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Amplitude death in Coupled Thermoacoustic Oscillators with Frequency Detuning

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ABSTRACT

Thermoacoustic oscillation is a harmful vibration in combustors of a gas turbine engine because it can damage the combustors. Development of a suppression method of thermoacoustic oscillation is urgent necessity. Recently, a coupling-induced amplitude death phenomenon was reported experimentally and numerically in thermoacoustic oscillators of Sondhauss type. Since the coupling was established only by using hollow tubes that connects two oscillators, this method can be a simple and reliable method to stop the oscillations, compared to active control method involving electronics and acoustic drivers. In this study, the tube coupling is further tested for the coupled thermoacoustic oscillators with frequency detuning, in order to explore the possibility of stronger suppression effect. The linear stability of the coupled oscillators is numerically obtained based on hydrodynamic equations. We describe that the amplitude death is realized with much smaller tube diameter than that of coupled thermoacoustic oscillators with no detuning. Also, we describe that the tube length resulting in the amplitude death is different from that of coupled identical oscillators. These numerical results are compared with experiments to confirm the validity.

Keywords: Amplitude death, Thermoacoustics, Coupling

1. INTRODUCTION

Thermoacoustic oscillation is a self-sustained gas oscillation when a heat source exists in a duct. Combustion oscillation (1) is a thermoacoustic oscillation in combustors of a gas turbine engine. Lean premixed combustion method is employed to reduce the emission of NOx in gas turbine engine. In this method, the emergence of combustion oscillation is a troublesome issue because the combustion oscillation leads to serious damage to the combustors. Suppression methods of thermoacoustic oscillation such as active control method have been studied. These suppression methods, however, sometimes can be complex because of requirement of the equipment to control an external forcing. Development of simple and reliable suppression method of thermoacoustic oscillation is an urgent necessity.

The stabilization of oscillation can be realized by the coupling of two or more oscillators. The annihilation of the oscillation is called amplitude death. The presence of the amplitude death has been reported theoretically and experimentally in various oscillation systems (2-4). Recently, the stabilization of coupled Sondhauss type thermoacoustic oscillators has been reported (5). Two identical Sondhauss oscillators were coupled using a hollow tube. The Sondhauss oscillator were modeled based on the hydrodynamic equations. The stability analysis has shown the amplitude death occurs when the coupling tube lengths are one-quarter or three-quarters of the wavelength of fundamental acoustic oscillations in uncoupled oscillators.

In this study, the tube coupling is further tested for the coupled Sondhauss type thermoacoustic oscillators with frequency detuning, in order to explore the possibility of stronger suppression effect. Frequency detuning is introduced by using two oscillators with different resonance tube length. The conditions realizing amplitude death is numerically obtained and presented on the parameter plane on the detuning and coupling tube length. We have found that the tube length resulting in the amplitude death is different from that of coupled identical oscillators. We also have found that the coupling tube diameter required to cause the amplitude death is much smaller tube diameter than that of coupled thermoacoustic with no detuning.

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2. COUPLED THRERMOACOUSTIC OSCILLATORS

2.1 Thermoacoustic Oscillator

A Sondhauss type thermoacoustic oscillator used in this study is made of a resonance tube, a stack, and cold and hot heat exchangers. The working gas is air at atmospheric pressure. A circular tube of diameter D = 40 mm is used as a resonance tube. The stack is made of many circular pores of 1.12 mm diameter and 40 mm length. The porosity of the stack is 0.78. The stack is located in the resonance tube with its central position separated from the end by 180 mm. The stack is sandwiched between hot and cold heat exchangers, each of which is made of parallel plates with 0.5 mm thickness and 20 mm length, placed with 1 mm spacing. The temporal mean gas temperature, T_C, in the cold heat exchanger is constant at room temperature (297 K). The temperature in the hot heat exchanger is a control parameter to induce thermoacoustic oscillations. In the other parts of the oscillator, the temporal mean temperature is equal to T_C.

