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Study of the relation between entropy flux density production and thermal efficiency of a thermoacoustic engine

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ABSTRACT

A traveling wave thermoacoustic device, which traveling wave performs energy conversion with the regenerator (a bundle of narrow channels) having a temperature gradient instead of a piston. This device can realize Carnot cycle, which is an isothermal reversible cycle, ideally. However, irreversibility exists in the actual traveling wave thermoacoustic device, and the isothermal reversibility of energy conversion can be considered by the ratio of the thermal penetration depth δ_α and the channel radius r in the regenerator. In order to make the actual traveling wave thermoacoustic device approach the Carnot cycle, it is necessary to set r/δ_α in the regenerator to realize the small irreversibility due to viscous dissipation and the isothermal thermodynamic cycle. In this study, entropy production is focused as an indicator of reversibility / irreversibility in the regenerator. The thermal efficiency and the entropy production in the regenerator were analyzed by changing r/δ_α, and the relationship between irreversibility and thermal efficiency was investigated. In analysis results, it was found that r/δ_α of the maximum point of the thermal efficiency and the minimum point of the entropy production almost agrees, and the thermal efficiency can be evaluated by using the minimum point of the entropy production.

Keywords: Thermoacoustic engine, Thermal efficiency, Entropy production

1. INTRODUCTION

A traveling wave thermoacoustic device that performs energy conversion with a regenerator (a bundle of narrow channels) having a temperature gradient using traveling wave sound waves instead of pistons can realize Carnot cycle, which is ideally a isothermal reversible cycle (1). This device has the advantage of high efficiency (2) and simple structure. Recently, many research is promoted as a waste heat recovery device actively (3, 4). The reversibility of the energy conversion of the traveling wave thermoacoustic device can be considered by the thermal penetration depth δ_α and the channel radius r of the regenerator (5). In the case of $r/\delta_\alpha >> 1$, the heat from the channel wall is not transferred to the gas in the channel, resulting in an adiabatic reversible cycle. Because of this reason, the energy conversion cannot be performed. On the other hand, in $r/\delta_\alpha << 1$, the thermodynamic cycle is isothermal because the heat of the channel wall is easily transferred to the gas in the channel. However, if r/δ_α is too small, viscous dissipation becomes dominant and the irreversibility due to dissipation becomes large (5, 6). Therefore, it is necessary to set r/δ_α appropriately in order to bring the actual traveling wave thermoacoustic device closer to the Carnot cycle.

In general heat engines, the entropy production is used as an indicator of the reversibility/irreversibility. If the entropy production is large, the irreversibility becomes large. Since the thermoacoustic device is also heat engine, it is possible to discuss reversibility / irreversibility by using the entropy production.

In this study, the thermal efficiency and the entropy production in the regenerator were analyzed by changing r/δ_α, and the relationship between irreversibility and thermal efficiency was investigated.

2. ANALYSIS METHOD

In this study, analysis of the entropy production and the thermal efficiency under the constraint

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condition is based on the linear thermoacoustic theory (5, 6, 7). In analysis, only one flow channel of the regenerator is considered. By using the linear thermoacoustic theory, dP/dx (gradient of complex pressure amplitude P), dU/dx (gradient of cross-sectional mean complex velocity amplitude U) and cross-sectional mean complex temperature amplitude T are expressed as follows. x is defined as the propagation direction of the acoustic wave as positive along the axial direction of the flow channel.

$$\frac{dP}{dx} = \frac{-j\omega\rho_{\mathrm{m}}}{1-\chi_v} U, \tag{1}$$

$$\frac{dU}{dx} = -\frac{j\omega}{P_{\mathrm{m}}}\left[1-(\gamma-1)\chi_\alpha\right]P + \frac{\chi_\alpha - \chi_v}{(1-\chi_v)(1-\sigma)}\frac{1}{T_{\mathrm{m}}}\frac{dT_{\mathrm{m}}}{dx}U, \tag{2}$$

$$T = \frac{1}{C_P\rho_{\mathrm{m}}}(1-\chi_\alpha)P - \left(\frac{(1-\chi_\alpha)-\sigma(1-\chi_v)}{(1-\chi_v)(1-\sigma)}\right)\frac{1}{j\omega}\frac{dT_{\mathrm{m}}}{dx}P. \tag{3}$$

Here, j: imaginary unit, ω : angular frequency, ρ_{m} : mean density, γ : specific heat ratio, P_{m} : mean pressure, T_{m} : cross-sectional mean temperature, C_{p} : isobaric specific heat, σ : Prandtl number. Furthermore, the parameters χ_α, χ_v are thermoacoustic functions related to thermal diffusion and viscosity depending on $r / \delta_{\alpha,\, v}$ (5, 6). The energy conversion between heat flux density Q and work flux density I in the regenerator of the thermoacoustic engine can be written as Equation (4) corresponding to the first law of thermodynamics.

$$\frac{d}{dx}(Q+I) = 0\,, \tag{4}$$

$$I = \frac{1}{2}\mathrm{Re}\left[\tilde{P}U\right]\,, \tag{5}$$

$$Q = \frac{1}{2}C_P\rho_{\mathrm{m}}\mathrm{Re}\left[T\tilde{U}\right] - I\,. \tag{6}$$

