Numerical Approach for Aerodynamics around a tone hole of woodwind instruments: an example solving moving boundary problems with topologically change

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Abstract
In this paper, we discuss the numerical reproducibility of the compressible fluid behavior around a tone hole of woodwind instruments by using compressible Large Eddy Simulation (LES). In particular, we focus on the situation that the tone hole is opened and closed with moving a pad above the tone hole, which is regarded as a moving boundary problem with topology change. Our numerical model is based on a 2D analog of Keefe’s experimental model, which has a tone hole placed at the center of the resonance tube. To reproduce the opening and closing the tone hole, the pad is moved continuously. Actually, the position of the pad was continuously changed in the order of “open - close - open”. Our numerical results agree rather well with Keefe’s experimental results. We solved the moving boundary problem with topology change under the situation of acoustics of fluid-structure interaction, and reproduced transitions between resonance and non-resonance states. In addition, the details of complex vortices near the tone holes clarified by this numerical calculation will be reported.

Keywords: Flue instruments, Tone hole, Moving boundary conditions, Numerical simulation

1 INTRODUCTION
The transient phenomena around tone holes of woodwind musical instrument have been studied as a long standing problems in the field of musical acoustics[1, 2, 3, 4, 5, 6, 7, 8]. The main difficulty in studying the function of tone holes comes from the highly complicated interaction around compressible fluid dynamics (non-linear dynamics), aerodynamic sound as a linear wave motion and a moving boundary problem with topology change.

In order to achieve a rigorous study on it, we had developed an compressible LES solver resolving the moving boundary problem[9]. Then we had studied the function of a tone hole by using a 2D model of a small recorder-like-instrument with a tone hole[10]. In that study, the pad above a tone hole is set to a fixed position, that is, the distance between the pad and the tone hole is taken at fixed values, 0, 0.5, 1, 2, 3, 5, 10, 20mm, and the change of pitch depending on the position of the pad is observed. The results is consistent with Keefe’s experimental results. In the next study, we had developed numerical models in order to manipulate moving mesh without topology change[11].

In this paper, we give a short review of recent results about this series of study. We reproduce numerically the compressible fluid behavior around a tone hole of woodwind instruments by using compressible LES. We focus on the situation that the tone hole is opened and closed with moving a pad above the tone hole, which is regarded as a moving boundary problem with topology change. In order to achieve seamless numerical calculation of moving boundary problem with topology change, we develop compressible LES solver and a method to move and remap the mesh structure. Numerical results are consistent with Keefe’s experimental results, and error rates of pressure between mesh remapping with/without topology change are in the same order ~ 10⁻⁹.
2 MODEL AND NUMERICAL METHOD

We continuously use the same model which is a 1/4 scaled 2D analog of Keefe’s experimental model (see Figure 1 left), which has a tone hole placed at the center of the resonance tube[11]. To reproduce the opening and closing the tone hole, the pad is moved continuously.

![Figure 1](image1). (left) Dimensions of the model. (right) typical mesh structure. “dgap” is the distance between the pad and the top of the tone hole. The unit is mm.

We employ a compressible Large Eddy Simulation (LES) with the one-equation sub-grid-scale (SGS) model[12]. In practice, we have developed a extended compressible LES solver “rhoPimpleMyDyMFoam” which includes a function of “DynamicMesh” for the numerical calculation of moving mesh. This solver “rhoPimpleMyDyMFoam” is based on “rhoPimpleFoam” in the open source software, OpenFOAM ver.4.1. The pressure and temperature at the rest are taken as $p_0 = 10^5$Pa and $T_0 = 300$K, respectively. The smallest mesh size around the tone hole is $\Delta r = 0.17$mm and the time step of the numerical integration is $\Delta t = 1.0 \times 10^{-7}$s. The number of mesh cells is 24125. To excite an acoustic wave in the resonance tube at a given frequency, the velocity “U” at the left end of the tube is changed periodically. The amplitude of $U$ is taken as $|U| = 0.024$ m/s in the steady state. The frequencies of $U$ are taken at 1269Hz and 2538Hz as the resonance frequencies of fundamental mode and 2nd higher mode, respectively.

![Figure 2](image2). Mesh structures around the pad and tone hole. The distance between the pad and the top of the tone hole is 1 to 0mm.

To numerically solves the moving boundary problem with topology change, we use a combination of “Dy-
"namicMesh" and "mapFields", which controls the pad movement and the mesh manipulation. In practice, we configure moving velocity of mesh in the configuration file "pointMotionU" and "cellMotionU". In order to relieve the distortion of mesh structure, we use "mapFields" every interval of 0.2mm for the pad movement. We configure this mesh structure remapping in the configuration file "mapFieldsDict".