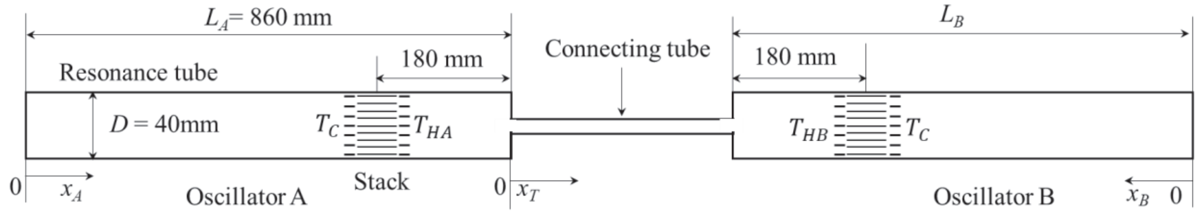

Figure 1 – Coupled thermoacoustic oscillators

2.2 Coupled Oscillators

Two thermoacoustic oscillators are coupled using a connecting tube, as shown in Figure 1. Two oscillators, Oscillator A and Oscillator B, are different only in the resonance tube length. The resonance tube length, L_A, of Oscillator A is fixed at L_A = 860 mm, whereas the tube length, L_B, of Oscillator B is a control parameter to make a frequency detuning. The temperature T_{HA} in the hot heat exchanger of Oscillator A is fixed at T_{HA} = 537 K. The temperature T_{HB} in the hot heat exchanger of Oscillator B is controlled as explained in section 3. These two oscillators are coupled at their ends by using the connecting tube, which is modeled by a circular channel with diameter d and length L.

3. CALCULATION METHOD

3.1 Basic Equation

The hydrodynamic equations are used as basic equations for the analysis. The equations of continuity, momentum, and energy are linearized for a purely oscillating flow with angular frequency ω (6). Taking the cross-sectional average along a radial direction of a flow channel, these basic equations are reduced to a pair of quasi-one-dimensional differential equations of complex pressure amplitude P and complex velocity amplitude U as

$$\frac{dP}{dx} = -ZU, \tag{1}$$

$$\frac{dU}{dx} = -YP + GU. \tag{2}$$

In these equations, Z, Y, and G are given as

$$Z = i\omega \frac{\rho_m}{A} \frac{1}{1-\chi_\nu}, \tag{3}$$

$$Y = i\omega \frac{A\left[1+(\gamma-1)\chi_\alpha\right]}{\gamma P_m}, \tag{4}$$

$$G = \frac{\chi_\alpha - \chi_\nu}{(1-\chi_\nu)(1-\sigma)} \frac{1}{T_m} \frac{dT_m}{dx}, \tag{5}$$

where ρ_m, P_m, and T_m respectively denote the temporal mean density, pressure and temperature of the gas. Also, γ and σ are the specific heat ratio and Prandtl number of the gas.

For the gas with thermal diffusivity α and kinematic viscosity v, the thermoacoustic function χ_j (j=α or v) is given using the first-order and zeroth-order Bessel functions J_1 and J_0 of the first kind as

$$\chi_j = \frac{2J_1\left[(j-1)\sqrt{\omega\tau_j}\right]}{(i-1)\sqrt{\omega\tau_j}J_0\left[(j-1)\sqrt{\omega\tau_j}\right]}. \tag{6}$$

In equation (6), τ_j is given

$$\tau_j = r^2/(2j), \tag{7}$$

where r is the radius of the flow channel.

