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Sound field reconstruction in a room from spatially distributed measurements

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Abstract

The full characterization of the sound field in a room from experimental data is a challenging problem, as the direct measurement of the sound field in the volume requires considerable experimental effort. In this study, the sound field in a room at low frequencies is characterized using a representation in wavenumber domain that infers the modal structure of the sound field. The modal structure is used to synthesize frequency response functions, to characterize the spatio-temporal properties of the wave field. The proposed method makes it possible to infer frequency response functions between source-receiver pairs that have not been measured, exploiting the representation of the sound field as a modal superposition, and thus reducing the dimensionality of the reconstruction problem

Keywords: Acoustic holography, room acoustics, array processing, sound field analysis

1 INTRODUCTION

In many applications, knowledge of the room impulse response functions throughout a room is valuable - e.g., source localization [1], dereverberation [2], room equalization [3, 4], sound field analysis [5, 6], etc. However, measuring experimentally the spatio-temporal properties of the sound field in an entire room, i.e. measuring impulse responses in the three-dimensional domain point by point, requires extensive experimental effort. The alternative of interpolating and extrapolating the sound field - i.e. estimating room impulse response functions at positions that have not been directly measured, is a powerful approach that has been explored in multiple studies [7, 8, 9, 10, 11, 12]. The rationale behind most approaches is to formulate a physical propagation model, often based on elementary wave expansions (in frequency or time domain), onto which a limited set of measurements is mapped. This constitutes an inverse acoustic problem [13], where the model is fitted to experimental data, making it possible to predict or reconstruct the sound field anywhere in the domain [14, 15, 16]. These inverse approaches typically do not require explicit knowledge of the geometry or boundary properties of the room, as they are essentially concerned with the propagation of sound waves in the medium. Yet, prior information can be incorporated into the problem, to promote meaningful solutions that lead to better predictions [17, 18, 19, 20].

Several studies have addressed the reconstruction of the sound field in a room from a limited set of measurements [7, 8, 9, 10, 11, 12], typically at mid and low frequencies or over small apertures - due to the sampling requirements. When considering the sound field in a room at low frequencies, the most distinct property of the sound field is its marked modal structure: i.e. the sound field results from the superposition of normal modes (eigenfunctions). When addressing sound field reconstruction in small or medium enclosures (low modal densities), this physical property can be exploited, i.e. [10, 21, 22, 23]. These studies propose to decompose the normal modes into a sparse set of plane-wave functions, a case which is well-suited to rectangular enclosures, but is less suited to rooms of arbitrary shape. In the latter case, it is necessary to assume that an over-complete set of basis functions can approximate the eigenfunctions sufficiently well.

In the present study we propose a method to reconstruct the sound field in a room, via incorporating the modal structure of the sound field into the problem. The approach serves to predict the spatio-temporal properties of the wave field at low frequencies over the volume of the room. As a result of introducing the normal modes into the model, the method presented here enables to predict/synthesize frequency response functions between any source-receiver pair in the room, including those that have not been measured.

2 THEORY

The solution to the inhomogeneous wave equation in an enclosure, i.e. the Green's function, has the well known solution [24]

$$G(\mathbf{r}|\mathbf{r}_0) = -\frac{1}{V}\sum_{n=0}^{\infty}\frac{\Psi_n(\mathbf{r}_0)\Psi_n(\mathbf{r})}{k^2 - k_n^2 - jk/c\tau_n}, \tag{1}$$

where V is the room's volume, Ψ_n are the normal modes (eigenfunctions) of the room, k is the wavenumber, k_n the modal frequency of the mode of index n, and τ_n is its time constant. The sound pressure in the room is $p(\mathbf{r}) = \mathrm{j}\omega\rho Q G(\mathbf{r}|\mathbf{r}_0)$, and Q is the volume velocity of the source. The normal modes can be expanded into an elementary wave basis

$$\Psi_n(\mathbf{r}) = \sum_{m}^{\infty} A_{nm}\varphi_{nm}(\mathbf{r}), \tag{2}$$

