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Double Reflections from corrugated surfaces

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ABSTRACT

Reflections of acoustic waves from corrugated surfaces are discussed. A proposal made by Yi-Fan Zhu for a metasurface model considers that each groove on a corrugated surface is analogous to each individual source in a line array, where the phase difference between sources is determined by the depth of each groove. Zhu's original proposal suggests using the phase gradient of the arrangement as a design parameter and derives conditions for reflections without dispersion. In this paper, we expose the advantages of considering the time-delay gradient instead of the phase gradient. Also, designs that combine two corrugated surfaces interspersed with different gradients to produce double reflection patterns are presented. The angles and delays of both reflections can be selected independently.

Keywords: Dispersionless reflection; Acoustic metasurfaces; Reflection control.

1. INTRODUCTION

In the last few years, there is raising interest in the study of artificial structures with subwavelength characteristics which could enable wave manipulation features that could hardly be obtained with conventional materials. These structures are known as metamaterials and the possibility of obtaining practical solutions to acoustical problems applying these designs continues to increase. A metamaterial is a structure obtained by adding units with a particular design related to its geometry, orientation, and internal organization, which enables acoustical properties that are not present in materials found in nature (1). The term 'metasurface' is used to refer to a bi-dimensional metamaterial structure.

In this article, an analysis of a particular kind of structure is presented as a possible link between classic theory and recently proposed designs. The work presented by Zhu et al. (2,3) is used as a reference, where the characteristics of a corrugated surface that enables dispersionless wavefront manipulation are analyzed. The original process requires taking the phase gradient as a parameter of design. This forces to choose a reference frequency for the calculations, even when the design is intended to achieve has no dispersion, so that it is independent of the frequency. The proposal presented here suggests that the design becomes even easier if you take the time delay gradient delays as a design parameter, instead of the phase gradient. On the other hand, it will be shown that the time delay gradient is not altered if we add a constant delay to the whole design, which allows selecting independently a specific reflection profile and an arbitrary delay of the reflected waves. In addition, it is proposed to combine two different profiles of reflection in the same design. This is accomplished by performing the calculations for each profile independently and alternating then the values of depth of grooves of each one of them. A proof-of-concept of this proposal is simulated using COMSOL Multiphysics.

1.1 Wavefronts Emitted by a Linear Array.

The proposal presented by Zhu begins with the well-known theory that describes the behavior of a linear array of n elements separated by a distance Δx. A cylindrical wavefront will be formed in the near field by a linear array with n number of sources radiating in phase, within a certain bandwidth, and in a direction that will be perpendicular to the array (this is, with a radiating angle of 0°). The lower cutoff frequency of the band is related to the total length of the array $L=n.\Delta x$, while the upper cutoff frequency depends on the distance Δx between sources.

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Constant phase delays between adjacent sources will result in a wavefront with radiating angle θ (Figure 1), which is related to the distance Δr among the source B and a point A' with the same phase as the adjacent source A, as shown in Eq. (1).

$$\theta = \arcsin\left(\frac{\Delta r}{\Delta x}\right) \tag{1}$$

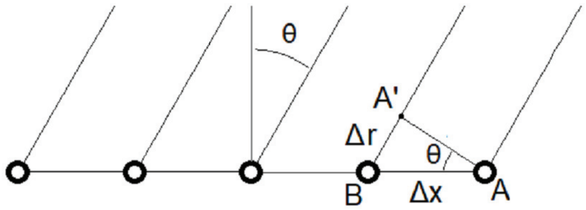

Figure 1 – A linear array of sources with constant phase difference will radiate a wavefront with angle θ.

Points labeled with A' and A in Figure 1 are in phase if B is ahead of A by a value of Δϕ. For a given wavelength, the relationship between the parameters is expressed by Eq. (2).

$$\theta = \arcsin\left(\frac{\lambda}{2\pi}\cdot\frac{\Delta\phi}{\Delta x}\right) \tag{2}$$

Considering a continuous arrangement of sources, Δϕ /Δx turns into dϕ /dx, this is the phase gradient of the array. Eq. (2) allows analyzing the necessary conditions for the angle of emission θ to be non-dispersive (that is, to be independent of the frequency). As the argument of the arcsine depends explicitly on the wavelength it is necessary that the gradient of phase dϕ/dx be inversely proportional to λ.

1.2 Wavefront reflected by a corrugated surface

Figure 2 shows a corrugated surface formed by a set of rectangular grooves of different depths. A plane wave incident will face each open inlet of the grooves; propagate along its length and return to be emitted by the open outlet with a change on its phase. The phase shift derives from the difference in the wave path which will result in a phase-gradient inversely proportional to λ, satisfying the condition indicated by Zhu to ensure dispersion-less reflection.