![Figure 3. Time evolution of the pad movement. (a) the velocity, (b) "dgap": the distance between the pad and the top of tone hole](image)

The actual pad movement is shown in Figure 3. The quantity "dgap" as a function of time must be continuously differentiable in order to avoid shock waves. And furthermore, at the point (dgap=0mm) where the topological change of the mesh-boundary arises, the velocity of the pad as a function of time must be also continuously differentiable. The maximum velocity of the pad is set to 0.1m/s.

<table>
<thead>
<tr>
<th>Region</th>
<th>Time [s]</th>
<th>Time of mapping mesh [s]</th>
<th>Default &quot;dgap&quot;</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>0~0.053</td>
<td>0.053</td>
<td>1mm</td>
</tr>
<tr>
<td>(2)</td>
<td>0.053~0.055</td>
<td>0.055</td>
<td>0.8mm</td>
</tr>
<tr>
<td>(3)</td>
<td>0.055~0.057</td>
<td>0.057</td>
<td>0.6mm</td>
</tr>
<tr>
<td>(4)</td>
<td>0.057~0.059</td>
<td>0.059</td>
<td>0.4mm</td>
</tr>
<tr>
<td>(5)</td>
<td>0.059~0.06198</td>
<td>0.06198</td>
<td>0.2mm</td>
</tr>
<tr>
<td>(6)</td>
<td>0.06198~0.07</td>
<td>0.07</td>
<td>0mm</td>
</tr>
<tr>
<td>(7)</td>
<td>0.07~0.073</td>
<td>0.073</td>
<td>0.2mm</td>
</tr>
<tr>
<td>(8)</td>
<td>0.073~0.075</td>
<td>0.075</td>
<td>0.4mm</td>
</tr>
<tr>
<td>(9)</td>
<td>0.075~0.077</td>
<td>0.077</td>
<td>0.6mm</td>
</tr>
<tr>
<td>(10)</td>
<td>0.077~0.079</td>
<td>0.079</td>
<td>0.8mm</td>
</tr>
<tr>
<td>(11)</td>
<td>0.079~0.082</td>
<td></td>
<td>1mm</td>
</tr>
</tbody>
</table>

Table 1 shows the mesh dimensions and the time of mesh structure remapping.

3 NUMERICAL RESULT

3.1 Resonance conditions

Our numerical model has a tone hole placed at the center of the resonance tube. Thus, in the case of fundamental frequency resonance mode, stable oscillation can be obtained irrespective of the position of pad, because of the node of standing wave of pressure is placed at the center of resonance tube at which the tone hole is also
placed. On the other hand, in the case of 2nd higher mode, due to an antinode of standing wave of pressure is placed at the center of resonance tube, stable oscillation is prohibited at the condition of opening tone hole.

3.2 Numerical result of fundamental mode (1269Hz)
Numerical results of fundamental mode are shown in Figure 4 and 5.

Figure 4. Snapshots of spatial distributions of pressure $p$ (upper line) and magnitude of velocity $U$ (lower line).

Figure 4 shows snapshots of spatial distributions of pressure $p$ (upper line) and magnitude of velocity $U$ (lower line) in a steady state. Because of the antinode of standing wave of velocity at the center of resonance tube, the velocity around the pad and tone hole is relatively active.

Figure 5. Time evolution of pressure $p$ (upper line) and horizontal particle velocity $U_x$ (lower line) oscillations. Each line color corresponds to each mesh dimension in Table 1.

Figure 5 shows time evolution of pressure $p$ (upper line) and horizontal particle velocity $U_x$ (lower line) oscillations. The sampling points of $p$ and $U_x$ are as follows: All sampling points are on the center line of the tube.
“Left” is the point at 1mm right from the left end of the tube. “Center” is the point at the center of the tube. “Right” is the point at 1mm left from the right end of the tube.

As shown in Figure 5, our numerical method for moving mesh with topology change works very well, during the position of the pad is continuously changed in the order of "open - close - open". Amplitudes of \( p \) and \( U_z \) are large at the antinode positions and small at the node positions, i.e. behaviors of \( p \) and \( U_z \) obey the resonance condition of fundamental mode.

### 3.3 Numerical result of second higher mode (2538Hz)

Numerical results of fundamental mode are shown in Figure 6 and 7.

![Snapshots of spatial distributions](image)

Figure 6. Snapshots of spatial distributions of pressure \( p \) (upper line) and magnitude velocity \( U \) (lower line).

