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A new technique for measuring sound particle velocity and sound pressure using face-to-face cardioid microphones

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ABSTRACT
Generally sound particle velocity can be measured by using a particle velocity sensor directly or by two sound pressure type microphones. Impedances and absorption coefficients of materials can be measured by these sensors. We have proposed a new technique for measuring sound particle velocity and sound pressure using face-to-face cardioid microphones, which is called the C-C method. A cardioid microphone responds to both sound pressure and particle velocity. Directivity of the cardioid microphone is formed by a combination of both sound pressure and particle velocity. In our new technique, the sound pressure and particle velocity can be obtained simply by summation and subtraction of responses of the face-to-face cardioid microphones. However, errors of measured sound pressure and sound particle velocity occur due to differences of directivities of the cardioid microphones. Therefor we have developed a calibration method of the cardioid microphones. By this calibration, accurate sound pressure and particle velocity can be measured even by using any kinds of cardioid microphones.

Keywords: Sound particle velocity, Sound intensity, Impedance, Cardioid microphone, C-C method

1. INTRODUCTION

The authors have proposed the C-C method which can measure sound pressure and sound particle velocity using face-to-face cardioid microphones [1, 2]. Sound intensity, sound energy density, acoustic impedance and so on can be measured by using the C-C method [3, 4]. This paper describes a new algorithm of the C-C method. Previous C-C method was premised on cardioid microphones have ideal cardioid directivity patterns. However, the directivities are not always ideal cardioid patterns in reality. We cannot measure accurately using such directivities.

Therefore we developed the new algorithm of C-C method which can accurately measure the sound pressure and the sound particle velocity even in conditions where the microphone doesn't have an ideal cardioid pattern. This paper describes the new algorithm of the C-C method and a method for calibrating a cardioid microphone used for the C-C method.

2. PREVIOUS ALGORITHM OF THE C-C METHOD

2.1 Formulation in far sound field

First version of the C-C method was premised on both far sound field and the ideal cardioid directivity patterns represented by Eq. (1).

$$C(\theta) = 0.5 + 0.5\cos(\theta) \tag{1}$$

We assumed a condition where a plane wave comes from angle θ relative to the x-axis as shown in Figure 1. In such far sound field, sound particle velocity $u(t)$, a x-component of the particle velocity $u_x(t)$, and a x-component of sound intensity can be represented by Eqs. (2) to (4) respectively.

$$u(t) = p(t) / \rho c \tag{2}$$

$$u_x(t) = -\{p(t)\cos\theta\} / \rho c \tag{3}$$

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$$I_x(t) = p(t)u_x(t) = -\{p^2(t)\cos\theta\}/\rho c \tag{4}$$

When this sound field is measured by face-to-face cardioid microphones, Mic.1 and Mic.2, responses by the two microphones are represented as

$$M_1(t) = p(t)(0.5 + 0.5\cos\theta) \tag{5}$$

$$M_2(t) = p(t)(0.5 - 0.5\cos\theta) \tag{6}$$

From Eqs. 5 and 6, sound pressure can be obtained by a sum of the two responses as Eq. (7).

$$M_1(t) + M_2(t) = p(t) \tag{7}$$

And a difference of the two responses detects a bi-directional component as Eq. (8).

$$M_1(t) - M_2(t) = p(t)\cos\theta \tag{8}$$

The x-component of particle velocity can be obtained as following Eq. (9).

$$u_x(t) = -\{M_1(t) - M_2(t)\}/\rho c \tag{9}$$

Therefore the x-component of the sound intensity can be calculated as

$$\begin{aligned} I_x(t) &= p(t)u_x(t) \\ &= -\{M_1(t) + M_2(t)\}\{M_1(t) - M_2(t)\}/\rho c \end{aligned} \tag{10}$$

Furthermore, the sound intensity can also obtained by the following equation.

$$I_x(t) = -\{M_1^2(t) - M_2^2(t)\}/\rho c \tag{11}$$

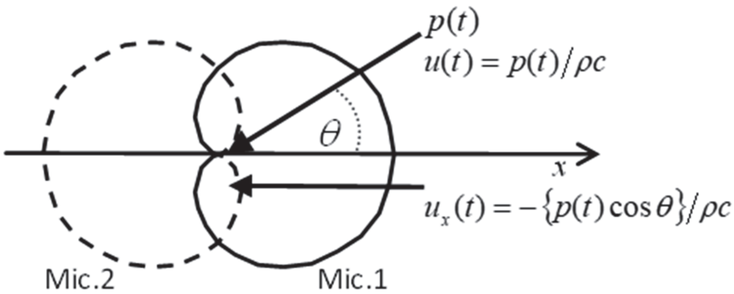

Figure 1 – C-C method in far sound field

2.2 Formulation in near sound field

For the formulation for near sound fields, a response of the cardioid microphone is redefined using a sound pressure $p(t)$ and a x-component of the particle velocity $u_x(t)$.

$$M(t) = 0.5p(t) - 0.5u_x(t)\rho c \tag{12}$$

where ρ and c represent a density of medium and a speed of sound.

