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Acoustic radiation modes and active structural acoustic control in coupled enclosure

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ABSTRACT

There are two problems in the active structural acoustic control (ASAC) of coupled enclosure. Firstly, the structural mode information is required in order to obtain the acoustic radiation mode (ARM) of coupled enclosure. Secondly, only the radiation efficiency is included in conventional ASAC strategy of coupled enclosure, which has not enough good precision. To overcome these problems, by analogy to ARM theory of free space, the acoustic potential energy is expressed in quadratic form of normal velocity on coupling surface and ARM of coupled enclosure is redefined. So the proposed ARM theory of coupled enclosure is consistent with that of free space and is convenient for application. The control strategy based on the dominant acoustic radiation mode (dARM) is proposed, and corresponding ASAC model of the coupled enclosure is also established. Simulation analysis and experimental research on elastic plate-rectangular coupled enclosure are carried out. Both simulation and experimental results show that better control effect can be obtained by controlling APE contributed by dARM. Therefore, the feasibility and effectiveness of the proposed dARM control strategies are proved.

Keywords: ASAC, Acoustic radiation mode, Coupled enclosure

1. INTRODUCTION

Acoustic Radiation Mode (ARM) theory, which was put forward in 1990s (1,2), is an effective structural acoustic analytical method. The ARM is a specific radiation pattern of a radiator surface, and all normal velocity or surface pressure on the vibrating surface can be expressed by a linear combination of ARMs. An important feature of ARM applied in structure radiation research is that coupling terms of structural modes may be avoided and acoustic radiation power can be reduced by controlling an arbitrary order of ARM coefficient amplitude. Therefore, the calculation and control of acoustic radiation can be performed more effectively (3-5).

The problems of noise control in coupled enclosure exist widely in industry and life environments, such as cabins of plane, ship and vehicle, rooms for working and living. A lot of researches at home and aboard have been conducted on this topic. Among them, the applications of Active Structural Acoustic Control (ASAC) attracted extensive attention, due to avoidance of secondary sound sources by actuators applied on structure directly instead and good performance with a few actuators. Point force control was applied on a cylindrical shell to reduce the transmitted sound power by Simpson (6), and internal noise control in DC-9 cabin was achieved successfully. The mechanism of ASAC in coupled enclosure was studied in detail by Pan (7-9) via both theoretic analysis and experiment research. Snyder (10) firstly proposed a formulation to derive the eigenvectors of the error weighting matrix of coupled enclosure, and corresponding ASAC model was built. Similar researches on ARM of weak coupled enclosure were also conducted by Cazzolato (11), Hill (12) and Bagha (13), and great achievements were made.

However, it's worthy to note that the "ARMs of weak coupled enclosure" in previous studies are incompatible with ARMs in free space in physical significance, namely, the former are a set of basis functions of structural mode amplitude whereas the latter are a set of basis functions of structural surface normal vibration or pressure. On the other hand, structural mode is still necessary in calculation and control process. However, accurate structural modes cannot be obtained conveniently due to complexity of structures in practice and dependence on the boundary conditions. In addition, only radiation efficiency is considered in the ASAC, which may fail to achieve the expected control

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effect or lead to low control efficiency.

To overcome these problems, by analogy to ARM theory of free space, the acoustic potential energy was expressed in quadratic form of normal velocity on coupling surface and ARM of coupled enclosure was redefined. Then, the active control strategy based on the dominant acoustic radiation mode (dARM) is proposed, and the corresponding ASAC model is established.

2. AOUSTIC RADIATION MODE THEORY

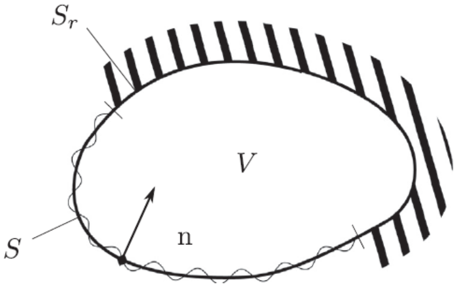

Figure 1 –The coupled enclosure

Considering the minimization problem of sound transmission into a coupled enclosure. The Acoustic Potential Energy (APE) of an enclosure is chosen as the global error criterion, which is written as

$$E_p = \frac{1}{4\rho_0 c_0^2}\int_V |p(\boldsymbol{r})|^2\, d\boldsymbol{r}, \tag{1}$$

where ρ_0 is the density and c_0 the speed of sound within the acoustic fluid enclosed by the enclosure, $p(r)$ is the acoustic pressure and the integral is evaluated over the enclosure volume V, and $r = (x, y, z)$.

