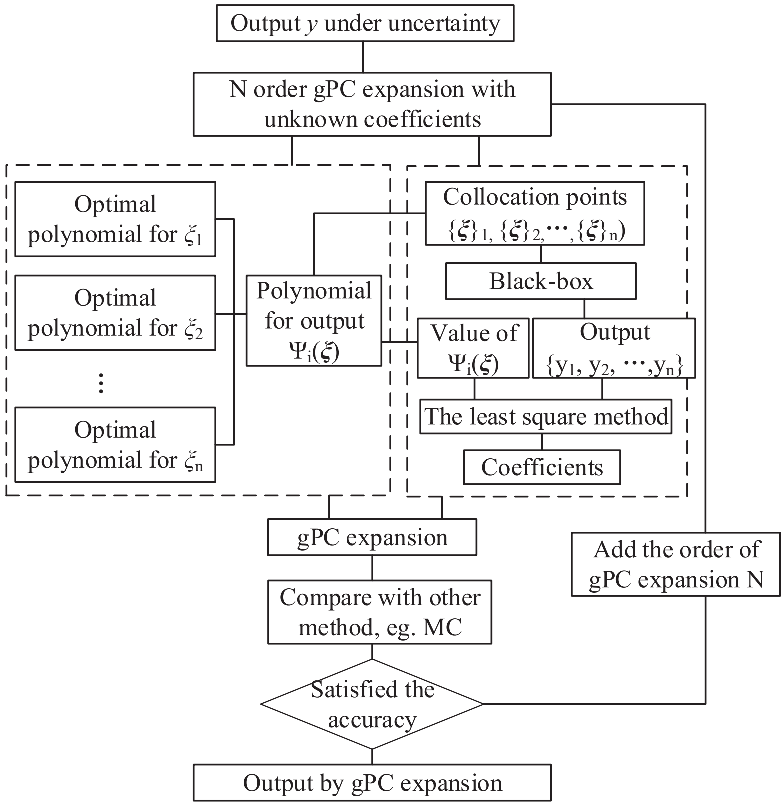

Figure 3 – Flowchart of gPC expansion based on collocation method

4. NUMERICAL ANALYSIS

The analyzed model is a kind of modified Jeffcott rotor model with offset disc, and the parameters of this model are provided in Table 1.

Table 1 Parameters for analyzed rotor-bearing model

Parameter	Value
Length of shaft, m	0.88
Distance from end of shaft to disc, m	0.3
Diameter of shaft, m	0.03
Diameter of disc, m	0.24
Thickness of disc, m	0.04
Viscosity of oil, Pa·s	0.074
Length of bearing, m	0.06

4.1 Deterministic Analysis

It can be concluded from Eq. (9) – (10) that the stiffness and damping coefficients of bearing are the function of rotating speed, which can be illustrated in Figure 4. Then, the unbalance response in x-direction is solved for the rotor model with rigid support. The result is shown in Figure 5. It can be noted that the critical speed is 36.17 Hz. When considering the oil-film bearing, the response in x-direction is different compared with the rigid support situation, which is shown in Figure 6. Comparing these two types of supports, the critical speed is different, and the amplitude of response under flexible support is significantly lower than that under rigid support.