where $\varphi_{nm}(\mathbf{r})$ are the elementary waves and A_{nm} a set of unkown coefficients. In the present study, plane propagating waves are chosen, $\varphi_{nm}(\mathbf{r}) = \mathrm{e}^{-\mathrm{j}\mathbf{k}_{nm}^T\cdot\mathbf{r}}$, where $\mathbf{k}_{nm} = \left(k_{x,nm}, k_{y,nm}, k_{z,nm}\right)$ is the wavenumber vector of mode n discretized into a basis of M plane waves. It should be noted that no assumption is made on the geometry of the room or the shape of the normal modes. Assuming that all modes are excited (i.e. the source is not placed on a nodal plane), the sound field in the room can be written as

$$p(\mathbf{r}) = -\frac{\mathrm{j}\omega\rho Q}{V}\sum_{n}^{N}\left(k^2 - k_n^2 - jk/(c\tau_n)\right)^{-1}\sum_{m}^{M} A_{nm}\varphi_{nm}(\mathbf{r}) \tag{3}$$

where N is the number of normal modes considered, and M is the number of plane waves into which each mode is discretized. As the modal frequencies are not available, and the modal shapes are unknown, this problem leads to a three-dimensional discretization of the wavenumber space or k-space, as shown in Fig. 1. In the figure, two lattice sampling patterns are shown, one in concentric circles and another in a cubical lattice, where the wavenumber domain is sampled at equal intervals in the k_x, k_y and k_z directions.

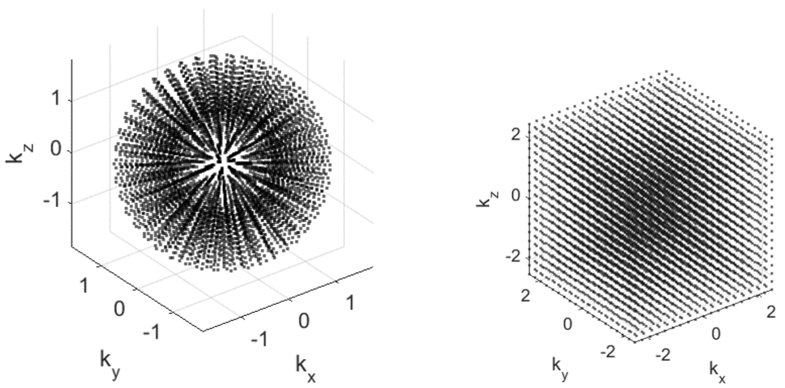

Figure 1. Sampling of wavenumber domain. Spherical lattice sampling (left). Cubical lattice sampling (right)

Problem (3) can be written in matrix form as $\mathbf{p} = \mathbf{\Phi}\mathbf{b}$ where $\mathbf{p} \in \mathbb{C}^M$, $\mathbf{b} \in \mathbb{C}^{N\cdot M}$. This problem is highly underdetermined, given the multi-dimensional basis used to model the sound field, and the inversion is not trivial. We

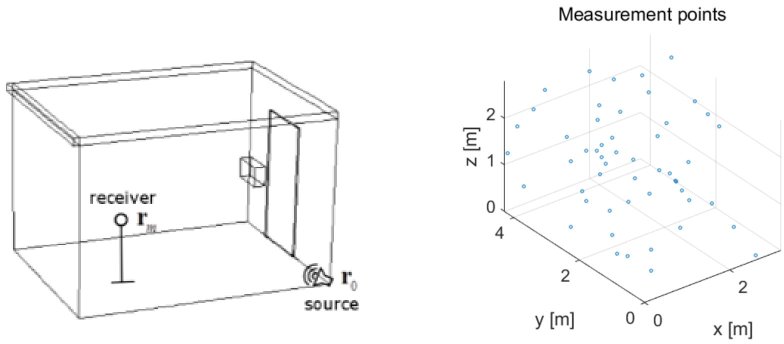

Figure 2. Finite element model of the room, with the source placed near the origin, at $(x, y, z) = (0.1, 0.1, 0.1)$ m (left). Distribution of measurement points in the volume of the room (right)

propose to make use of a synthesis prior in the solution [25, 26], which promotes solutions where the non-zero plane wave functions are close to the driving frequency,

$$\tilde{\mathbf{b}} = \arg\min_{\mathbf{b}} \|\mathbf{\Phi}\mathbf{b} - \mathbf{p}\|_2 + \lambda\|\mathbf{L}\mathbf{b}\|_1 \tag{4}$$