With the assumption of weak coupling between the flexible structure and enclosure, the acoustic pressure at an any location in the enclosure can be expressed as a linear combination of the rigid wall acoustic mode shape functions of the enclosure

$$p(\boldsymbol{r}) = \sum_{i=1}^{\infty} P_i \boldsymbol{\varphi}_i(\boldsymbol{r}), \tag{2}$$

where P_i is the i-th acoustic mode amplitude and $\varphi_i(\boldsymbol{r})$ is the associated mode shape function at the location $\boldsymbol{r}$ in the enclosure.

Assuming that there is no fluctuating sound source in the closed cavity, the sound pressure $p(\mathbf{r})$ at any point $\mathbf{r}$ in the enclosure generated by the elastic structure vibration can be defined as the Green's function response equation

$$p(\boldsymbol{r}) = j\rho_0\omega \int_S G_a(\boldsymbol{x}|\boldsymbol{r})\boldsymbol{v}(\boldsymbol{x})d\boldsymbol{x}, \tag{3}$$

where $\boldsymbol{v}(\boldsymbol{x})$ is the normal velocity on the structural boundary S at location $\boldsymbol{x}$, and G_a is the Green's function for the enclosure satisfying rigid walled boundary conditions

$$G_a(\boldsymbol{x}|\boldsymbol{r}) = \sum_{i=1}^{\infty} \frac{\boldsymbol{\varphi}_i(x)\boldsymbol{\varphi}_i(r)}{\Lambda_i(\kappa_i^2 - k^2 + 2j\kappa_i k\xi_i)}, \tag{4}$$

where κ_i and k are acoustic wave number of the i-th order acoustic mode frequency ω_i and the analysis frequency ω, ξ_i is the modal damping ratio of i-th enclosure mode, Λ_i is the volume normalization factor of the i-th acoustic mode with $\Lambda_n = \int_V \boldsymbol{\varphi}_n(\mathbf{r})^* \boldsymbol{\varphi}_n(\mathbf{r})\, d\mathbf{r}$, and the superscript "*" denotes the conjugate operator.

Suppose that the surface of the elastic structure is discretized into M elements. Considering the first N-th order acoustic modes, and using the orthogonality between the acoustic modes, the APE of enclosure can be expressed as the form of the normal velocity of the elastic structure surface (14)

$$E_p = \boldsymbol{v}^{\boldsymbol{H}}\boldsymbol{\Omega}\boldsymbol{v}, \tag{5}$$

where the superscript "H" denotes the Hermite operator, $\boldsymbol{v}$ is an $(M \times M)$ vector of normal velocity on the discrete elastic surface, $\boldsymbol{\Omega}$ is an $(M \times M)$ error weighting matrix, and the (m, n)-th element expression is

$$\boldsymbol{\Omega}(m,n) = \sum_{i=1}^{N} \frac{\rho_0 k^2 s_m s_n}{4\Lambda_i|\kappa_i^2 - k^2 + 2j\kappa_i k\xi_i|^2} \boldsymbol{\varphi}_i(x_m)^* \boldsymbol{\varphi}_i(x_n), \tag{6}$$

where s_m and s_n are the areas of the m-th and n-th discrete elements of the elastic structure surface respectively.

