Field test of resonant frequency monitoring utilizing background vibration in various buildings

Yoshinori Takahashi(1), Naru Sato(2), Yasutaka Nakajima(3)

(1) Tokyo Metropolitan College of Industrial Technology, Japan, yoshinori@ieee.org
(2) RION, Japan, n-satou@rion.co.jp
(3) RION, Japan, yasutaka@rion.co.jp

Abstract

A number of big intermittent aftershocks often follows a great earthquake. Even if a certain building escapes from the collapse due to the first main quake, there is no guarantee that the building can survive from the aftershocks. If it is possible to monitor the condition of the buildings constantly using some faint random vibration caused by background vibration such as wind, weak earthquakes and traffic vibration, the safety will be more secured. The authors have proposed cumulative harmonic analysis that emphasizes the hidden resonances in a time-invariant transfer function, and conducted the experimental monitoring of the resonant frequency in an actual building from the analysis of the faint random vibration on the building caused by the background vibration. From the results of comparison with the measured values of the resonant frequency by an active diagnosis it is confirmed that the estimation of the resonant frequency by the proposed method obtains an accurate value only including few percent error. This report describes about the results of field tests on various buildings using the proposed method.

Keywords: Structural Health Monitoring, Cumulative Harmonic Analysis, Microtremor Measurement, Resonant Frequency, Signal Processing

1 INTRODUCTION

In recent years, the earthquake resistance standards are defined in Japan so that any buildings do not cause their collapse to harm to human life at the intensity 7 on the Japanese scale[1]. However, the buildings deteriorate by being exposed to the natural environment for a long period of time. Before having a fatal damage caused by natural disasters such as earthquake, it is necessary to find the degradation by diagnosing the structural health of the buildings. In addition, it is known that a number of big intermittent aftershocks often follow a great earthquake. Even if a building escapes from the collapse due to the first main quake, there is no guarantee that the building can survive from the aftershocks. However, it is difficult to judge whether to evacuate from the building to prepare for aftershocks right after the big earthquake. If it was possible to monitor the structural health of high buildings and to build the network named Structural Triage Information Sharing Network (Figure 1) that can share the damaged situation of buildings in a city, evacuation sites will be easier to secure. tunnels and bridges just like fire alarms continuously, it would be possible to ensure the residents’ safety [2].
As the most basic diagnostic method, there is a method of observing vibration transmitted to a distant position by exciting a certain point using a test signal [3]. Abnormal conditions in buildings can be detected as changes in damping ratio and resonant frequency. With such a method, it is possible to diagnose the building only when the building is excited by the test signal. If it is possible to estimate the characteristics of the building vibration utilizing some faint random vibration caused by environmental noise such as wind or traffic noise, the structural health can be monitored constantly. In a field of machinery diagnostics, although several diagnostic methods using vibration based on environmental noise have been proposed, those methods require that the characteristics of the noise must be known since the resonance of the object may be buried by the environmental noise spectrum [4, 5]. In the field of architectural engineering, there are researches such as the structural diagnosis based on the microtremor measurement[6] performed in a time zone where there is little human activity in the building or little wind or environmental noise. However, it is difficult to apply the structural diagnosis based on the microtremor measurement in the city center where there are many environmental noises day and night.

On the other hand, the diagnostics methods utilizing unknown random vibration have been proposed without requiring the excitation by a test signal. These techniques emphasize the resonant frequency of the building buried in the observed noise spectrum using faint vibration due to environmental noise. In the past, a method to monitor invisible changes in building has been proposed, which is based on the distribution of the dominant frequency for the short-interval periods (SIP) in observed nonstationary vibration excited by unknown source [7]. By utilizing the frequency distribution of the dominant spectrum excluding the amplitude value, SIP can elucidate the harmonic structure of stationary transfer function buried in time variant source spectrum. However, the efficiency of SIP is not clearly shown for resonant frequency estimation of transfer function under the condition of stationary noise with little temporal change in the spectrum distribution of the source signal. Therefore, the authors have proposed passive diagnostics [9] (shown in Fig.1) of buildings by using cumulative harmonic analysis (CHA) [8]. CHA is an analysis method to emphasize the time invariant spectral peaks by utilizing windowing function with the effect of spectrum accumulation over a period of time and was originally proposed for howling prediction problem. In the study of howling prediction utilizing CHA, it is confirmed that the method can detect a faint spectrum peak which may cause the howling even for stationary noise input[10]. Therefore, CHA should be applicable to passive structural diagnostics. Previous works have shown that the change of resonant frequency and equivalent bandwidth in the buildings can be estimated through model experiments [11]. Furthermore, they have attempted to estimate the resonant frequency of an actual building by observing the random vibration [12].