The differential equations (1) and (2) are solved analytically. The solution gives the relation of P and U between two different points x and $x+l$ as

$$\begin{bmatrix} P(x+l) \\ U(x+l) \end{bmatrix} = M \begin{bmatrix} P(x) \\ U(x) \end{bmatrix}, \tag{8}$$

where M is the transfer matrix given as

$$M = e^{\frac{Gl}{2}} \begin{bmatrix} -\frac{G}{b}\sinh\lambda + \cosh\lambda & -\frac{2Z}{b}\sinh\lambda \\ -\frac{2Y}{b}\sinh\lambda & \frac{G}{b}\sinh\lambda + \cosh\lambda \end{bmatrix}. \tag{9}$$

In equation (9), $b = \sqrt{G^2 + 4YZ}$ and $\lambda = bl/2$.

In the following, the transfer matrices M_A and M_B denote the matrix of Oscillator A and Oscillator B, respectively. These matrices are created by taking a product of the transfer matrices of the circular tube, cold heat exchanger, stack, hot heat exchanger, and the circular tube. The transfer matrix M_T denotes that of the coupling tube.

3.2 Frequency Equation

Transfer matrices M_A and M_B link acoustic variables ($P_{A,B}(0)$, $U_{A,B}(0)$) at $x_{A,B} = 0$ with ($P_{A,B}(L_{A,B})$, $U_{A,B}(L_{A,B})$) at $x_{A,B} = L_{A,B}$ on both ends of the oscillator:

$$\begin{bmatrix} P_{A,B}(L_{A,B}) \\ U_{A,B}(L_{A,B}) \end{bmatrix} = M_{A,B} \begin{bmatrix} P_{A,B}(0) \\ U_{A,B}(0) \end{bmatrix}. \tag{10}$$

Transfer matrix M_T links the acoustic variables ($P_T(0)$, $U_T(0)$) with ($P_T(L)$, $U_T(L)$) at the ends of the coupling tube as

$$\begin{bmatrix} P_T(L) \\ U_T(L) \end{bmatrix} = M_T \begin{bmatrix} P_T(0) \\ U_T(0) \end{bmatrix}. \tag{11}$$

When the two oscillators are uncoupled, the boundary conditions require $U_{A,B} = 0$ at the closed ends ($x_{A,B} = 0$ and $x_{A,B} = L_{A,B}$). To have a nonzero solution,

$$\left(M_{A,B}\right)_{21} = 0 \tag{12}$$

must be satisfied. This equation serves as the frequency equation of the uncoupled oscillator, where the two subscript numbers denote the transfer matrix component.

For the coupled case, the connecting conditions between the oscillators and the connecting tube are the continuity of the pressure amplitude

$$P_A(L_A) = P_T(0),\ P_B(L_B) = P_T(L) \tag{13}$$

and the continuity of the velocity amplitude:

$$U_A(L_A) = U_T(0),\ U_B(L) = -U_T(L). \tag{14}$$

As a condition to have a nonzero solution of P and U, we obtain the frequency equation of coupled oscillators as

$$\begin{aligned}&\left(M_T\right)_{21}\left(M_A\right)_{11}\left(M_B\right)_{11}+\left(M_T\right)_{11}\left(M_A\right)_{11}\left(M_B\right)_{21}\\&+\left(M_T\right)_{22}\left(M_B\right)_{11}\left(M_A\right)_{21}+\left(M_T\right)_{12}\left(M_A\right)_{21}\left(M_B\right)_{21}=0\end{aligned} \tag{15}$$