The second term has a same dimension as the first term by multiplying the ρc and represents sound pressure gradient proportional to the sound particle velocity. As shown in Figure 2, the sound pressure and the particle velocity at a distance r from a sound source can be represented as the following equations respectively.

$$p(t,r) = (A/r)\exp[i(\omega t - kr)] \tag{13}$$

$$u(t,r) = [p(t,r)/\rho c](1 - i/kr) \tag{14}$$

where A, i, ρ and k represent a complex amplitude, an imaginary unit, an angular frequency and a wavelength constant respectively.

The x-component of particle velocity can be obtained as the following equation.

$$u_x(t,r) = -\{p(t,r)(1 - i/kr)\cos\theta\}/\rho c \tag{15}$$

Therefore the x-component of the sound intensity can be calculated as Eq. (16).

$$I_x(t,r) = -\{p^2(t,r)(1 - i/kr)\cos\theta\}/\rho c \tag{16}$$

When this sound field is measured by face-to-face cardioid microphones, responses by the two microphones are represented as

$$M_1(t,r) = p(t,r)\{0.5 + 0.5(1 - i/kr)\cos\theta\} \tag{17}$$

$$M_2(t,r) = p(t,r)\{0.5 - 0.5(1 - i/kr)\cos\theta\} \tag{18}$$

Sound pressure can be obtained by a sum of the two responses as the following equation.

$$M_1(t,r) + M_2(t,r) = p(t,r) \tag{19}$$

And a difference of the two responses detects a bi-directional component as

$$M_1(t,r) - M_2(t,r) = p(t,r)(1 - i/kr)\cos\theta\ . \tag{20}$$

The x-component of particle velocity can be obtained as following Eq. (21).

$$u_x(t,r) = -\{M_1(t,r) - M_2(t,r)\}/\rho c \tag{21}$$

Therefore the x-component of the sound intensity can be calculated as

$$\begin{aligned} I_x(t,r) &= p(t,r)u_x(t,r) \\ &= -\{M_1(t,r) + M_2(t,r)\}\{M_1(t,r) - M_2(t,r)\}/\rho c \\ &= -\{M_1^{\,2}(t,r) - M_2^{\,2}(t,r)\}/\rho c \end{aligned} \tag{22}$$

In the C-C method, as described above, sound pressure and particle velocity can be obtained simply by a sum and a difference of responses of the face-to-face cardioid microphones. And sound intensity can be obtained by multiplying both results of sum and difference.

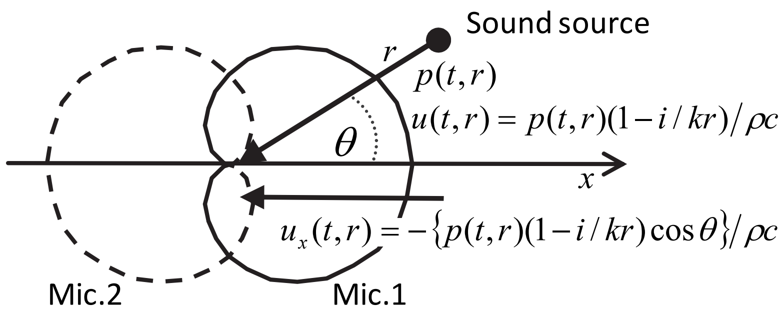

Figure 2 – C-C method in near sound field

3. IMPROVED ALGORITHM OF THE C-C METHOD

3.1 Improved algorithm

First and second terms of Eq. (12) represent omnidirectional and bidirectional components respectively. The cardioid directivity pattern is come up by combining both components in ratios of just 0.5 and 0.5 in the far sound field. However, an actual cardioid microphone has always different directivity from ideal cardioid pattern depending on frequency and individual differences of the microphone. When using such directivities, we cannot measure accurately based on the algorithms mentioned in Chapter 2.