The matrix $\mathbf{L} = \text{diag}(\mathbf{w})$ is a diagonal matrix that contains the distance between the angular frequency of each plane wave and the modal frequency that is being excited, with elements $w_i = k^2 - \|k^2_{nm,i}\|$. The estimation can be further refined by normalizing the magnitude of the modes to match the measured sound pressure levels in a broadband sense, i.e. minimizing the functional $\arg\min_{a_n} \|p(\mathbf{r}_m, \omega) - \tilde{p}(\mathbf{r}_m, \omega, a_n)\|$ where a_n is a scaling factor associated to each mode, $\mathbf{r}_m$ are the measurement positions and $\tilde{p}$ the reconstructed pressure. Once the eigenfunctions are estimated from (4), it is possible to predict the sound field at any point in the volume, through a reconstruction matrix $\mathbf{\Phi}_r$ that maps the estimated coefficients $\tilde{\mathbf{b}}$ and the reconstruction points

$$\tilde{\mathbf{p}}_r = \mathbf{\Phi}_r \tilde{\mathbf{b}} \tag{5}$$

with $\tilde{\mathbf{p}}_r \in \mathbb{C}^R$ containing the predicted sound pressure at the reconstruction points.

A significant result derived from this model is the fact that based on the expansion of the normal modes into plane waves (retaining the modal structure of the sound field), it becomes possible to predict the sound field in the room at different source positions than the measured one. From Eqs. (2)-(5), the sound pressure field in the room for any source at position $\mathbf{r}_x$ can be synthesized as

$$p(\mathbf{r}_r) = -\frac{\mathrm{j}\omega\rho Q}{V} \sum_{n=0}^{N} \frac{\Psi_n(\mathbf{r}_x)\Psi_n(\mathbf{r}_r)}{k^2 - k_n^2 - jk/c\tau_n}.$$

It is apparent that the proposed model enables to predict the sound pressure in the room for any combination of source-receiver pairs, including those that have not been measured.

3 RESULTS

A numerical study is presented to test the proposed methodology. A Finite Element Method (FEM) simulation of an approximately rectangular room of dimensions 3.3×3×4.5 m^3 and reverberation time of $T_{60} = 2.2$ seconds is performed in COMSOL (Fig. 2 left). The sound pressure estimated from the FEM study is used as reference "true" data to assess the accuracy of the reconstruction method. The simulated measurement consists of 64 measurement points pseudo-randomly distributed throughout the volume of the room, with a minimum spacing

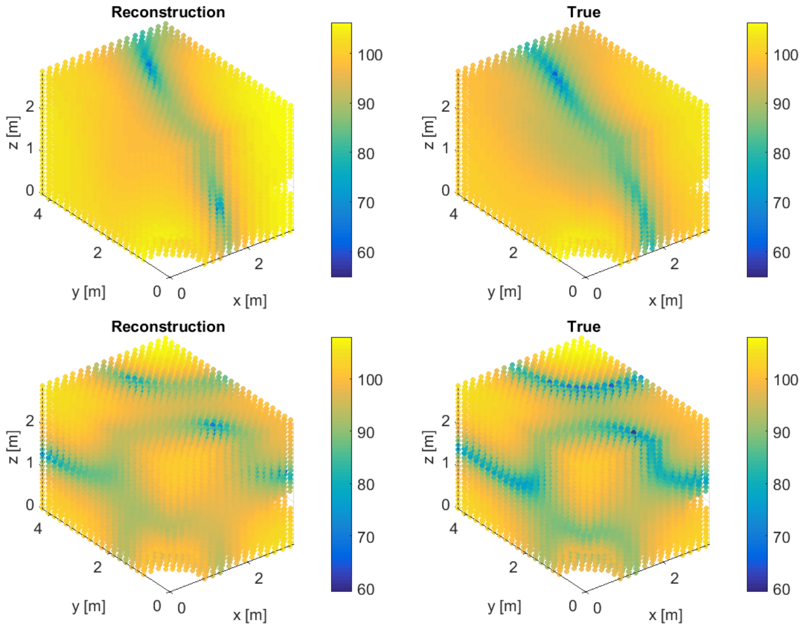

Figure 3. Reconstruction of the sound field in the room at 44 Hz (top row) and 78 Hz (bottom row). The reconstructed field is shown in the left column, and the true reference sound field in the right column [dB SPL].

of 10 cm, and an average spacing of 75 cm (see Fig. 2 (right)). A wave basis is constructed consisting of 29000 plane waves, chosen to sample the wavenumber spectrum in a cubical lattice of dk=0.18 rad/m, and spanning the first 25 modes in the room. The reconstruction of the sound field is analyzed in the frequency range between 30 Hz and 150 Hz, and the reconstruction domain is the entire volume of the room, discretized in 43500 points.