When T_{HA}, T_{HB}, d, L, and L_B are given, equations (12) and (15) can be solved for the complex frequency $f = (\omega/2\pi)$. The real part of solution represents the frequency of oscillation. The imaginary part represents the stability of the equilibrium; a positive imaginary part means that the equilibrium state is linearly stable, whereas a negative one means that the equilibrium is unstable. In this way, the stability of the uncoupled oscillator is investigated. By solving equation (12) with changing the hot end temperature, the critical temperature above which the oscillations are excited are searched. The critical temperature $\Delta T_A = T_{HA} - T_C$ of the uncoupled Oscillator A is obtained as ΔT_A = 190 K. When ΔT_A is set to 240 K, the complex frequency f_A (= $f_{RA} + i\, f_{IA}$) of the uncoupled Oscillator A is obtained as f_{RA} = 206 Hz and f_{IA} = −1.20 (1/s) which indicates the excitation of the fundamental acoustic oscillations whose wavelength is essentially twice the length of L_A, and the characteristic number of cycles Q_A(= f_{RA} / f_{IA}) is 172 before the oscillation amplitude grows substantially. When the temperature ΔT_B (= $T_{HB} - T_C$) and the resonance tube length L_B is given, the complex frequency f_B (= $f_{RB} + i\, f_{IB}$) of the uncoupled Oscillator B can be obtained as well. When the resonance tube length L_B differs from L_A, also the frequency f_{RB} differs from f_{RA} although the oscillation mode is persistently the fundamental one. The frequency detuning $\Delta f_R = f_{RB} - f_{RA}$ is chosen as a parameter. It is to be noted that the linear stability f_{IB} is also affected by changing L_B. Therefore, the temperature ΔT_B is controlled so that the Q_B(=f_{RB} / f_{IB}) of Oscillator B becomes the same value as Q_A.

Figure 2 presents the resonance tube length L_B dependence of the frequency f_{RB} of uncoupled Oscillator B. The red curve represents f_{RB} and the horizontal black line represents the frequency with $L_B = L_A$. The frequency f_{RB} monotonically decreases as the tube length L_B increases. The difference between the red curve and black line represents the frequency detuning Δf_R. In the following calculation, the length L_B is changed in the range 790 mm < L_B < 950 mm so that the frequency detuning range becomes −20 Hz < Δf_R < 20 Hz.

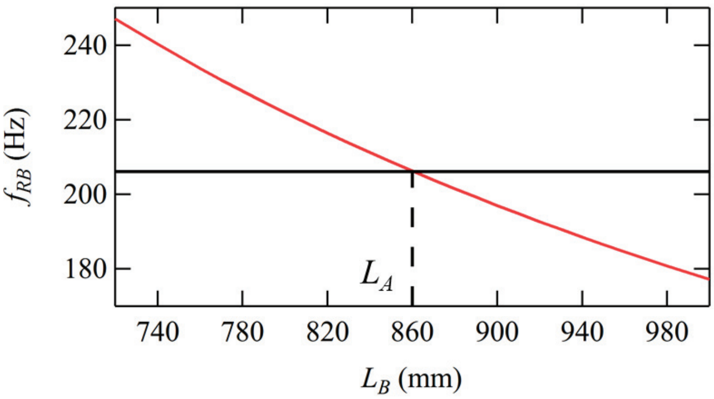

Figure 2 – Resonance tube length L_B dependence of oscillation frequency f_{RB}

4. RESULTS AND DISCUSSION

4.1 Calculation Result

By solving equation (15), the stability of the coupled oscillators is investigated by changing L_B and the coupling tube length L while keeping the coupling tube diameter d = 8 mm. Figure 2 presents a contour map of f_I on the plane of Δf_R and L. Broken curves represent the contour line with f_I = 0. Inside of the broken curves, the f_I is positive. In these areas, the amplitude death occurs, i.e. both two oscillators are stabilized. The death regions are located near L = 0.85 m and L = 1.7 m. Two lengths L = 0.85 m and L = 1.7 m are close to a half or one wavelength of 206 Hz fundamental acoustic waves. Near L = 1.7 m, the death regions are created when the detuning Δf_R is larger than 10 Hz which is 5% of the fundamental oscillation frequency. Near L = 0.85 m, the detuning Δf_R must be larger than 15 Hz to realize the amplitude death. These death regions are slightly asymmetric. In the center of the death