2 TIME-ININVARIANT POLES ESTIMATION UTILIZING CUMULATIVE HARMONIC ANALYSIS

For the passive diagnosis proposed in this paper, it is necessary to estimate the spectrum of the time-invariant transfer function hidden in the harmonic structures embedded in the harmonic structure of stationary noise. This section introduces cumulative harmonic analysis (CHA) [8] which is effective to solve such a problem.

CHA is formulated by introducing a spectrum accumulation into cumulative spectral analysis (CSA) which was proposed by Barman and Fincham [13]. As shown in Figure 2 (a), CSA is a time frequency analysis that expresses the generation process of spectrum for each sample of input signal, and it was proposed to visualize the generation process of the harmonic structure in the impulse response of loudspeaker. When the input signal is defined as s(n) and the angular frequency is \( \Omega \), CSA can be expressed as

\[
\text{CSA}(n, \Omega) \equiv \sum_{m=0}^{n} w(m)s(m)\exp(-j\Omega m)
\]  

(1)

using the step function \( w(m) = 1 \ (m \leq n), \ w(m) = 0 \ (m > n) \).

On the other hand, CHA is an analytical method to emphasize time invariant spectrum peak by accumulating
Figure 2. Schematics of the CSA (a) and CHA (b), (c), (d) for a N point input signal. The CSA (a) shows
the generating process of the spectrum according to the input signal by enumerate these spectra. The CHA
(b) accumulates these spectra obtained by the CSA. The result of CHA for \( n = N - 1 \) in (b) is equivalent to
(c) performing a single \( N \)-point DFT by applying a triangular window to the input signal. When a triangular
window that increases according to time is used, the results of the cumulative spectrum will be weighted to the
decay part of the input signal (d).

\[
\text{CHA}_D(n, \Omega) \equiv \sum_{m=0}^{n} (n-m+1)s(m)e^{-j\Omega m}
\]  

Figure 3. An example of impulse response and the amplitude spectra computed by the DFT and the CHA. (a): Input signal has one pair of complex conjugate poles and zeros. (b): The pole and zero are confirmed as a
spectral peak and a dip in the amplitude spectrum. (c): The CHA sharpens the steeper spectral peaks without
the spectral dips.

spectra obtained in every sample using CSA. CHA algorithm shown in Fig. 2 (b) requires, \( N \) times of the DFT
to analyze the accumulated spectrum of \( N \) points signal. However, since the DFT is a linear operation, the
analysis process shown in Fig. 2 (b) is equivalent to Fig. 2 (c). This means that only one DFT calculation is
required to obtain the accumulated spectrum of points signal. The calculation of accumulated spectrum for the
input signal \( s(n) \) can be expressed as

\[
\text{CHA}_D(n, \Omega) \equiv \sum_{m=0}^{n} (n-m+1)s(m)e^{-j\Omega m}
\]  

(2)
by the DFT of the signal \( s(n) \) that the triangular window was applied. The triangular windowing shown in equation (2), which decreases on the time axis, can be regarded as weighting to old signals on the time axis. On the contrary, if the triangular window (Fig. 2(d)) increases along the time axis such as

\[
\text{CHA}_H(n, \Omega) = \sum_{m=0}^{n} (m+1)s(m)\exp(-j\Omega m)
\]

(3)

it becomes weighting to the latter part of the signal. It is known that the increasing triangular window has the emphasizing effect of the decaying part of the input signal. It has also been demonstrated that CHA corresponds to the derivative of \( S(\Omega) \) by the angular frequency \( \Omega \) as,

\[
\sum_{m=0}^{n} (m+1)s(m)\exp(-j\Omega m) \approx j\frac{dS(\Omega)}{d\Omega}
\]

(4)

where \( S(\Omega) \) is the DFT of \( s(n) \) [14].