The definition of error weighting matrix $\mathbf{\Omega}$ shows that $\mathbf{\Omega}$ is Hermitian. Additionally, from the physic meaning of the APE E_p in Equation 5, it can be deduced that $E_p > 0$ will be always true except the normal velocity vector equals 0. Therefore, $\mathbf{\Omega}$ is positive definite and Hermitian, and it can be expressed by the eigenvalue decomposition as

$$\boldsymbol{\Omega} = \boldsymbol{D}\boldsymbol{\Lambda}\boldsymbol{D}^H, \tag{7}$$

where $\mathbf{\Lambda}$ is a diagonal matrix with eigenvalues $\lambda_i(i = 1,2,\cdots I)$ decreasing monotonically along the diagonal, $\mathbf{D}$ represents an $(M \times M)$ matrix whose columns are eigenvectors $\mathrm{d}_i(i = 1,2,\cdots M)$. Since $\mathbf{\Omega}$ is positive definite and Hermitian, all of its eigenvalues λ_i are positive real and its eigenvectors d_i are orthogonal to each other. Thus, it can be seen that eigenvectors $\mathrm{d}_i$are a set of basis functions in the M dimensional space, so they're termed as the Acoustic Radiation Modes of coupled enclosure specifically. It can be found that no structural modal information of the elastic structure is required in the calculation of ARM.

Similar to the ARM in free space, the ARM of coupled enclosure represents a particular velocity pattern on the coupling surface. Both of them are dependent on the exciting frequency and the geometry of the flexible surface, but independent of the surface material and its boundary condition. However, different from the ARM in free space, the ARM of coupled enclosure is dependent on the geometry, natural frequencies and damp of the enclosure in addition. On this basis, the ARM theory of coupled enclosure which is consistent with that of free space and convenient for application was formed.

3. ACTIVE CONTROL STRATEGY

From Equation (7), it can be seen that eigenvectors $\mathrm{d}_i(i = 1,2,\cdots M)$ are a set of basis functions in M dimensional space, and then all arbitrary normal velocity $\mathbf{v}$ on the elastic surface can be expanded uniquely by the eigenvectors $\mathbf{d_i}$ as

$$\mathbf{v} = \mathbf{D}\mathbf{y} = \sum_{i=1}^{M} y_i \mathbf{d_i}, \tag{8}$$

where $\mathbf{y}$ is the ARM amplitude vector.

Substituting Equations (7, 8) into Equation (5), the APE of coupled enclosure can be further written as follows

$$E_p = \mathbf{y}^H \mathbf{\Lambda} \mathbf{y} = \sum_{i=1}^{M} \lambda_i |y_i|^2 = \sum_{i=1}^{M} Ep_i, \tag{9}$$

It can be seen from Equation (9) that the APE of coupled enclosure can be decoupled by the ARMs, that is, the APE can be reduced by control of an arbitrary order of ARM amplitude.

In ASAC, the primary problem is the determination of the dominant acoustic radiation modes (dARM). In most previous studies, dARMs are obtained only based on the radiation efficiency λ_i. As can be seen from Equation (9), the APE is not only related to the radiation efficiency λ_i, but also affected by the ARM amplitude $|y_i|$. Therefore, the conventional control strategy of ASAC is not comprehensive enough. Combining radiation efficiency and the ARM amplitude, a new dARM determination method is proposed in the Reference (15). Therefore, once the dARM contributed APE is cancelled, ideal ASAC effect can be obtained. When the APE contributed by dARM is used as the control target, an active control strategy based on the dARM may be designed.

Suppose there are G dARMs g_1 , $g_2 \cdots, g_G$, and the dARM contributed APE can be written as

$$E_{pG} = \mathbf{y}_G^H \mathbf{\Lambda}_G \mathbf{y}_G = \sum_{i=1}^{G} \lambda_{g_i} \left| y_{g_i} \right|^2, \tag{10}$$

where $\mathbf{\Lambda}_G = diag(\lambda_{g_1}, \lambda_{g_2}, \cdots, \lambda_{g_G})$ is a diagonal matrix consisting of the radiation efficiency of dARM, and $\mathbf{y}_G$ is a ARM amplitude vector of G dARMs, according to Equation (8),

$$\mathbf{y}_G = \mathbf{D}_G \mathbf{v}_n, \tag{11}$$

where $\mathbf{D}_G = [\mathbf{d}_{g_1} \quad \mathbf{d}_{g_2} \quad \cdots \quad \mathbf{d}_{g_G}]$ is a $(M \times G)$ dimensional matrix, with G dARMs vectors in its columns.