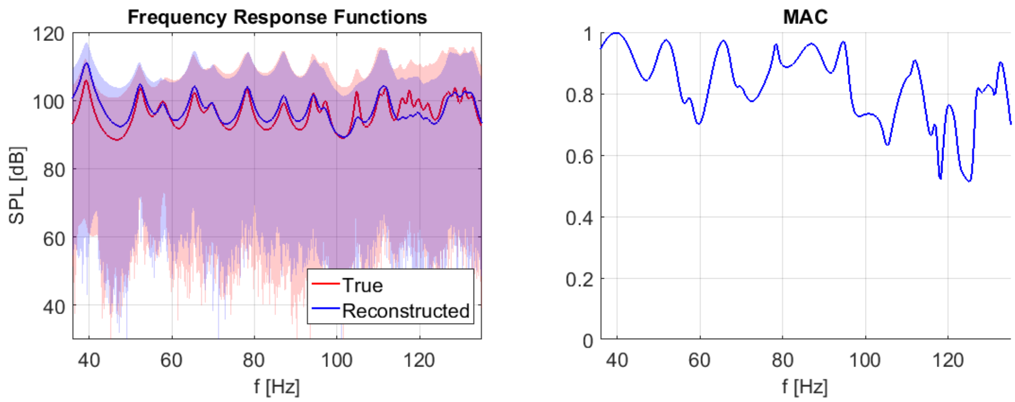

Figure 4. Frequency response functions (averaged and min/max values in shaded area [dB SPL]) and MAC for the frequency range between 30 and 135 Hz, when the room is driven with the source in the corner $(x_0, y_0, z_0) = (0.1, 0.1, 0.1)$ m.

Figure 2 shows the shape of the examined room, which is a model of one of the rooms at the DTU acoustics labs which is used for flanking transmission measurements. The figure also shows the measurement points distributed throughout the volume.

Figure 3 shows the reconstructed and true sound pressure levels inside the volume of the room at 44 Hz (top), which is close to an axial mode, as well as the response at 78 Hz (bottom) which is between two modal frequencies. The reconstructed results and the true reference sound field are shown for comparison. At 44 Hz the pressure response is dominated by mode (1,0,0). In the case at 78 Hz, the resulting sound pressure is due to the superposition of several modes - as expected due to the higher modal overlap, including the oblique (1,1,1) mode. It is apparent from the figure that the estimation is fairly accurate in both cases, and the sound field in the room is reconstructed correctly.

Figure 4 (right) shows the reconstructed frequency response functions in the room up to 135 Hz (averaged FRF as well as max/min values in the shaded area), and the modal assurance criterion (MAC), defined as

$$\mathrm{MAC} = \frac{||\mathbf{p}^H \hat{\mathbf{p}}||^2}{(\mathbf{p}^H \mathbf{p})(\hat{\mathbf{p}}^H \hat{\mathbf{p}})}, \tag{6}$$

which is used to quantify the spatial similarity of the modes (MAC= 1 indicates maximal similarity, and MAC= 0 indicates maximal dissimilarity). The FRF's and the MAC results show that the quantitative accuracy of the method is good, particularly close to modal frequencies. This is in agreement with the fact that the estimation problem (4) is solved at these frequencies.