Considering the input signal having one zero at the position of \( r\exp(j\Omega_z) \) (0 < \( r < 1 \)) on the z-plane, the spectrum of the input signal is shown as \( S(\Omega) = 1 - r\exp(-j(\Omega - \Omega_z)) \), where \( \Omega_z \) is the angular frequency \( r \) is the distance from the origin. In this case, CHA power spectrum becomes as

\[
|\text{CHA}(\Omega)|^2 = \left| \frac{d\{1 - r\exp(-j(\Omega - \Omega_z))\}}{d\Omega} \right|^2 = r^2
\]

(5)

This means that the power spectrum is no longer the function of \( \Omega \).

On the other hand, for a signal having one pole \( S(\Omega) = \frac{1}{1 - r\exp(-j(\Omega - \Omega_p))} \), CHA power spectrum becomes as

\[
|\text{CHA}(\Omega)|^2 = \frac{r^2}{\{1 - 2r\cos(\Omega - \Omega_p) + r^2\}^2},
\]

(6)

where \( \Omega_p \) denotes the angular frequency of the pole, and the distance between the origin and the pole \( r \) is 0 < \( r < 1 \). Consequently, CHA sharpens the steeper spectral peaks and removes the spectral dips.

Figure 3 shows an example of a comparison between CHA and the DFT. The input signal shown in Fig. 3 (a) is given as the inverse Fourier transform of the transfer function \( S(\Omega) \) with complex conjugate poles and zeros,

\[
S(\Omega) = \frac{\{1 - r\exp(-j(\Omega - \Omega_z))\}}{\{1 - r\exp(-j(\Omega - \Omega_p))\}} \cdot \frac{\{1 - r\exp(j(\Omega - \Omega_z))\}}{\{1 - r\exp(j(\Omega - \Omega_p))\}},
\]

(7)

where \( r = 0.9 \), \( \Omega_z = \pi/4 \) and \( \Omega_p = \pi/3 \). The amplitude spectra based on the DFT and CHA are shown in Fig. 3 (b) and (c), respectively.

3 ESTIMATION ALGORITHM FOR RESONANT FREQUENCY OF BUILDING

This chapter explains the estimation method of resonant frequency of a building utilized in passive diagnosis [11, 12]. Figure 4 shows an outline of a proposed method for estimating the resonant frequency from faint random vibration in a building. The acceleration pickup installed in the building observes faint random vibration excited by the environmental noise. The observed vibration is divided into frame signals by rectangular window function. In this paper, the frame length is 5 seconds, and the frame shift length is 2.5 seconds. The time invariant spectral peak of each frame signal is emphasized by CHA. The frequency of the spectral peak above a fixed value is selected from CHA spectrum of each frame signal, in order to create a frequency histogram.

In this paper, the CHA spectral peaks more than 25% of the maximum value in the amplitude are selected to create histograms in every hour. The mode of the frequency on the histogram monitored in every hour is the estimated value of the resonant frequency in the building.
4 MONITORING OF RESONANT FREQUENCY IN ACTUAL BUILDING

4.1 Experiment in reinforced concrete building and its evaluation

A monitoring experiment of the resonant frequency over a long period of time was conducted at 8th floors of a reinforced concrete building (Arakawa campus in Tokyo Metropolitan Industrial Technology) as shown in Figure 5. The target building has a whole structure where three rectangular spaces are connected to the triangle center part (See Fig. 5 (a), (b)). The monitoring experiment was conducted at one of the rectangular spaces. Fig. 5 (c) shows the photos of two installed acceleration pickups (PV-87, RION) and recorders (DA-21, RION). The acceleration pickups were installed on the rooftop so that the vibration amplitude of the building was the largest and the acceleration was observed easier. In addition, as the acceleration pickups were installed for the long side direction (X - axis) and the short side direction (Y - axis) respectively along the rectangular structure, the vibrations were observed towards each direction. The monitoring experiment for the resonant frequency had been carried out for a month from September 6, 2016 to October 5, 2016.