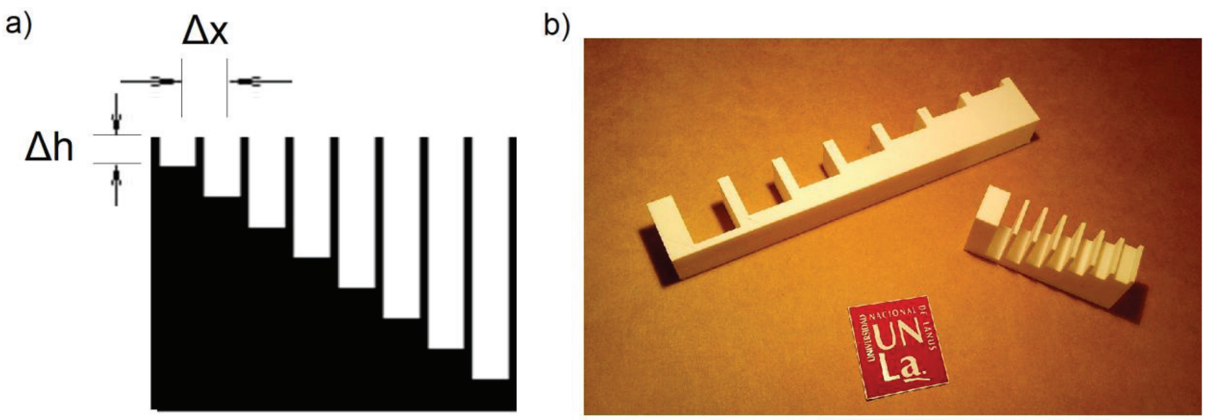

Figure 2 – Corrugated surface. (a) section drawing, (b) samples of 3D printed surfaces

The lower cutoff frequency of the bandwidth will be related to the total length of the array (in the x-direction of Figure 2b), and its upper frequency with the gap between grooves (Δx).

Given a groove depth Δh, the necessary time to travel through it with propagation velocity c is shown in Eq. (3), and the phase change in Eq. (4), where c stands for the propagation velocity.

$$\Delta t = \frac{2 \cdot \Delta h}{c} \tag{3}$$

$$\Delta \phi = \frac{2\pi}{\lambda} \cdot 2 \cdot \Delta h \tag{4}$$

Replacing equation (4) in Eq. (2) cancels the factor $\lambda/2\pi$. Eq. (5) relates the depth gradient of the grooves $\Delta h/\Delta x$ with the reflection angle without dispersion.

$$\theta = \arcsin\left(2 \cdot \frac{\Delta h}{\Delta x}\right) \tag{5}$$

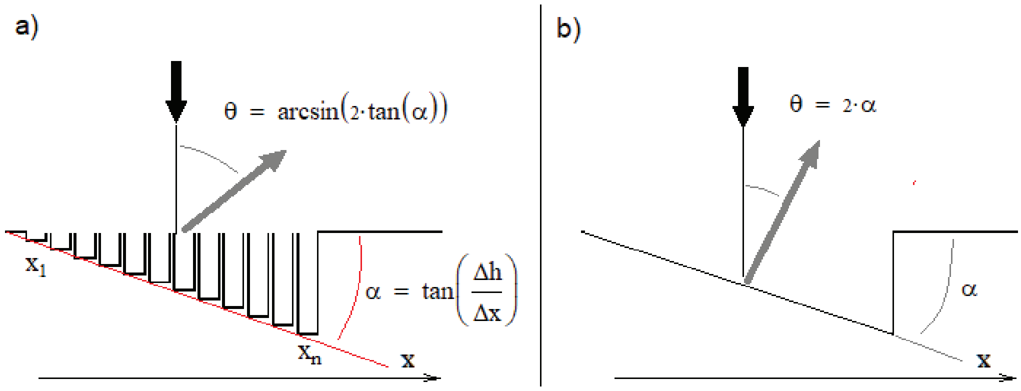

Figure 3 – Reflection angle comparison on a corrugated surface and a plane surface

Figure 3a shows the reflection angle θ produced by a corrugated surface. The reflected angle is equal to arcsin(2.tan(α)), because $\alpha = \tan(\Delta h/\Delta x)$. On the plane surface (Figure 3b), reflected angle is equal to twice the angle α. Figure 4 shows a simulation of surfaces reflecting a plane wave, both with the same value of α.