Considering the coupling effect of the acoustic enclosure, the normal vibration velocity of the elastic structure surface can be expressed as

$$\mathbf{v}_n = \mathbf{T}(\mathbf{F} - \mathbf{p}(s)), \tag{12}$$

where $\mathbf{T}$ is the admittance matrix of the elastic structure without considering the coupling of the enclosure, $\mathbf{F}$ is the excitation force vector applied to the elastic structure surface, and $\mathbf{p}(s)$ is the surface acoustic pressure vector of the elastic structure in the enclosure. According to Equation (3), $\mathbf{p}(s)$ is expressed as matrix form is

$$\mathbf{p}(s) = \mathbf{G}\mathbf{v}_n, \tag{13}$$

Inserting Equation (13) into Equation (12), the relationship between the normal vibration velocity of the elastic structure surface and the external force can be expressed as

$$\mathbf{v}_n = (\mathbf{I} + \mathbf{T}\mathbf{G}_s)^{-1}\mathbf{T}\mathbf{F} = \mathbf{T}'\mathbf{F}, \tag{14}$$

where $\mathbf{T}' = (\mathbf{I} + \mathbf{T}\mathbf{G}_s)^{-1}\mathbf{T}$ is the admittance matrix of elastic structure considering coupling effect.

Assuming that m primary excitation forces and n secondary control forces act together on the elastic structure of the coupled enclosure. According to the superposition principle, the normal vibration velocity of the elastic structure surface can be written as

$$\mathbf{v}_n = \mathbf{v}_{np} + \mathbf{v}_{ns} = \mathbf{T}_p'\mathbf{F}_p + \mathbf{T}_s'\mathbf{F}_s, \tag{15}$$

By substituting Equations (11, 15) into Equation (10), E_{p_G} can be written as a quadratic form of the secondary control force complex amplitude

$$E_{p_G} = \mathbf{F}_s^H\mathbf{a}_A\mathbf{F}_s + \mathbf{F}_s^H\boldsymbol{b}_A + \boldsymbol{b}_A^H\mathbf{F}_s + \mathbf{c}_A, \tag{16}$$

where

$$\mathbf{a}_A = (\mathbf{T}_s')^{\mathbf{H}}\mathbf{D}_{\mathbf{G}}\mathbf{A}_{\mathbf{G}}\mathbf{D}_{\mathbf{G}}^{\mathbf{H}}\mathbf{T}_s', \tag{17}$$

$$\mathbf{b}_A = (\mathbf{T}_s')^{\mathbf{H}}\mathbf{D}_{\mathbf{G}}\mathbf{A}_{\mathbf{G}}\mathbf{D}_{\mathbf{G}}^{\mathbf{H}}\mathbf{T}_p'\mathbf{F}_p, \tag{18}$$

$$\mathbf{c}_A = \mathbf{F}_p^H\left(\mathbf{T}_p'\right)^{\mathbf{H}}\mathbf{D}_{\mathbf{G}}\mathbf{A}_{\mathbf{G}}\mathbf{D}_{\mathbf{G}}^{\mathbf{H}}\mathbf{T}_p'\mathbf{F}_p, \tag{19}$$

According to the quadratic optimal theory, the optimal secondary control force can be obtained by minimizing the dARM contributed APE

$$\mathbf{F}_{s,opm} = -\mathbf{a}_A^{-1}\mathbf{b}_A, \tag{20}$$

The minimum value of the dARM contributed APE is

$$E_{pG,min} = \boldsymbol{c}_A - \boldsymbol{b}_A^H\boldsymbol{a}_A^{-1}\boldsymbol{b}_A, \tag{21}$$

4. SIMUALTION ANALYSIS

In the following, the effectiveness of the dARM contributed APE control strategy is analyzed. The same numerical calculation model in Reference (7) is used here. The dimension of the coupled enclosure is given by $L_x \times L_y \times L_z = 0.868\ m \times 1.150\ m \times 1.000\ m$, and its top surface consists of a 6mm-thick simply supported aluminum panel and other five surfaces are perfectly rigid walls. The constants used in the calculation can also be found in reference 7. The primary excitation force is a single simple harmonic force with a amplitude $F_p = 1N$, applied to the lower left corner $(L_x/8, L_y/8)$ of the elastic plate to ensure that all the modes of the elastic panel in the analysis band can be excited; Three simple harmonic forces are arranged as secondary control forces, their amplitudes are denoted as F_s^1, F_s^2 and F_s^3, the excitation positions are $(0.434m,\ 0.575m,\ 1.000m)$, $(0.434m,\ 0.863m,\ 1.000m)$, and $(0.651m,\ 0.575m,\ 1.000m)$ respectively.