Figure 6 shows snapshots of spatial distributions of pressure \( p \) (upper line) and magnitude of velocity \( U \) (lower line) in a steady state. Because of the resonance condition, i.e. node of standing wave of velocity at the center of resonance tube, the velocity around the pad and tone hole is inactive.

Figure 7 shows time evolution of pressure \( p \) (upper line) and horizontal particle velocity \( U_z \) (lower line) oscillations. The sampling points of \( p \) and \( U_z \) are as follows: All sampling points are on the center line of the tube. “Middle-Left” is the point at 34.375mm right from the left end of the tube. This point is a node of pressure in the 2nd higher mode condition. “Center” is the point at the center of the tube. “Model-Right” is the point at 34.375mm left from the right end of the tube. This point is also a node of pressure in the 2nd higher mode condition.

As shown in Figure 7, our numerical method for moving mesh with topology change also works very well, during the position of the pad is continuously changed in the order of "open - close - open". Amplitudes of \( p \) and \( U_z \) are large at the antinode condition and small at the node condition, i.e. behaviors of \( p \) and \( U_z \) obey the resonance condition of 2nd higher mode. In this case, transitions between resonance and non-resonance states are observed clearly, when the pad is closing and opening.

### 3.4 Error estimations of mesh manipulation

Table 2 shows errors of pressure at the “Center” sampling point. In the case of “increase mesh (0.8mm \( \rightarrow \) 1mm)”, error is bigger than the case of “decrease mesh (1mm \( \rightarrow \) 0.8mm)”. We assume that this increasing of error rate is caused by a higher order complement of physical quantities when they are mapped from on small mesh onto large one.

Other error rates are in the same order \(~ 10^{-9}\). Thus, the function of moving boundary calculation with topology change in our numerical model is ensured.
sampling position: “Middle-Left”, “Center”, “Middle-Right” side in the pipe

Figure 7. Time evolution of pressure \( p \) (upper line) and horizontal particle velocity \( U_x \) (lower line) oscillations. Each line color corresponds to each mesh dimension in Table 1.

<table>
<thead>
<tr>
<th>Mesh manipulation</th>
<th>Default mesh dimension</th>
<th>Pressure [Pa]</th>
<th>Error rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>decrease mesh (1mm → 0.8mm)</td>
<td>1mm(Before)</td>
<td>100004.3572</td>
<td>9.0 \times 10^{-9}</td>
</tr>
<tr>
<td></td>
<td>0.8mm(After)</td>
<td>100004.3581</td>
<td></td>
</tr>
<tr>
<td>increase mesh (0.8mm → 1mm)</td>
<td>0.8mm(Before)</td>
<td>100003.5042</td>
<td>-2.1 \times 10^{-8}</td>
</tr>
<tr>
<td></td>
<td>1mm(After)</td>
<td>100003.5021</td>
<td></td>
</tr>
<tr>
<td>topology change (Open → Close)</td>
<td>0.2mm(Before)</td>
<td>100077.2729</td>
<td>6.0 \times 10^{-9}</td>
</tr>
<tr>
<td></td>
<td>0mm(After)</td>
<td>100077.2735</td>
<td></td>
</tr>
<tr>
<td>topology change (Close → open)</td>
<td>0mm(Before)</td>
<td>100120.6458</td>
<td>1.0 \times 10^{-9}</td>
</tr>
<tr>
<td></td>
<td>0.2mm(After)</td>
<td>100120.6459</td>
<td></td>
</tr>
</tbody>
</table>

4 CONCLUSIONS

In this paper, we study the numerical reproducibility of the compressible fluid behavior around a tone hole of woodwind instruments by using compressible LES. In particular, we focus on the situation that the tone hole is opened and closed with moving a pad above the tone hole, which is regarded as a moving boundary problem with topology change.

Our numerical model is a 2D analog of 1/4 scaled Keefe’s experimental model, which has a tone hole placed at the center of the resonance tube. For seamless numerical calculation of moving boundary problem with topology change, we have also developed our own compressible LES solver “rhoPimpleMyDyMFoam” on an open source software OpenFOAM ver.4.1. This solver is applied to our numerical tone hole model and ensured that the functions of mesh manipulation as moving boundary problems with/without topology change work very well. From the view point of numerical reproducibility, the most important result is as follows. The transitions between resonance and non-resonance arise for the calculation of the second mode, when the position of the pad is continuously changed in the order of “open - close - open”. They are consistent with Keefe’s experimental results. Error rates of pressure at the mesh manipulation is also consistent with this reproducibility.
It is necessary to check that the same results occurs for a 3D model, and reproduce the pitch change in the opening and closing the tone hole of more practical woodwind instrument model.

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