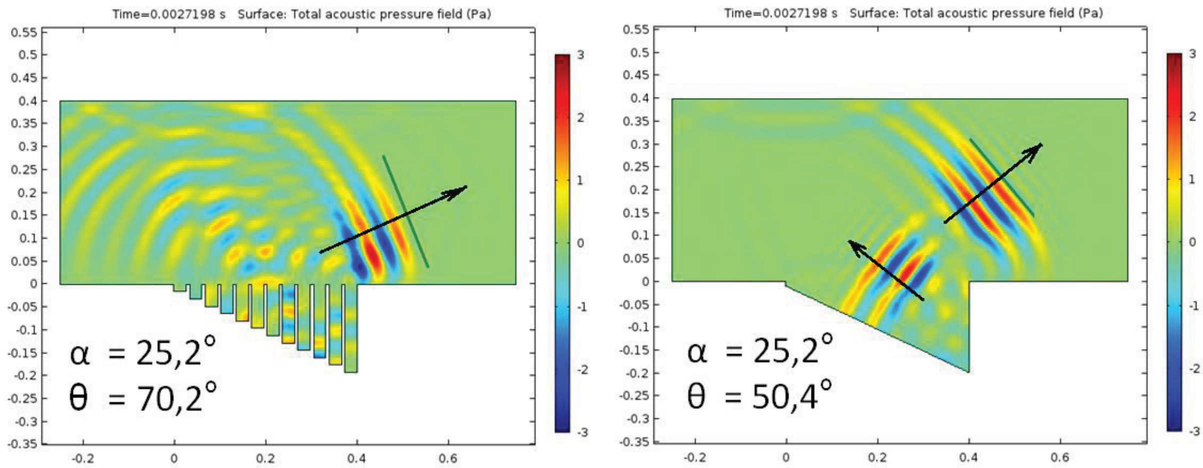

Figure 4 – Wave reflection simulation on a corrugated surface and a plane surface

2. DESIGN OF A SPECIFIC PROFILE

In the previous descriptions, it has been assumed a constant value $\Delta h/\Delta x$ between adjacent grooves. In general, the non-dispersive condition can be satisfied by deriving the Eq. (4) for a continuous case in which the groove depth profile h depends on x (Eq. 6).

$$\frac{d\phi}{dx} = \frac{4\pi}{\lambda} \cdot \frac{dh}{dx} \tag{6}$$

By integration, Eq. (7) is obtained which allows the design of a specific profile h(x) from the desired phase gradient.

$$h(x) = \frac{\lambda}{4\pi} \cdot \int \frac{d\phi}{dx}\, dx \tag{7}$$

The Eq. (7) expresses in general terms the proposal of Zhu. This process starts by selecting a reference frequency f_0 to determine λ, and hence the depth profile h(x).

There is an alternative that does not require using a reference frequency f_0 for the design expressing the phase gradient dh/dx in terms of time-delay gradient dτ/dx (Eq. 8), to obtain Eq. (9)

$$\frac{d\phi}{dx} = \frac{2\pi}{\lambda} \cdot c \cdot \frac{d\tau}{dx} \tag{8}$$

$$h(x) = \frac{c}{2} \cdot \int \frac{d\tau}{dx}\, dx = \frac{c}{2} \cdot \tau(x) + C \tag{9}$$

Eq. (9) derives the groove-depth profile h(x) from the time-delay of the array τ(x). In practice, the constant of integration C corresponds to a parallel displacement of the entire depth profile h(x) and it results in a constant time delay for all elements of the array. This delay can be controlled independently from the previously selected reflection angle. Figure 5 shows simulations of profiles with the same angle of reflection and different delays.