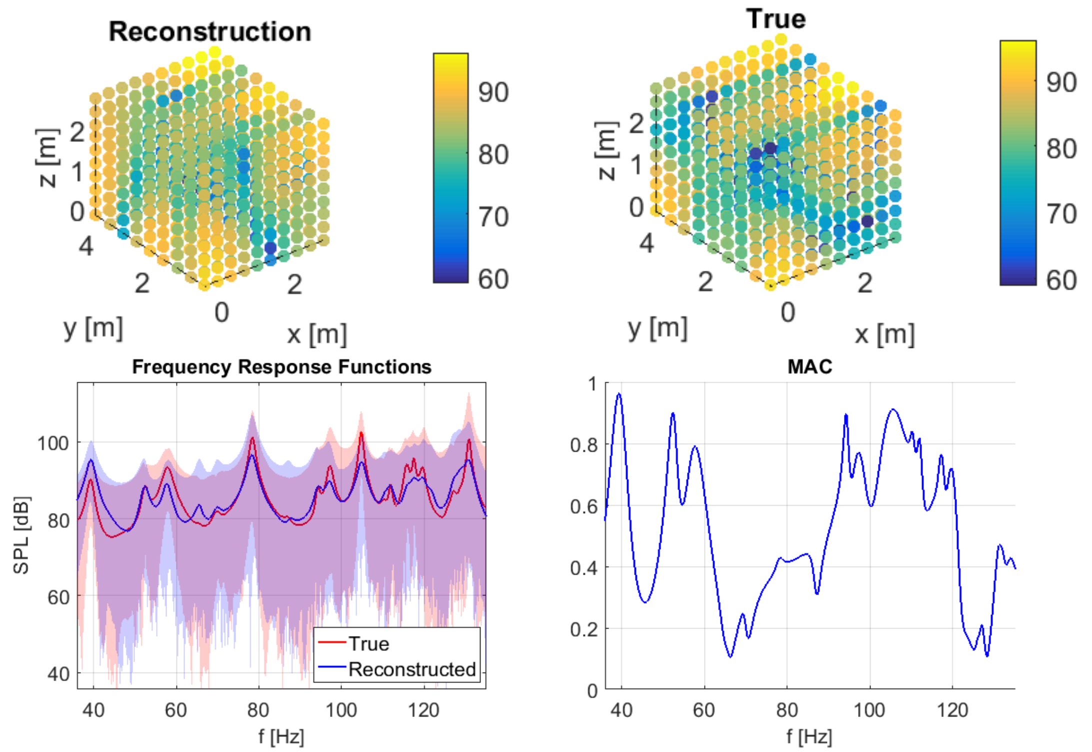

Figure 5. Synthesized frequency response functions when the source is placed at position $(x, y, z) = (1.5, 2, 1)$ m, not too far from the center of the room. Reconstructed sound pressure levels [dB] at 87 Hz, over 869 points in the room. Frequency response functions (averaged and min/max values in shaded area) and MAC for the frequency range between 30 and 135 Hz.

It is perhaps of greater interest to consider the reconstruction of the sound field for source-receiver combinations that have not been measured, i.e. if the presented model is used to synthesize or predict the frequency response function for an arbitrary source position $\mathbf{r}_x$. Figure 5 (top) shows the predicted/reconstructed sound field if a source was placed at position $(x_x, y_x, z_x) = (1.5, 2, 1)$ m, not too far from the center of the room. In this case the reconstruction is done over 869 points (the lower number of points in this test was chosen for computational efficiency). The figure shows the reconstruction at 87 Hz, along with the reference true pressure, which was estimated from a new FEM simulation with the source placed at $(1.5, 2, 1)$ m. It is clear that the method predicts well the actual sound field in the room. Figure 5 (bottom) shows the frequency response functions (averaged as well as max-min values) as well as the MAC, for the reconstruction of the sound field in the case that the source is placed in a new position. It is noticeable that several of the modes with nodal planes near the new source position contribute less to the frequency response than in the previous case (source in the corner). On average, the reconstructed responses and true reference response are fairly similar throughout the room and the MAC is higher at frequencies where a single mode dominates, or at higher modal densities, as the distance between modes is smaller. The accuracy is somewhat lower than the case that the source is in the corner (Figs. 3 and 4), which is to be expected since the frequency responses are based on predicting the resulting sound field if the source was placed in a different position. Nonetheless, the dominant spatial properties of the sound field seem to be recovered correctly.

4 CONCLUSIONS

A method is presented to reconstruct the sound field in a room at low frequencies. The sound field is modeled using a wavenumber domain representation that infers the modal structure of the sound field. The method is tested numerically at low frequencies, showing that the sound field in the volume of the room can be accurately reconstructed. Due to incorporating the normal modes into the problem, it is possible to predict frequency response functions between any source-receiver pair in the room, including those that have not been measured. This result has interesting potential, as it leads to a model of the room that is independent of source position, reducing the dimensionality of the reconstruction problem.

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