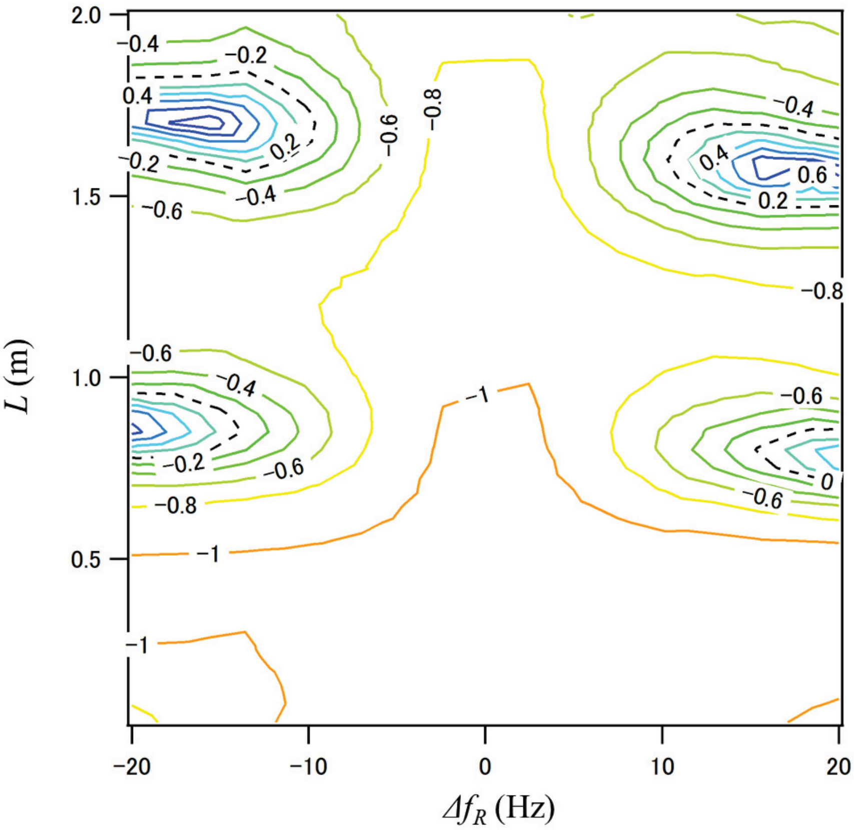

Figure 3 – Contour map of f_I of the coupled thermoacoustic oscillators

region with L = 1.7, Δf_R = −16 Hz, f_R and f_I are f_R = 198 Hz, f_I = 1.16 (1/s). This means that the oscillation ceases after 171 cycles $Q(=f_R / f_I=170.7)$. Outside of the death regions, the value of f_I is not extremely smaller than that of uncoupled oscillator, which means that the instability is not enhanced by the coupling.

It should be noted here that the frequency detuning reduces the tube diameter and changes the tube length necessary for the amplitude death in the coupled thermoacoustic oscillators. By changing and comparing the contour maps of f_I when d value is reduced less than 8 mm, we have found that the smallest d that stabilize the oscillators is d = 3mm. On the other hand, our previous study (5), conducted using the same oscillator as Oscillator A in figure 1, demonstrated that two identical thermoacoustic oscillators without detuning were stabilized only when the coupling tube diameter is larger than 25 mm. The tube diameter difference of 3 mm and 25 mm is significantly large, considering that the resonance tube diameter is 40 mm. In this study, the death regions appeared with L = 0.85 m and L = 1.7 m. These lengths correspond to a half wavelength and one wavelength of the fundamental oscillation. In the case with zero detuning, the necessary lengths were L =0.42 and L = 1.27 m, corresponding to one-quarter and three-quarters of the acoustic wavelength. The coupled thermoacoustic oscillators with frequency detuning is tested by experiments in the next section, to confirm the amplitude death conditions.