Figure 6 shows CHA peak frequencies in every hour obtained by the passive diagnostics method shown in Section 3. The black dots in each histogram indicate the mode, which corresponds to the estimated resonant frequency of the building. Table 1 shows the average, the standard deviation and the relative error of estimations in each direction. However, the estimated values deviating from the average value (larger than ±5 Hz from the average value) in the result of Fig. 6 were excluded. These results have indicated that almost stable estimation results were obtained throughout the experiment.

For evaluate the estimated value, it is discussed about the measurement of the resonant frequencies in the building by active diagnostics utilizing the impact by a weight, and compared the estimated values obtained in
the last chapter with the measured values in the active diagnostics. In this study, the active diagnostics were performed utilizing an impactor that can excite a structure faintly by hitting with a suspended weight. A void tube, which is filled with 20kg of sand bag as a weight, was put in the suspended green bag to prevent the sandbag from absorbing the shock at the impact. The weight was hanged with a rope so that the length from the fulcrum to the center of the weight becomes about 1.2m. Then, the weight was lifted to such an extent that the angle between the pillar of structure and the rope becomes 30 degrees, and the collision was caused with the pillar in the manner of a pendulum. For the purpose of obtaining as large moment of force as possible with a small impact, the excitations were given at a pillar on the top floor. Excitations were performed 12 times, and the average and standard deviation were obtained. In the experiment, the resonant frequency of the building was measured as $2.03 \pm 0.13$ Hz along the X-axis and $2.79 \pm 0.12$ Hz along the Y-axis. Assuming that the measured values by the experiment are the real resonant frequency of the building, the relative error with the estimation value given in the previous chapter was less than 2%. Therefore, it can be confirmed that the proposed method is effective for the monitoring of the resonant frequency for the building.

4.2 Experiment in steel building

Furthermore, this work conducts a monitoring experiment of resonance frequency in a 5th floors of steel building (including several reinforced concrete columns) was also conducted as shown in Figure 7. In this experiment, two servo vibration pickups (RION, LS-10C) were installed for the long side direction (X-axis) and the short side direction (Y-axis) respectively along the rectangular structure. The monitoring experiment had been carried intermittently for a total of 22 days from November 22, 2018 to February 21, 2018.
Figure 8 shows CHA peaks frequencies in every hour obtained by the same analysis shown in section 4.1. Table 2 shows the average, the standard deviation and the relative error of estimation in each direction. It can be confirmed that the estimation results are stable as in the experimental results of reinforced concrete building. The Building Standard Low of Japan uses a formula \( T = (0.02 + 0.01\alpha)h \) to obtain the approximated value of the natural period of a building, while \( h \) is the height of the building and \( \alpha \) is the ratio of the height of steel structures to the total height of the building. Although the actual resonant frequency is unknown because no active diagnostics have been performed for this building, the approximated value of the resonant frequency for the steel building can be obtained \( 1/T = 1.36\text{Hz} \), where substituting \( h = 24.5\text{m} \) and \( \alpha = 1.0 \) for the formula. From this calculation, it is conclude that the adequate estimation result is obtained in this experiment.

**Table 2. Average and standard deviation of the estimation for the steel building.**

<table>
<thead>
<tr>
<th></th>
<th>X-axis</th>
<th>Y-axis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average</td>
<td>1.23Hz</td>
<td>1.27Hz</td>
</tr>
</tbody>
</table>
| Standard deviation | \( 0.75 \times 10^{-2}\text{Hz} \) | \( 2.96 \times 10^{-2}\text{Hz} \)
| Relative error  | 0.61%      | 2.33%     |

**5 CONCLUSIONS**
In this paper, the resonant frequency of the actual building was monitored for a long period of time using the proposed passive diagnostics method. This monitoring test was conducted for reinforced concrete building.
and steel building. As a result, although it was found that the estimated values occasionally deviated from 
the estimated average, it was confirmed that the resonant frequency were stably estimated. In addition, for the 
reinforced concrete building, the resonant frequencies of the building were measured by active diagnostics using
the impact by a weight, and the estimated values were evaluated with the measured values. In the experiment 
of steel building, the adequacy of the result was confirmed by calculation of the natural period of a building 
based on the Building Standard Low of Japan. It is concluded that the proposed method is effective for an 
estimation of the resonant frequency of buildings.

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