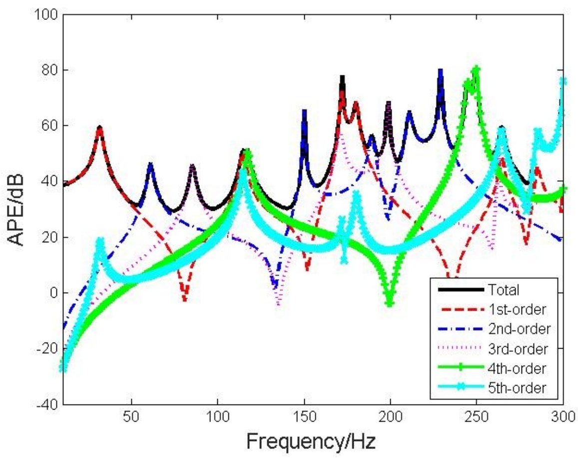

Figure 2 – First five order ARM contributed APE curves

The first five order ARM contributed APE curves by the primary excitation force are given Figure 2, in which the dARM at a given frequency can be observed clearly.

Figures 3-5 show the APE curves of coupled enclosure before and after control when using one secondary control force to cancel the APE contributed by the 1st ARM (i.e. $G = 1$and$g_1 = 1$) , using two secondary control forces to cancel the APE contributed by the 1st and 2nd ARMs (i.e. $G = 2$ and $g_1 = 1,\ g_2 = 2$), and using two secondary control forces to cancel the APE contributed by the 2nd and 3rd ARMs (i.e. $G = 2$ and $g_1 = 2,\ g_2 = 3$) respectively.

As illustrated by Figure 2 and Figure 3, the APE is significantly reduced only in the frequency band where the 1st ARM is the dARM, and there is almost no control effect or even control overflow in other frequency bands, such as 132 Hz (the dARM at this frequency is the fourth-order). Similarly, from Figure 2 and Figure 4, when the 1st and 2nd ARMs are controlled, the APE is greatly reduced in the frequency band where the 1st and 2nd ARMs are the dARMs and there is no control effect in other frequency bands. Also, from Figure 2 and Figure 5, when 2nd and 3rd ARMs are controlled, significant control effect is obtained in the frequency band where the 2nd and 3rd ARMs are the dARMs. But there is no longer reduction effect or even a large control overflow in the frequency band where the 1st ARM is the dARM.

Through the above analysis, it can be concluded that by controlling the dARM contributed APE, a satisfactory control effect can be obtained. Conversely, if any dARM is not cancelled, it is usually impossible to obtain the desired control effect. This shows that the proposed dARM based ASAC strategy is effective.