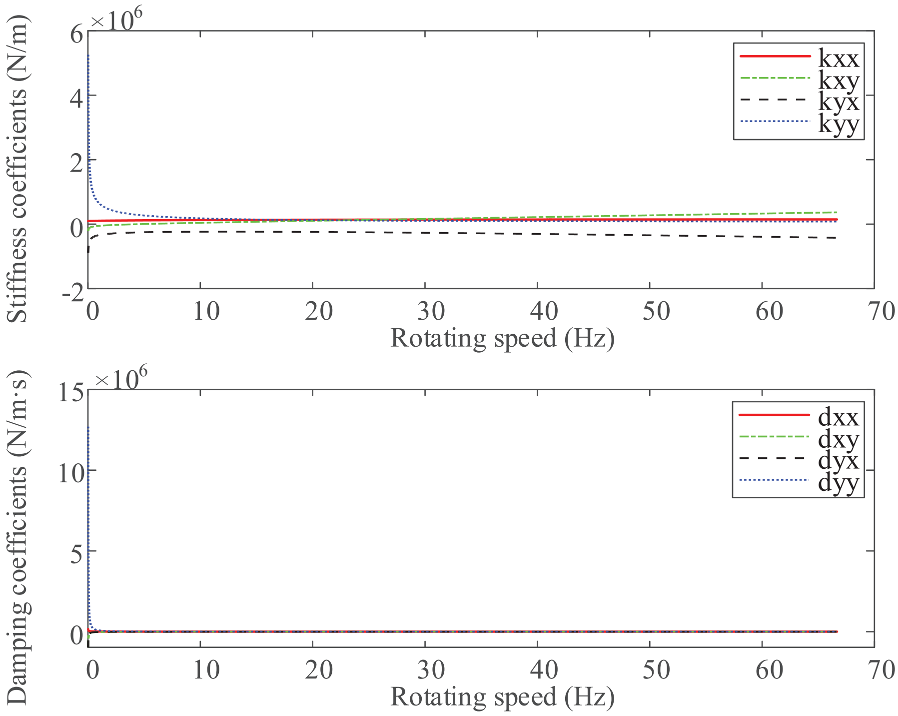

Figure 4 – Stiffness and damping coefficients of oil-film bearing

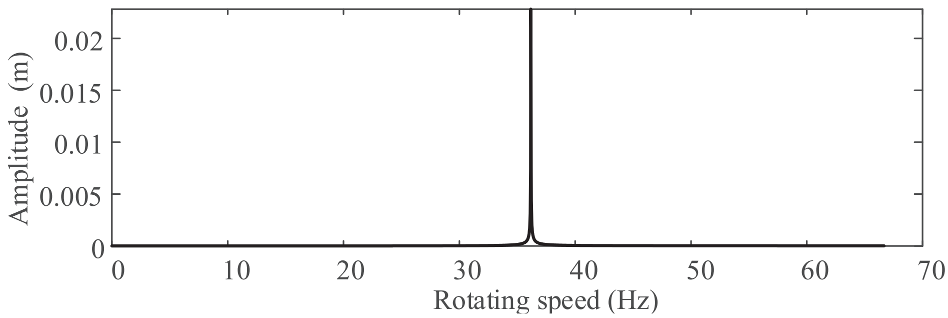

Figure 5 – Unbalance response in x direction for rotor with rigid support

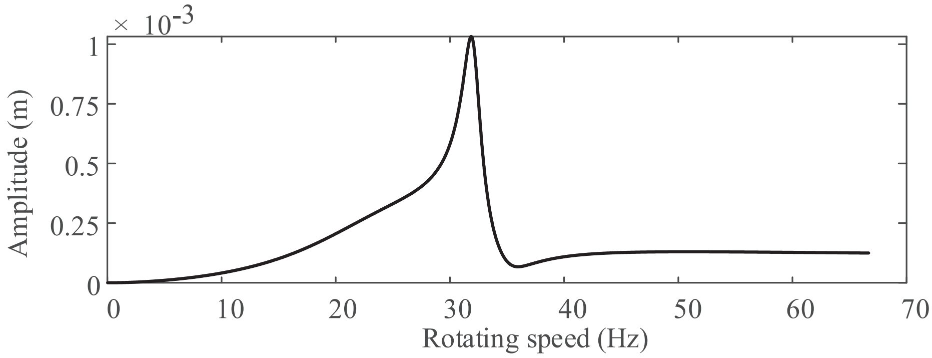

Figure 6 – Unbalance response for rotor with short bearing model

4.2 Random Parameters and Uncertainty Analysis

In this work, the parameters of oil-film are viewed as the random parameters. The length of bearing and clearance are assumed as uniform distribution, and the viscosity is assumed as normal distribution. The detailed information can be seen in Table 2.

Table 2 Random parameters and distribution

Random Parameter	Distribution
Viscosity of oil, Pa·s	$\mu \sim N(0.074,\ 0.003)$
Length of bearing, m	$L_b \sim U(0.05,\ 0.07)$
Clearance, m	$c \sim U(0.0008,\ 0.0012)$

The choice of collocation points is significant for gPC expansion. It can affect the convergence and convergent rate of the construction of gPC expansion. Here, the combination of specific roots of higher order polynomial are used to build the sample space, and the corresponding response under these sample points can be obtained by deterministic analysis. Three order gPC expansion is built in this work. The response under these points are shown in Figure 7. It can be seen that the responses are different due to the different input parameters, which means the input parameters like viscosity can affect the characteristic of oil-film so that can affect the response of rotor.