Therefore we propose a new algorithm of the C-C method which can accurately measure the sound pressure and the sound particle velocity even with microphones which don't have an ideal cardioid pattern. In the new algorithm, a response of the cardioid microphone $M(t)$ is redefined as Eq. (23). Both omnidirectional and bidirectional components are combined in ratios of α and β in the Eq. (23).

$$M(t) = \alpha p(t) - \beta\{u_x(t)\rho c\} \tag{23}$$

The α means a sensitivity coefficient of the microphone for sound pressure component, and the β means a sensitivity coefficient for sound particle velocity component. If both α and β are the same, the directivity becomes a cardioid pattern. However, because the α and β of an actual cardioid microphone are not the same, we need to know both α and β to obtain measurement results accurately. If both α and β are the known, it is not necessary that the directivity is a cardioid pattern. When calibrating the cardioid microphone, it is not enough to calibrate only maximum sensitivity of the microphone. Calibrations of both α and β of the microphone are needed.

Eq. (23) can be used for both near and far sound field. When sound fields shown in Figures 1 or 2 are measured by face-to-face cardioid microphones, responses by the two microphones are represented as

$$M_1(t)=\alpha_1 p(t)-\beta_1\{u_x(t)\rho c\} \tag{24}$$

$$M_2(t)=\alpha_2 p(t)+\beta_2\{u_x(t)\rho c\}. \tag{25}$$

where α_1 and β_1 for Mic.1, α_2 and β_2 for Mic.2.

Summation and subtraction of $M_1(t)$ and $M_2(t)$ are written as

$$M_1(t)+M_2(t)=(\alpha_1+\alpha_2)p(t)-(\beta_1-\beta_2)\{u_x(t)\rho c\} \tag{26}$$

$$M_1(t)-M_2(t)=(\alpha_1-\alpha_2)p(t)-(\beta_1+\beta_2)\{u_x(t)\rho c\}. \tag{27}$$

According to Eqs (26) and (27), $p(t)$ and $u_x(t)$ cannot be obtained simply by summation and subtraction when $\alpha_1\neq\alpha_2$ or $\beta_1\neq\beta_2$. The term of particle velocity is eliminated from the simultaneous equations of Eqs. (26) and (27).

$$\frac{M_1(t)}{\beta_1}+\frac{M_2(t)}{\beta_2}=\left(\frac{\alpha_1}{\beta_1}+\frac{\alpha_2}{\beta_2}\right)p(t) \tag{28}$$

The sound pressure can be determined as follows.

$$p(t)=\left\{\frac{M_1(t)}{\beta_1}+\frac{M_2(t)}{\beta_2}\right\}\Bigg/\left\{\frac{\alpha_1}{\beta_1}+\frac{\alpha_2}{\beta_2}\right\} \tag{29}$$

Similarly, when the term of sound pressure $p(t)$ is eliminated, the following equation is obtained.

$$\frac{M_1(t)}{\alpha_1}-\frac{M_2(t)}{\alpha_2}=-\left(\frac{\beta_1}{\alpha_1}+\frac{\beta_2}{\alpha_2}\right)\{u_x(t)\rho c\} \tag{30}$$

From Eq. (30), the particle velocity $u_x(t)$ can be obtained as follows.

$$u_x(t)=-\left\{\frac{M_1(t)}{\alpha_1}-\frac{M_2(t)}{\alpha_2}\right\}\Bigg/\left\{\frac{\beta_1}{\alpha_1}+\frac{\beta_2}{\alpha_2}\right\}\rho c \tag{31}$$

As described above, in the new algorithm, the directivity of the microphone does not have to be exactly cardioid. If α and β of two cardioid microphones are known, sound pressure and particle velocity can be obtained even if there are individual differences among microphones. The next section describes how to measure α and β.

3.2 Calibration algorithm of the C-C method

First, it is needed to prepare a sound field with known sound pressure and particle velocity, such as a plane wave sound field in a free sound field or a one-dimensional sound field in an acoustic tube. Here, as shown in Figure 3, a calibration method using a sound field in which a single plane wave (calibration signal) comes from the + direction of the x-axis will be described. Let $p_0(t)$ and $u_0(t)$ be the sound pressure and particle velocity of the calibration signal at the microphone position, respectively. Since it is a single plane wave, $p_0(t)=u_0(t)\rho c$. Let $M^+(t)$ and $M^-(t)$ be the responses measured with the sensitivity maximum direction of the cardioid microphone directed to the sound source and its opposite direction. The next section describes how to measure The α and β can be obtained by Eqs. (32) and (33).

$$\alpha = \frac{\sqrt{\frac{1}{T}\int_0^T \left\{M^+(t)+M^-(t)\right\}^2 dt}}{2\sqrt{\frac{1}{T}\int_0^T p_0^{\,2}(t)\,dt}} \tag{32}$$

$$\beta = \frac{\sqrt{\frac{1}{T}\int_0^T \left\{M^+(t)-M^-(t)\right\}^2 dt}}{2\rho c\sqrt{\frac{1}{T}\int_0^T u_0^{\,2}(t)\,dt}} \tag{33}$$