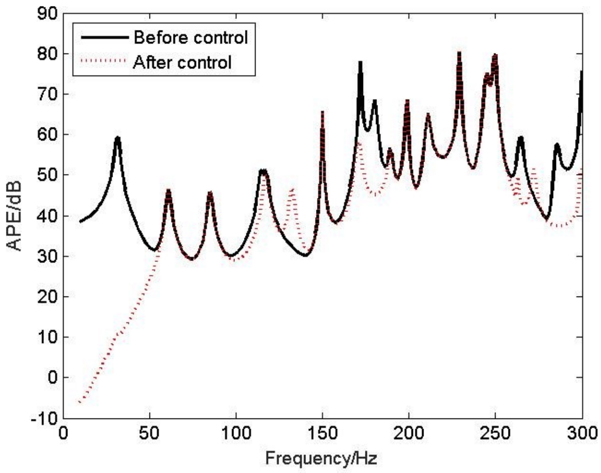

Figure 3 – Results of controlling the 1st ARM using one secondary control force

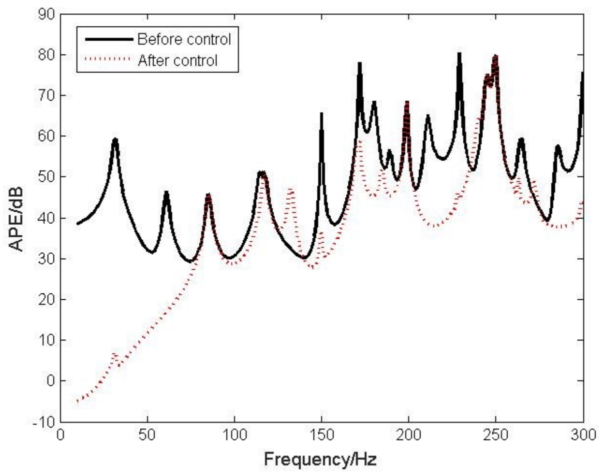

Figure 4 – Results of controlling the 1st and 2nd ARMs using two secondary control forces

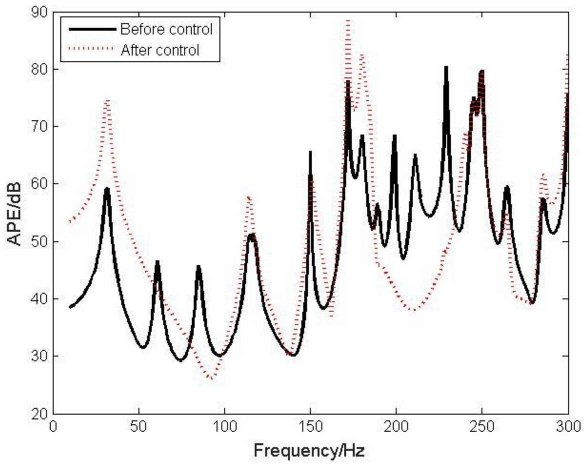

Figure 5 – Results of controlling the 2nd and 3rd ARMs using two secondary control forces.

5. EXPERIMENTAL RESEARCH

A fixed elastic plate-rectangular coupled enclosure model was designed to carry out experimental research. The inner wall dimension of the rectangular acoustic enclosure is $L_x \times L_y \times L_z = 0.65m \times 0.50m \times 0.45m$.The upper surface is an elastic steel plate with thickness $h_p = 0.0026m$, and the other five surfaces are steel plates with thickness $h_c = 0.03m$ (equivalent to a rigid wall). The elastic steel plate is fastened the rectangular enclosure by bolts, and is equivalent to the fixed boundary condition. The experimental system is shown in Figure 6.

In the plane of upper surface, with the lower left corner of the surface as the coordinate origin, a HEV-50 vibration exciter is installed at the position $(0.155m$, 0.05m)as the primary excitation, and another HEV-50 vibration exciter is at the position $(0.325m,\ 0.250m)$ as the secondary force source. The elastic plate is evenly meshed into 4×3 small area elements, 12 acceleration sensors are evenly arranged in the center of the 4×3 elements to measure the normal vibration velocity, which is used to determine the dARM and its amplitude (15). Four MPA416 microphones are arranged in the four corners of the bottom of the rectangular cavity to measure the APE of coupled enclosure before and after control (16). The adaptive controller is installed at the NI Compact RIO embedded measurement and control platform, and the Filtered-x LMS algorithm is used as the adaptive control algorithm.