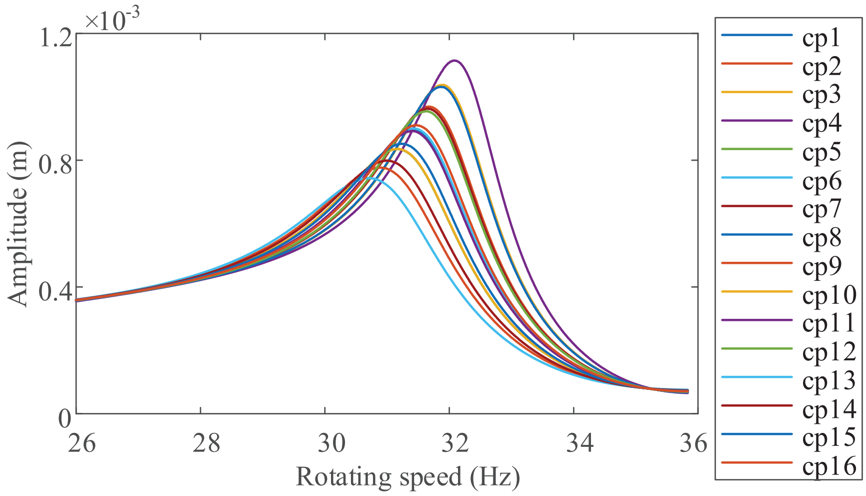

Figure 7 – Unbalance response of sample points

After these responses obtained, the gPC expansion can be built by choosing the optimal polynomials and calculating the related coefficients. In order to give a comparison, the maximum amplitude and corresponding rotating speed are extracted to construct their gPC expansion. To validate the precise of constructed gPC expansion, a comparison with the MC simulation is carried out. The PDF of maximum amplitude and corresponding rotating speed under gPC expansion and MC simulation are obtained and shown in Figure 8 and Figure 9. The PDF obtained by gPC expansion are well fitted with that by MC simulation.

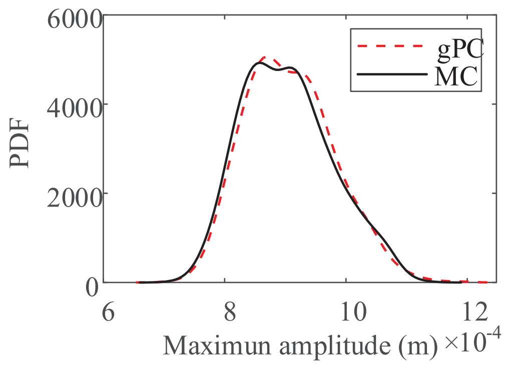

Figure 8 – PDF of maximum amplitude

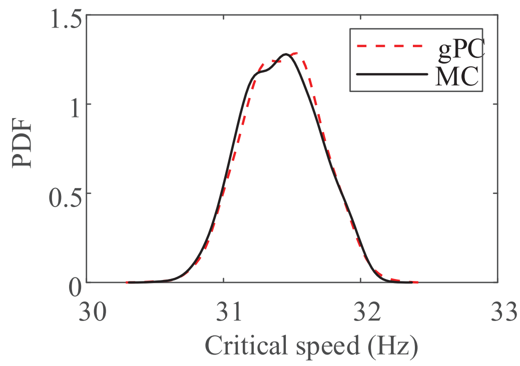

Figure 9 – PDF of critical speed

5. CONCLUSIONS

Rotor dynamics analysis under the impact of oil-film bearing is conducted considering the uncertainties in bearing. In this work, a short bearing model is employed to study its impact to rotor dynamics. The pressure of oil-film is calculated by solving Reynolds' Equation, and the oil-film force is obtained by integrating the pressure on length direction. Then, the dynamics coefficients including stiffness and damping are calculated by differencing the oil-film force. After that, the rotor-bearing model is established by taking the parameters of bearing into consideration. It can be concluded that the flexible bearing has a significant influence on dynamics response of rotor.

Parameters of oil-film such as viscosity are viewed as random parameter to study the influence on rotor dynamics caused by their uncertainties. The gPC expansion based on collocation method is developed to study the uncertain problem in this rotor-bearing model. Results show that the stochastic process of response can be well described by the constructed gPC expansion in a higher accuracy compared with MC simulation. It was also shown that the selected random parameters can affect the response of rotor system.

ACKNOWLEDGEMENTS

The authors acknowledge the support from China Scholarship Council (File No. 201806450023).

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