4.2 Comparison with Experimental Result

Two thermoacoustic oscillators shown in figure 1 were constructed in the same way as Reference (5). In Oscillator B, one of the ends of the resonance tube was replaced with the movable plug to change L_B. When the resonance tube length L_B was changed, not only the frequency f_B of uncoupled Oscillator B but also the pressure amplitude of uncoupled Oscillator B was changed. Therefore, the temperature difference ΔT_B was controlled so that the pressure amplitude of oscillator A and B became the same before the coupling was introduced. The two oscillators were connected by thick-walled flexible tube with the diameter 8 mm.

Figure 3 presents the oscillation states after the coupling on the plane of the frequency detuning $\Delta f = f_B - f_A$ and coupling tube length L. The amplitude death regions (crosses) appeared with L = 0.8 m and L = 1.6 m when the frequency detuning was employed. For L = 0.8 m, the frequency detuning

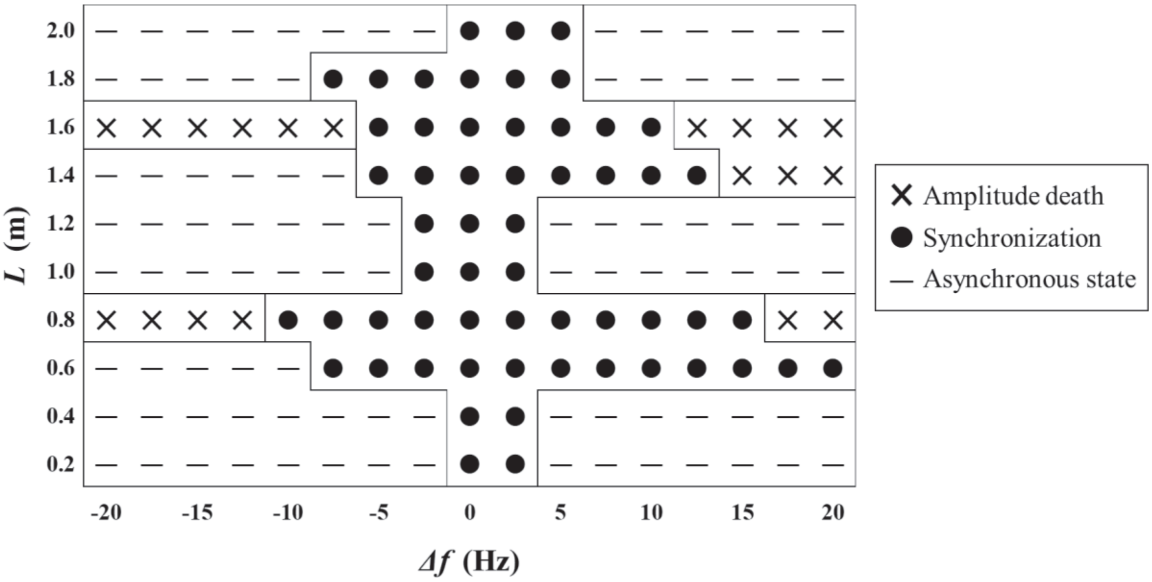

Figure 4 – Experimental bifurcation diagram

required to cause the amplitude death is larger than that of L = 1.6 m. We observed the oscillation states outside of the death regions. When the frequency detuning was small, the two oscillators were synchronous (circles). When the frequency detuning was increased, the synchronization was broken (horizontal lines). The pressure amplitude of coupled oscillators did not greatly exceed that of uncoupled oscillators. These experimental results are consistent with the calculation results. The stability analysis is useful for examining the amplitude death conditions even if the frequency detuning is introduced.

5. CONCLUSIONS

The amplitude death conditions in the coupled thermoacoustic oscillators of the Sondhauss type with frequency detuning were explored with respect to the coupling tube length and the detuning. We found that the tube coupling with frequency detuning realize the amplitude death with much smaller tube diameter than that of coupled thermoacoustic oscillators with no detuning. The frequency detuning was found to greatly reduce the tube diameter necessary to cause the amplitude death, indicating that the stronger suppression effect is expected than in the coupling with zero detuning.

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