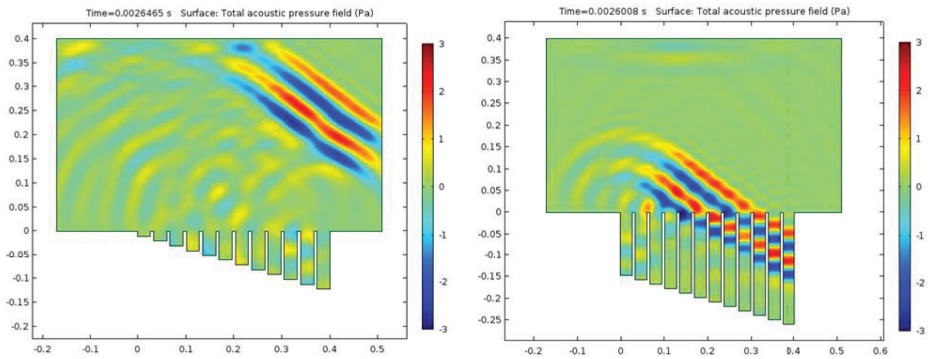

Figure 5 – Different delays in corrugated surfaces

3. DOUBLE REFLECTION PROFILES

A surface with double reflection can be obtained with two depth profiles h1(x) and h2(x), with alternating values for the depth of each slit, by distributing, for example, to the odds slits the profile h1(x) and to the evens h2(x). On the basis of what has been discussed in the previous section, different delays for each reflection can be selected. Each of the reflections will be generated by sets of slots (odd or even) that are spaced further apart, so the maximum frequency limit, which depends on the separation between the slits, will be lower. Figure 6 shows a simulation of a profile designed to have a double reflection (one matches to θ1 = 45° and the other to θ2 = 0°).

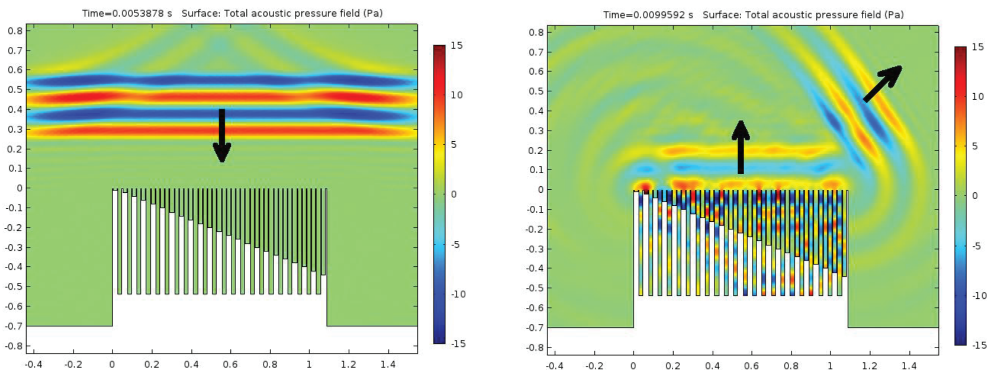

Figure 6 – First two reflections of plane wave on a corrugated surface

A double-reflection design generates, in fact, multiple sets of reflections in sequence due to the interaction between adjacent slots. Each wavefront reflected by the set of slots of order even (corresponding to θ2 = 0° in the example of Figure 6) will cause new pressure variations in the inputs of the adjacent slots odd-numbered. This will cause extra waves crossing odd slots, and drive to a further reflection with a new time delay, as shown in Figure 7.

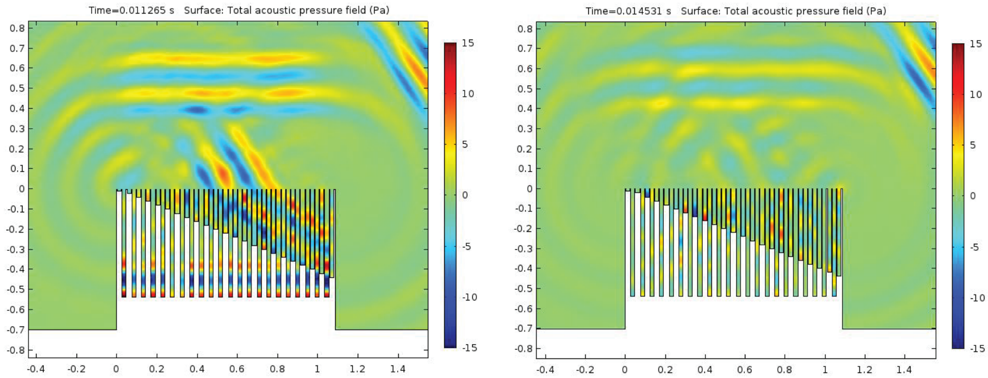

Figure 7 – Later reflections (reverberant tail) generated by a corrugated surface

4. CONCLUSIONS

The proposal of Zhu et al. allows for the design of corrugated surfaces to achieve reflection without dispersion in a controlled manner within a range of frequencies from knowing the gradient of the phase desired. The proposal presented here replaces the phase gradient for a time-delay gradient, which avoids the selection of a reference frequency. On the other hand, it is possible to select a time delay of the reflected pattern in an independent way of the angle of reflection, simply by shifting horizontally the depth profile of the grooves. The preliminary simulations carried out test the behavior of surfaces with a double reflection pattern, which in turn causes that the reflections do not become extinct immediately, but have an exponential decay. Such profiles with double reflection and with exponential decay of reflections may have interesting applications in room acoustics. Currently, the authors are developing physical models of such profiles in order to complement the study of these profiles with experimental measurements.

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