Here, if the entropy flux density S is Q/T_{m}, the entropy flux density production per unit length dS/dx is expressed as follows (8);

$$\frac{dS}{dx} = \frac{d}{dx}\left(\frac{Q}{T_{\mathrm{m}}}\right) = \frac{1}{T_{\mathrm{m}}}\frac{dQ}{dx} - \frac{Q}{T_{\mathrm{m}}^2}\frac{dT_{\mathrm{m}}}{dx} = -\frac{1}{T_{\mathrm{m}}}\frac{dI}{dx} - \frac{Q}{T_{\mathrm{m}}^2}\frac{dT_{\mathrm{m}}}{dx}. \tag{7}$$

Therefore, thermal efficiency η is given by the ratio of increment of I and amount of heat input Q in the regenerator.

$$\eta = \left(\frac{dI}{-Q+\frac{dI}{2}}\right). \tag{8}$$

dS/dx and η can be analyzed by substituting Equations (1)~(6) to Equations (7) and (8). In the analysis of dS/dx and η, it is assumed that the phase difference between P and U is zero in order to consider the energy conversion by traveling wave. In this study, η and dS/dx are analyzed by changing r/δ_α, their dependence on r/δ_α and the relationship between them was investigated. The analysis conditions are shown in Table1.

Table 1 – Parameter in analysis

T_m	450 K	ρ_m	1.08 kg/m^3
dT_m	300 K	P_m	1.01 MPa
dx	50 mm	P	5000 Pa
γ	1.65	U	0.40 m/s
λ	4.95	r	5.23×10^{-3}~5.23 mm (r / δ_α = 0.01 ~10)
C_p	5232 J/(kg・K)	ω	251.3 rad/s (= 40 Hz)

3. ANALYSIS RESULTS

Figure 1 shows the results of η and Fig. 2 shows dS/dx, respectively. The horizontal axis in both figures is r/δ_α. The maximum point of η is shown by black circle in Fig. 1. From this figure, η is maximized at r/δ_α=0.19. This reason is considered from dS/dx. In Fig. 2, the minimum point dS/dx is expressed by black triangle. The range in which dS/dx decreased is $0.05 < r/\delta_\alpha < 1.7$, and dS/dx becomes minimum value at r/δ_α=0.17. As described above, it is considered that dS/dx becomes large in $r/\delta_\alpha \leq 0.05$ because the viscous dissipation is dominant. On the other hand, dS/dx is small in $1.7 \leq r/\delta_\alpha$. However, in this region r / δ_α, the energy conversion between I and Q is not performed since the thermodynamic cycle is adiabatic cycle. For this reason, η is decreased in this region. Furthermore, by comparing the value r/δ_α in Fig. 1 and Fig. 2, the r/δ_α minimum point of dS/dx and maximum point of η are almost same. From these results, η can be evaluated by using the minimum point of dS/dx.

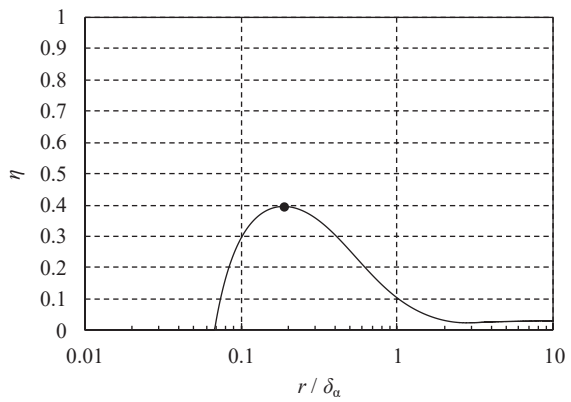

Figure 1 – η vs. r / δ_α

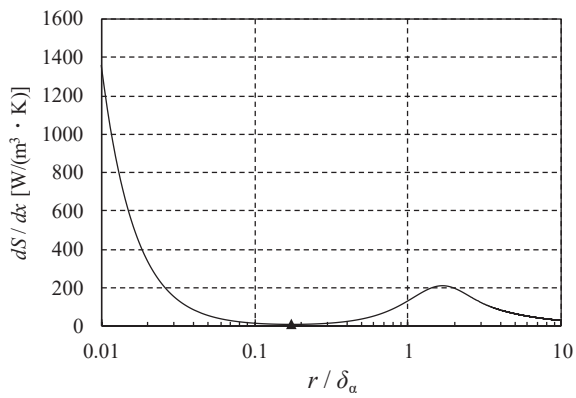

Figure 2 – dS/dx vs. r / δ_α

4. CONCLUSIONS

In this study, for the traveling wave thermoacoustic engine, the thermal efficiency and the entropy production under a constraint condition in the regenerator were analyzed and the relation between them was investigated. As results, the thermal efficiency is maximum at r/δ_α=0.19, and the entropy production became minimum at r/δ_α=0.17 in the analysis range of this study. Therefore, it was found that r/δ_α of the maximum point of the thermal efficiency and the minimum point of the entropy production almost agrees, and the thermal efficiency can be evaluated by using the minimum point of the entropy production.

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