When the sound field is a single plane sound field, the sound pressure can be used for the denominator.

$$\beta \approx \frac{\sqrt{\frac{1}{T}\int_0^T \left\{M^+(t)-M^-(t)\right\}^2 dt}}{2\sqrt{\frac{1}{T}\int_0^T p_0^{\,2}(t)\,dt}} \tag{34}$$

The α and β are determined separately for the face-to-face cardioid microphones, Mic.1 and Mic.2 used for the measurement, and are defined as α_1 and β_1 for Mic.1, α_2 and β_2 for Mic.2. In Eq. (23), if the microphone response $M(t)$ is the microphone output voltage [mV], the α and β can be interpreted as the microphone sensitivity [mV / Pa] for the sound pressure and particle velocity components, respectively. Therefore, if the α and β of the microphone used for measurement are obtained in advance for each measurement frequency (approximately, for each measurement band), the absolute value of the sound pressure and the particle velocity can be measured by the C-C method.

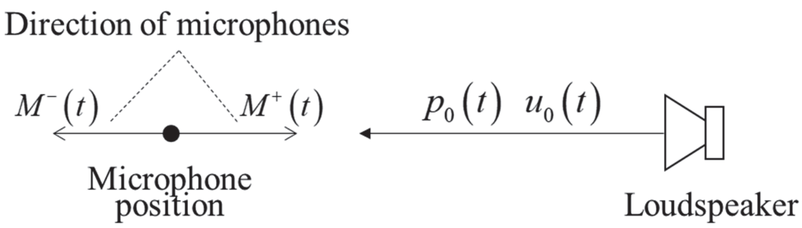

Figure 3 – Method for measuring α and β

3.3 The C-C method in frequency domain

Here, the C-C method in frequency domain with more accurate calibration in consideration of the phase will be described. The $\mathrm{P}_0(\omega)$, $\mathrm{U}_0(\omega)$, $\mathrm{M}^P(\omega)$ and $\mathrm{M}^m(\omega)$ are defined as Fourier transforms of $p_0(t)$, $u_0(t)$, $\left\{M^+(t)+M^-(t)\right\}$ and $\left\{M^+(t)-M^-(t)\right\}$ in Figure 3. The α and β in the frequency domain are complex functions including phase information with ω as a parameter, and can be obtained as in the following equations.

$$\alpha(\omega) = \mathrm{M}^P(\omega)/2\,\mathrm{P}_0(\omega) \tag{35}$$

$$\beta(\omega) = -\mathrm{M}^m(\omega)/2\rho c\mathrm{U}_0(\omega) \tag{36}$$

If the sound field for calibration is a single plane wave field, it can be calculated as follows.

$$\beta(\omega) \approx \mathrm{M}^m(\omega)/2\,\mathrm{P}_0(\omega) \tag{37}$$

Using $\alpha(\omega)$ and $\beta(\omega)$, sound pressure and particle velocity are obtained as in Eqs. (38) and (39).

$$
\mathrm{P}(\omega)=\left\{\frac{\mathrm{M}_1(\omega)}{\beta_1(\omega)}+\frac{\mathrm{M}_2(\omega)}{\beta_2(\omega)}\right\}\Big/\left\{\frac{\alpha_1(\omega)}{\beta_1(\omega)}+\frac{\alpha_2(\omega)}{\beta_2(\omega)}\right\} \tag{38}
$$

$$
\mathrm{U}_x(\omega)=-\left\{\frac{\mathrm{M}_1(\omega)}{\alpha_1(\omega)}-\frac{\mathrm{M}_2(\omega)}{\alpha_2(\omega)}\right\}\Big/\left\{\frac{\beta_1(\omega)}{\alpha_1(\omega)}+\frac{\beta_2(\omega)}{\alpha_2(\omega)}\right\}\rho c \tag{39}
$$

where $\mathrm{M}_1(\omega)$, $\mathrm{M}_2(\omega)$, $\mathrm{P}(\omega)$ and $\mathrm{U}_x(\omega)$ are Fourier transforms of $M_1(t)$, $M_2(t)$, $p(t)$ and $u_x(t)$ respectively.

By inverse Fourier transform of $\mathrm{P}(\omega)$ and $\mathrm{U}_x(\omega)$, sound pressure $p(t)$ and particle velocity $u_x(t)$ can be obtained as temporal responses.

4. CONCLUSIONS

We proposed a new algorithm of the C-C method that can measure sound pressure and particle velocity accurately even if the microphone does not have cardioid directivity. In the future, we plan to verify the validity of the algorithm by experiments.

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