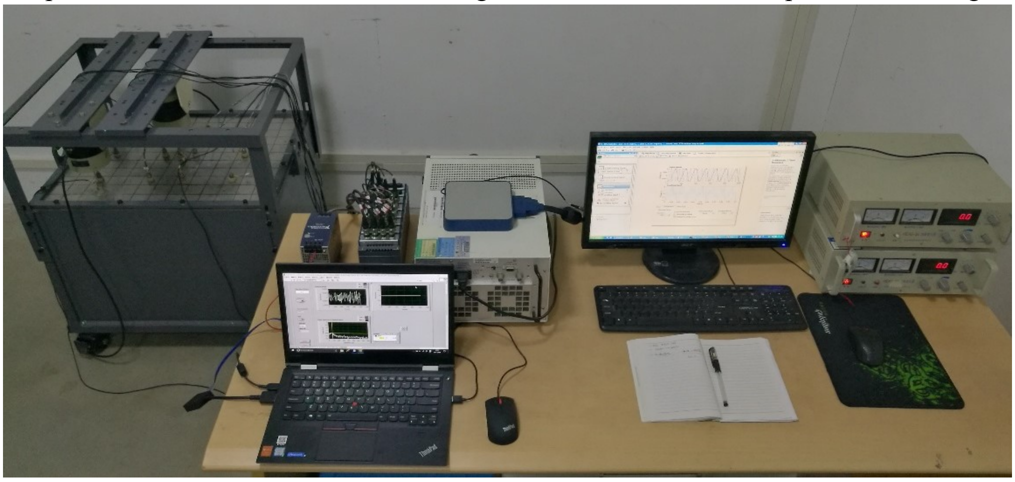

Figure 6 – Experimental system

The elastic plate (1,1) structure modal frequency 72 Hz was selected as the primary excitation frequency. The 1st ARM is determined as the dARM under the primary excitation by the method proposed in the Reference (15). Then the control system is turned on to cancel the determined the dARM contributed APE. Figure 7 shows the time history of the dARM amplitude during control process. The APE curves before and after control are compared in Figure.8. It can be seen that by cancelling the dARM contributed APE, the dARM amplitude is effectively reduced, and APE of coupled enclosure also achieves a satisfactory control effect, the APE reduction is as high as 14.7 dB. This further indicates that the ASAC strategy based on the dARM is effective.

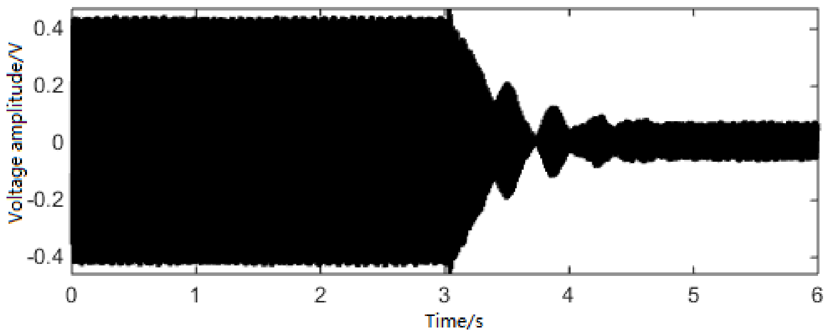

Figure 7 – Time history of dARM amplitude during control

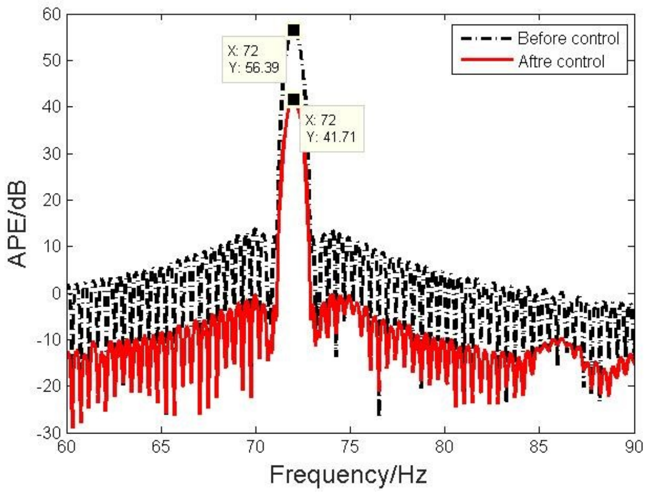

Figure 8 – The APE before and after control

6. CONCLUSIONS

(1) By analogy to ARM theory of free space, the acoustic potential energy is expressed in quadratic form of normal velocity on coupling surface and ARM of coupled enclosure is redefined, no structural modal information of the elastic structure is required in the calculation of ARM.

(2) Combining radiation efficiency and the ARM amplitude, a dARM based ASAC strategy is proposed, which can guarantee satisfactory control effect. The simulation and experimental results show that the control strategy is effective.

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